

Mathematics Activities from Diverse Cultures

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LEAH P. MCCOY
WAKE FOREST UNIVERSITY
mccoy@wfu.edu

<http://www.wfu.edu/~mccoy/mgames.pdf>

To make mathematics relevant and meaningful for all students, it is important that we embrace a wide variety of real-world applications. Diverse cultures provide rich and interesting contexts in which students can experience and explore mathematics. One of the five essential process standards is connections, and the following activities present probability activities in cultural contexts to help students connect to the mathematics through games of chance from diverse cultures.

Game 1. HUBBUB. Native American.

Background. Wa'lade hama'gan (Bowl and Dice) is one of many similar games played by early Native American tribes. The Penobscot Nation in New England called it Hubbub because of the chanting that accompanied the tossing of the dice. The dice were traditionally made from carved bone or antler, animal teeth, or small stones. They were engraved, burned, polished, or painted to distinguish the sides.

Description. This is a simple gambling game, where five or six dice were tossed in a bowl or basket. The game then involved getting as many as possible to land on the same side.

Materials. Each pair will need:

- 1 bowl
- 6 two-sided dice (lima beans painted on one side)
- 56 Narrow Sticks (toothpicks)
- 4 Flat Sticks (Popsicle sticks)

Rules.

- Place all sticks in a pile (4 Flats (popsicle sticks) and 56 Narrows (toothpicks)).
- Player one shakes the beans within the bowl and places the bowl back down
- If you toss:
 - 6-alike: Take one Flat stick and roll again.
 - A second 6-alike: Take 2 Flat sticks and roll again.
 - A third consecutive 6-alike: Take 3 Flat Sticks and pass turn.
- If you toss:
 - 5-alike: Take 3 Narrow sticks and roll again.
 - A second 5-alike: Take 6 Narrow sticks and roll again.
 - A third consecutive 5-alike: Take one Flat stick and pass turn.
- If you toss less than five alike, take no sticks and pass turn to your opponent.
- Continue in this fashion until the last stick is taken. The person who takes the last stick gets to take 2 Narrow sticks from their opponent's pile.
- The player with the most points wins! Add points as follows.
 - 1 point for every narrow stick
 - 3 points for every flat stick

Connections. Simple probability and compound probability.

1. $P(\text{all red})$
2. $P(\text{all red})$ and $P(\text{all red again})$
3. Find the probability of rolling all 6 red (or white), three times in a row. What would your score be?
4. Assuming you have beaten the odds and rolled 6 the same each time, how many times do you have to do this to win the game? Explain.

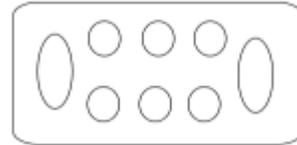
Game 2. MANCALA. African.

Background. Mancala (also called Wari) is a very common game that is played by hundreds of tribes throughout Africa. It is also popular in the Caribbean, where it was brought by slaves from Africa. There are many different variations and special names for this game. Mancala was played before 1400 BC in Ancient Egypt, and many historians believe that it is the oldest game in the world.

Description. The word Mancala means "to transfer" in Arabic, and the game is played by transferring small items from one bin to another. The playing pieces are usually small stones or seeds. In its simplest version, the "board" is a patch of ground outside, and the "pits" are depressions dug into the earth. Mancala "boards" were usually carved of stone, wood or ivory, and sometimes were elaborate and artistic.

Materials. Each pair will need:

- 1 egg carton (dozen or half dozen size)
- 2 bowls to place at each end of the carton
- Several beans or other counters



Rules.

- The player chooses one pit from which to "sow" the seeds.
- Each seed (bean) in the pit is then placed, one at a time, into the successive pits, moving counter-clockwise around the board. Seeds placed in a *Kalaha*, the pit at the end of board, are points for that player. Seeds are not sown in the opponent's *Kalaha*.
- If the last seed in a play is placed in the player's own *Kalaha*, they get another turn. If the last seed is placed in an *empty* pit on their *own* side of the board, then they Capture the seeds in the opposite (opponent's) pit.
- All captured seeds, as well as the capturing piece, are placed in the player's *Kalaha*.
- The game ends when all of the pits on one side of the board are empty.
- The player with seeds remaining gets to put them into their *Kalaha*. The winner is the player with the most seeds.

Connections. Experimental vs. theoretical probability.

1. In your pair, what is the probability that if player 1 starts, then player 1 will win?
2. In your pair, what is the probability that if player 1 starts, then player 1 will lose?
3. In our class, what is the probability that if player 1 starts, then player 1 will win?
4. In our class, what is the probability that if player 1 starts, then player 1 will lose?
5. Write a paragraph describing whether starting first or starting second is related to whether you win the game.

Game 3. FIVE-UP DOMINOES. Chinese.

Background. Dominoes are small tiles traditionally carved from ivory or bone with small, round pips of inset ebony. They are commonly called kwat p'ái or bone tablets. Dominoes were popular in China as early as 1120 A.D. Sometime in the early 18th century, dominoes became known in Europe, first appearing in Italy.

Description. In this game, points are scored when the sum of the pips at the ends of the structure on the board is a multiple of 5.

Materials. Each game will need a set of dominoes (double-six or double-nine) and a paper and pencil to keep score. Domino sets are available at most dollar stores.

Rules.

- There are 2-4 players per game. The first player may lead any of their bones and plays it face up in the center of the table.
- The second player must play a bone which matches either end of the bone led by the first player. Subsequently, bones are added to either end of the resulting chain.
- The first double is a special case. This is laid crosswise on the chain. Also, up to four bones may be laid against this double, known as the spinner. If at any point a player cannot add a bone to the end of one of the chains then they must draw bones from the boneyard until they draw a bone which can be played. All bones drawn which cannot be played immediately are added to their hand.
- The goal of the game is to score points by forming chains whose ends total up to some multiple of 5. The player who forms such a pattern scores that total.
- The first player to play all of their bones scores points for the bones remaining in the other players' hands.
- A game is normally 150 or 200 points. If more than one player or team passes this total in the same hand, then the team or player with the highest point total wins.

Connections. combinations, ${}_n C_r$.

1. Imagine picking out two dominoes completely at random from a set of “double 6” dominoes.
What is the probability that these two dominoes in the double-six set “match” (the end of one matches at least one of the ends of the other)?

Hint: You will need to use the following formula.

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}$$

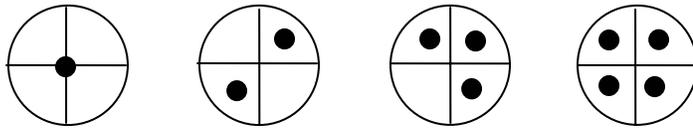
*Consider the cases involving “doubles” (a 6-6, 3-3, etc.) separately.
Notice, changing the order of a match **does not** create a new match.*

Game 4. LU-LU. Hawaiian.

Background. Early Hawaiians played lu-lu with disks of volcanic stone about 2.5 cm in diameter. These disks served as stone dice called u-lu. The word lu-lu means “to shake.”

Description. Four playing pieces, marked on one side only are tossed. The dots on the pieces falling face-up are counted.

Materials. The four playing pieces are marked as follows. Pieces can be made from clay, shell, or wood. Buttons can also be used.



Rules. A player shakes the playing pieces in both hands and tosses them. A turn consists of two tosses:

- If all four playing pieces fall face up, the player scores ten and then tosses all pieces for a second time.
- If all four playing pieces do not fall face up, the player counts and scores the dots of those face up and then tosses only the face-down pieces a second time. The dots that show on the second toss are counted with those of the first toss.
- The winner can be the player with the highest score in a single turn, or play can continue to an agreed-on score, such as 100

Connections. Listing of sample space and calculation of probabilities.

1. What are all the possible outcomes in one toss?
2. What are the possible outcomes of a toss of only part of the pieces (those landing face down the first time)?
3. What is the maximum number of points possible?
4. What other scores are possible?
5. How many different ways can each score be obtained?
6. What is the probability of each score?

Game 5. KAKAANAQ. Yup'ik Eskimo.

Background. The Yup'ik Eskimo people of western Alaska used this game to develop skills necessary for the hunt. It helped young men develop as hunters and learn important skills such as how to judge distances.

Description. A mat, traditionally an animal skin, was placed on the ground. It had a small dot in the center. Each of two players had five circular disks, made from carved wood, and they took turns attempting to hit the target.

Materials. Each pair will need:

- 1 mat (24 inches x 24 inches), felt fabric or posterboard, with a 1.5 inch circle painted at the center
- 5 (3-inch) wooden discs for each player, colored or decorated to distinguish sets. These can be purchased at a craft store.

Rules.

- The object of the game is to land the disks on the target. Players should stand three feet from the edge of the mat to play their disks.
- If a disk covers the whole target, the player scores 3 points. If it covers half the target, the player scores 2 points. If it only touches the target, the player scores 1 point.
- Players score points for themselves or their opponents and can change the score by sliding opponents' disks off or on the target.
- Players alternate tossing until each has played all five disks.

Connections. Experimental Probability.

1. In the course of one game, how many times is the opponent's disk moved?
2. How many of those moves result in more points for the opponent?
3. How many result in less points for the opponent?
4. What is the probability that if the opponent's disk is moved, that it will benefit the opponent?
5. Is it a good strategy to try to move the opponent's disk?

Game 6. MOCCASIN GAME. Native American.

Background. The Moccasin Game and its many variations was very popular in many Native American tribes. It was used to sharpen wit and thinking skills and test a person's ability to fool their opponent. The hand games usually involved gambling and were played to "hand game music" involving drums. More specific and complex "rules" existed and varied among tribes.

Description. Usually four or more moccasins were used. One player hid a small object under one moccasin. The other player or players made wagers as to where the object was.

Rules.

- The objective of the game is for one player to guess which moccasin the token is under.
- One player hides a small token under a moccasin while the second player looks away.
- The second player then attempts to guess which moccasin the token is under. He indicates his guess by touching the moccasin with a stick.
- Both players may put a wager on the results of the game or games.

Materials.

- moccasin
- token small enough to hide in your hand (ex. small rock)
- stick or pointer

Connections. Experimental probability.

1. After playing the game several times, are the placements of the tokens equally likely for all moccasins?
2. Given your findings for question one, which moccasin would you guess?

Game 7. DREIDEL. Jewish.

Background. One of the best known symbols of Chanukah is the *Dreidel*. A dreidel is a four sided top with a Hebrew letter on each side. They stand for "Nes Gadol Hayah Sham", which means "A great miracle happened there." In Israel the dreidel has *pey* instead of *shin*, for "Nes Gadol Hayah Po," which means "A great miracle happened here." The miracle is that the small flask of oil burned the menorah lamp for eight days. Dreidel is a popular game played during the Holiday.

Description. The dreidel is a four-sided top, with the following on the four faces.



(N) *or nun* stands for nicht or nothing. If the dreidel lands on nun, you do nothing.

(G) *or gimmel* stands for ganz or all. If the dreidel lands on gimmel, take everything in the middle.

(H) *or hay* stands for halb or half. If the dreidel lands on hay, take half of what's in the middle plus one if there's an odd number of objects.

(SH) *or shin* stands for shtel or put in. If the dreidel lands on shin, put two objects into the middle.

Materials. Dreidels can be purchased at holiday stores and toy stores. Inexpensive plastic or wooden ones are available. Students might also be interested in seeing quite expensive silver or crystal dreidels. Players use pennies, nuts, raisins, or chocolate coins (gelt) as tokens or chips.

Rules. Each player places an agreed-on number of tokens in the center of the table. Each player spins the dreidel in turn. When the dreidel stops, the letter that is facing up decides the turn. The game is over either when one player has won all of the tokens or after a designated number of rounds.

Connections. Theoretical probability and experimental probability.

1. If the dreidel is fair, what outcomes would you expect if you spin it 100 times?
2. Spin it 100 times and record the number of times you got each side?
3. Are these the same? Why or why not?

Game 8. TOMA TODO. Mexican.

Background. Toma todo or pirinola is a game of chance from Mexico. Children as well as adults play Toma todo with a six-sided spinner (pirinola) or top. Adults often play the game using money.

Description. Toma todo is a game of chance in which each of the six possible outcomes on the pirinola is equally likely to occur. Each player seeks to acquire all the beans.



Materials.

- Several beans or wrapped hard candies
- 1 six-sided pirinola (available in Mexican gift shops)

Rules.

- To begin the game, each player places one bean in a center pile. Turns are taken. A turn consists of spinning the pirinola and following the directions on the side that is facing up after the spin.
- The following are directions to be written on the pirinola:
 - Pon 1. Put 1.
 - Pon 2. Put 2.
 - Toma Todo. Take all.
 - Toma 1. Take 1.
 - Toma 2. Take 2.
 - Todos ponen. Each player puts 1 in the pile.
- If toma todo turns up, then in order to continue the game, each player must place one bean in a center pile again. Play continues until one player has all the beans or candies.

Connections. Theoretical probability and experimental probability.

- It appears that the pirinola is fair and each of the six sides is equally likely. What is the probability of each side (outcome) if this is true?
- Keep a tally of the number of times each side occurs. Calculate the probability of each outcome using this data.
- Are the probabilities the same? Explain the results.

Game 10. ASHBII. (Another Stick Game). Navajo.

Background. Ashbii (ash been) is a game played by Navajo women and children. The game was observed in the town of Chin Lee Arizona on the reservation there. The Navajo women sometimes played this game while sitting under a buffalo hide which had been staked up for drying.

Description. The game is played with three painted sticks and a basket. The sticks act as dice and are tossed.

Materials. Each game will require three sticks.

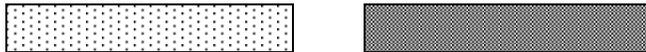
- The first is called the “tsi’i (zeen) head.” It is painted completely black on one side and half black on the other.



- The second stick is called “nezhi’ (nezshi), and is painted half red on one side and half black on the other side.



- The third stick is called “tqelli” (zelli) and is painted all red on one side and all black on the other.



- Large basket or paper plate to catch sticks.

Rules.

All players sit on the floor. The sticks are tossed upward, ideally against a blanket stretched overhead, and the sticks that land in the basket are scored. Only the highest scoring combination that occurs on any one toss will count. Players alternate throwing the sticks. The winner is that player who first scores 25 points.

Scoring:

- 1 point for all black or half black.
- 2 points for all red or half red.
- 3 points for crossing (all black and all red, all black and half red, all red and half black, half red and half black).
- 5 points for crossing all red and half red.

Connections. Theoretical probability.

1. Is this a fair way to score the game? How many different ways can the sticks fall. Make a table of all of the ways the sticks can fall. Don't forget those which do not earn points.
2. How many different outcomes are there? What is the probability of each outcome?

Game 11. NIM. Chinese.

Background. NIM is an Ancient game that originated in China.

Description. NIM is a simple game with few rules. Beginning with 11 beans, two players alternate turns of taking away 1 or 2 beans. The player to remove the last object loses.

Materials.

A pile of beans or other similar objects.

Rules.

- The two players alternate turns.
- There is a pile of 11 beans between the two players.
- Each player must remove either 1 or 2 beans in turn. A player may not skip a turn.
- The player to remove the last object loses.

Connections. Simple probability.

1. What strategies has your team discovered for winning?
2. Is it better to be the first or second player?
3. What is the probability of winning if you are the first player and you know the rules?

Game 12. Craps.

Background. Craps is the oldest confirmed game of chance in which dice were used. The game dates back to 1573 BC when it was played by the Egyptians. The modern game of craps is an American game that was developed on the Mississippi and other inland waterways during the great steamboat era of the 1800s.

Description. The basic idea behind Craps is to roll a pair of dice to establish a "point" number and roll that number again before rolling a 7 (craps). Only the numbers 4, 5, 6, 8, 9, or 10 can be a point number and all other rolls on the dice have special meanings depending on when they are rolled (before or after a point is established).

Materials.

A set of two dice that are each six-sided.

Rules. The idea behind craps is to roll a "point" (4, 5, 6, 8, 9, 10) on the come-out roll and then after that point is established, the dice thrower (shooter) wants to roll that same point number before he/she rolls a 7. If a 7 is thrown, then the point is lost and the process starts over. If the point is thrown, then the process also starts over again. It's that simple and the game can be played by making only those bets that play "with the dice".

You win on a 7 or 11, and you lose on "craps" 2, 3 or 12. Any other number becomes your "come-point" and you win if your "come-point" is made before a 7 is rolled.

Connections. Simple probability.

1. If the dice are fair, what is the probability of getting each number? Make a tree diagram to list all of the possibilities and then determine how many different ways you could get each one.
2. Explain the difference between probability and odds.
3. What are the odds you will win with the first roll (7 or 11)?
4. What are the odds you will lose with the first roll (2, 3, or 12)?
5. What are the odds of rolling each of the other points?