A commonly held notion is that students learning English will do well in a mathematics classroom because mathematics is not a subject dependent on language proficiency. However, a student who is an English language learner (ELL) may struggle to succeed in the mathematics classroom where only English is taught. In 2004, there were approximately 5 million children in grades K–12 who were considered ELLs (NCELA 2004). Students from various language backgrounds may be learning English at the same time that they are learning new concepts in mathematics taught through English. Further, students are also learning the academic language of mathematics. The NCTM states: “All students should have the opportunity and the support necessary to learn significant mathematics with depth and understanding” (NCTM 2000, p. 5). There are strategies for teaching mathematics to students who are ELLs. However, in addition to using these strategies, mathematics teachers also need to provide explicit language instruction for those students learning English (Rothenberg and Fisher 2007). By differentiating instruction in mathematics for English language learners, teachers can plan and provide access to mathematics curriculum for all students, with the added goal of language instruction for students learning English.

**LANGUAGE PROFICIENCY**

It is important for teachers to understand their students’ level of language proficiency to be able to plan instruction to support English language learning. Students who may seem fluent in spoken English may not be proficient in the academic language of English. Cummins’s theories of academic language and cognitively demanding communication defined Basic Interpersonal Communication Skills (BICS) and Cognitive Academic Language Proficiency (CALP) as two levels of language proficiency (Cummins 1979). BICS is language learned in highly contextualized situations. It is the ability to communicate with
friends, order lunch, or talk with the teacher. CALP is academic language. It is the language used in the classroom during discussions or lectures. Students who exhibit conversational fluency (BICS) may not have developed fluency in academic language (CALP) (Chamot and O’Malley 1994; Rothenberg and Fisher 2007). Since students use language to make sense of mathematics, those who are only conversationally fluent may have more difficulty understanding or accurately using the academic language of school mathematics (Rothenberg and Fisher 2007). When differentiating instruction in mathematics, special consideration must be given to the process of second language acquisition and the impact of learning on diverse cultural and linguistic backgrounds. Based on a review of decades of research and work helping ELL students succeed in the classroom, Rothenberg and Fisher (2007) have developed a set of four principles that support academic language acquisition:

1. Comprehensible input
2. Contextualized instruction
3. A low-anxiety learning environment
4. Meaningful engagement in learning activities

The teacher in the differentiated mathematics classroom needs to consider these principles when designing mathematics instruction for ELL students.

**COMPREHENSIBLE INPUT**
Comprehensible input was considered by Krashen (1988) to be the first step in developing language proficiency. For the mathematics classroom, it means that students must be able to understand the input of the lesson or what they are hearing and reading. To assist with verbal understanding, the teacher can intentionally speak at a slower rate, repeat ideas, and pause to check for understanding. Using gestures, real-life objects, graphic organizers, and other visual models together with oral explanation or discussion can also contribute to comprehensible input. This does not mean that teachers should simplify the mathematical goals for ELLs, only that they should monitor how well the students comprehend the information being presented. For instance, before students start a new task, the teacher can ask them to rephrase the task in their own words, and have several students share their interpretations. With this strategy, the teacher has the opportunity to ask clarifying questions.

**CONTEXTUALIZED INSTRUCTION**
Another principle for language acquisition is contextualized instruction. This means that students learn mathematics and associated academic language in a meaningful context that can be built on. Although many teachers think that defining new mathematical terms at the start of a lesson is essential, studies of language acquisition have found it is most effective to formally define new vocabulary after students have learned the associated concept and can connect the term with the known concept (Garrison and Mora 2003).
A Low- Anxiety Environment
Mathematics teachers are aware that many students have anxiety about mathematics. For the ELL, there is the additional anxiety of listening to, and having to communicate in, a new language. Providing a safe environment is essential in differentiating the mathematics classroom for the English language learner. One way to lessen students’ anxiety is to use tasks that have multiple entry points and provide scaffolds that support student participation. Cooperative learning is another strategy that reduces ELLs’ anxiety about using English, since they can ask a peer to clarify what was said or offer tentative English responses within a less threatening environment than that of whole-class communication.

Meaningful Engagement in Learning Activities
English language learners must have opportunities to listen, speak, read, and write about mathematics. The most effective way for this to occur is to engage students in the “authentic” use of language (Rothenberg and Fisher 2007). In a differentiated mathematics classroom, students should be given daily opportunities to discuss, defend, present, and ask questions about meaningful mathematics. For example, students may be given a problem-solving activity and asked to work as a group to make conjectures and present their findings to the class. Tasks should be interesting to adolescents and set in a real-world context to add further meaning to the mathematics.

Language acquisition principles, comprehensible input, contextualized instruction, a low-anxiety environment, and meaningful engagement can be used to help the teacher plan mathematics instruction for students who are learning mathematics while learning English. Table 1 summarizes language acquisition principles and strategies.

Investigations of Perimeter and Area
In a traditional mathematics classroom, a teacher might explore a lesson about perimeter and area by

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Table 1 Language acquisition principles and strategies

<table>
<thead>
<tr>
<th>Principle</th>
<th>Meaning</th>
<th>Strategies</th>
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| Promoting comprehensible input   | Students must be able to understand what they are hearing and what they are reading. | Speak slower  
Repeat ideas  
Pause  
Use gestures  
Avoid idioms and slang  
Scaffold (make clear connections)  
Introduce new vocabulary after the students have learned the concept |
| Contextualizing instruction      | English is learned in the context of mathematics.                       | Use Specially Designed Academic Instruction in English (SDAIE) strategies:  
Teach academic language  
Support realia (activities relate to real life)  
Use manipulatives  
Use visual tools  
Use graphic organizers  
Promote student interaction |
| Creating a low-anxiety learning environment | Students must have access to the mathematics content and feel comfortable participating in classroom activities and discourse. | Teacher establishes norms of behavior.  
Lessons are planned with multiple entry points.  
Students work in collaborative groups. |
| Engaging in meaningful learning activities | The lesson must provide students with authentic opportunities to listen, speak, read, and write about mathematics. | Students should make conjectures and present findings.  
Students should discuss, defend, and present their work.  
The teacher should design tasks that engage students.  
Tasks should be in a real-world context. |
writing the formulas on the board and demonstrating how to solve a few sample problems. Having been shown the formulas for perimeter and area, students are then expected to complete a few problems on their own to gain mastery. This traditional model of direct instruction provides little opportunity for meaningful engagement or the development of conceptual understanding; in addition, there is no support for students learning English.

In the reform mathematics classroom, perimeter and area are explored through problem-solving activities. The Curriculum and Evaluation Standards for School Mathematics (NCTM 1989) stated:

Problem solving should be the central focus of the mathematics curriculum.

As such, it is a primary goal of all mathematics instruction and an integral part of all mathematical activity. (p. 23)

The following investigations are based on the principles of language acquisition so as to give all middle school students opportunities to solve problems that lead to a deeper understanding of perimeter and area concepts and to develop meaningful understandings of familiar formulas for two-dimensional figures. Van de Walle noted that students should never use formulas without participating in the development of those formulas (2001).

For all the investigations, students are expected to work collaboratively. Group work supports English language learners by providing a low-anxiety environment and promotes opportunities for authentic conversation. Students can ask questions and receive support from their peers as well as their teacher, and they enjoy working with their peers. In addition, cooperative learning improves students’ attitudes about working in the mathematics classroom (Artzt and Newman 1997).

Investigation 1

Investigation 1 begins with students defining perimeter and area and then considering shapes with irregular sides (see fig. 1). Most middle school students are familiar with the terms perimeter and area, this review contributes to comprehensible input, which is a necessary scaffold for the discovery of formulas for other shapes.

**Fig. 1 Discussing area and perimeter**

**INVESTIGATION 1: AREA AND PERIMETER OF A LAKE**

1. Define the terms perimeter and area. Compare your answers with those of your team members.

2. Does your definition work for irregular objects? See the diagrams of the Little Lake and the Large Lake below. Modify or add to your answer if necessary.

3. Find an estimate of the perimeter and area of Little Lake and Large Lake. Based on the estimate of the perimeter and area of the lakes, are the names of the lakes appropriate? Explain your answers.
Investigation 1 highlights the teacher and student actions, placing emphasis on differentiating instruction for the student who is learning English.

Students are expected to struggle with the concepts of perimeter and area, discuss ideas, and explain solutions. One student commented, “I knew the math but did not know the words. Working in groups really helped me learn the words.” Another student said, “The picture helped me understand what the problem asked.” Students work together to find the estimated area of the lakes and decide if the names are appropriate. Each student is responsible for completing each investigation and writing a solution with an explanation. When students have completed the first investigation, groups should share answers with the class.

**Investigation 2**

Before beginning investigation 2, the teacher asks the students to define the term rectangle using examples and nonexamples (see fig. 2). Comprehensible input is achieved by letting students talk with peers and define a rectangle while looking at both examples and nonexamples. The teacher might notice that students are not including equal dimensions for the base and height of the rectangle and might ask questions that lead the students to understand that a square is also a rectangle. For instance, the teacher can ask, “Is there an example where the base of a rectangle is equal to the height? Do we have a name for that special rectangle?” Students are participating in the acquisition of language through contextualized instruction, namely, in the context of mathematics. The students decide on a group definition, share their group definitions with the class, and agree on a class definition for rectangle. The discussion, writing, and reading of the definition of rectangle within the group and the class provide meaningful engagement and an authentic opportunity for students to listen, speak, read, and write. The class definition is added to a word wall—a collection of vocabulary words, their definitions, and a picture posted on a wall in the classroom (Brassell and Flood 2004). Other examples and nonexamples can be found in *Discovering Geometry* (Serra 1993).

Investigation 2 asks students to compare the area and perimeter of many rectangles and write a generalization or formula (see fig. 3). Graph paper and tiles are given to students for these investigations. Using manipulatives helps students understand the mathematical concepts; manipulatives also support English language learning through contextualized instruction. For students who finish before others, the investigation extension challenges students to find the largest area, given a fixed perimeter. All students should be expected to complete the extension either for homework or at another time in class.

**SHARING THEIR FINDINGS**
At the end of each investigation, it is important for students to share their
INVESTIGATION 2: PERIMETER AND AREA OF A RECTANGLE
You are planning a vegetable garden. Each plot of land in your garden is in the shape of a rectangle. Find the perimeter of each plot of land. You need to know the perimeter because you want to place a fence around the garden. Also, find the area of each plot of land so that you can plan the number of plants you will need. Record the dimensions, perimeter, and area for each plot of land. Be sure to explain how you found the perimeter and area.

Plot 1
Dimensions: __________________
Perimeter: __________________
Area:  __________________
Explanation: 

Plot 2
Dimensions: __________________
Perimeter: __________________
Area:  __________________
Explanation: 

Plot 3
Dimensions: __________________
Perimeter: __________________
Area:  __________________
Explanation: 

Plot 4
Dimensions: __________________
Perimeter: __________________
Area:  __________________
Explanation: 

Plot 5
Dimensions: __________________
Perimeter: __________________
Area:  __________________
Explanation: 

INVESTIGATION 2: EXTENSION
You have been given 64 meters of fencing. You want to design a garden with the largest area. The garden must be in the shape of a rectangle. Draw possible gardens, and list the dimensions. What are the dimensions of the garden with the largest area you can fence? Defend your answer.
findings. Throughout all the investigations, the teacher must reflect on his or her instruction and ask whether it has been differentiated enough to provide access for all students and whether students who are learning English are given language instruction. For middle school students, the investigations are followed by similar activities that challenge them to find the area of parallelograms, triangles, and trapezoids, and the formulas for the circumference and area of a circle. Students should be expected to complete the Summary of Investigations chart (see fig. 4).

RESOURCES
Investigations exploring the perimeter and area of parallelograms, triangles, trapezoids, and circles would begin with students writing definitions for the shapes when given examples and nonexamples. The investigations for the parallelogram, triangle, trapezoid, and circle can be found in *Elementary and Middle School Mathematics* (Van de Walle 2001), in *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence* (NCTM 2006), or in *Discovering Geometry* (Serra 1993). The teacher assists students with going from the concrete to the abstract as they write formulas for the various shapes. As students complete the investigations, the teacher has an opportunity for formative assessment. If students are struggling with concepts, the teacher asks questions to assist the students’ understanding. Group or whole-class discussions are necessary to provide clarification of some tasks. Again, it is important for the class to discuss each investigation and have students demonstrate or explain each solution. Reading what students write about the investigations also gives insight into student understanding and guides the development of future lessons.

CONCLUSION
One goal of teaching mathematics should be to engage all students in meaningful problem-solving activities that lead to conceptual understanding of perimeter and area. Constructing knowledge leads to greater understanding. Developing formulas for perimeter and area provides students with the opportunity to construct knowledge about geometry.

“Excellence in mathematics education requires equity—high expectations and strong support for all students” (NCTM 2000, p. 11). Students who are English language learners are learning the language of mathematics while learning English. Mathematics teachers must keep in mind that they are not only teaching mathematics but are also assisting in language acquisition. The principles of language acquisition provide a framework to support differentiating the mathematics classroom. Such a classroom is designed to provide access to every student and assure high standards and support while students are learning English and mathematics.
BIBLIOGRAPHY