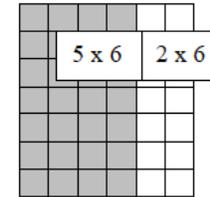


Connections to Previous Learning: Grade 2 students have worked primarily with addition and subtraction situations. They have begun extending the modeling of quantities to equal groups and arrays as a basis for multiplication. Students will model this operation using rectangles partitioned into equivalent squares.

Focus of the Unit: In this unit, Grade 3 students develop an understanding of the meanings of multiplication and division of whole numbers through activities and problems involving equal-sized groups, arrays, and area models and extends students' work with the distributive property. For example, in the picture the area of a 7 x 6 figure can be determined by finding the area of a 5 x 6 and 2 x 6 and adding the two sums. Students will continue their discovery of this concept and apply it to the composition and decomposition of shapes.



This unit will further address various problem-solving structures that students are expected to use while solving word problems involving multiplication and division. Students should use a variety of representations for creating and solving one-step word problems. They explain and apply properties of operations as strategies for finding their solutions to problems. In addition, students will solve two-step problems involving all four of the operations. Students will determine the unknown in a multiplication and division equation and understand division as an unknown-factor problem. Patterns within multiplication and division will be identified and students will explain them in more depth. This application unit will focus upon the multiplication and division situations of *Equal Groups* and *Arrays* (See Table 3 in K-5 Operations and Algebraic Thinking Progression Document.)

Fluency is also a focus of this unit. By studying patterns and relationships in multiplication facts and relating the operations of multiplication and division, students will build a foundation for fluency with multiplication and division facts. Students will demonstrate fluency with multiplication facts through 10 and the related division facts. Multiplying and dividing *fluently* refers to the skill of performing these operations accurately (using a reasonable amount of steps and time), flexibility (using strategies such as the distributive property), and efficiently.

NOTE: By the end of Grade 3, students will know from memory all products of two one-digit numbers.

Connections to Subsequent Learning: Grade 4 students will multiply and divide with multi-digit numbers and within multi-step problem situations including at this grade level multiplicative comparison. They will also extend multiplication and division concepts into factors and multiples. Patterns that flow from these operations will be generated and analyzed. Finally, Grade 4 students will multiply a fraction by a whole number.

Progression Citation: From the K-5 Operations and Algebraic progression document pp. 19, 23, 24, 26, 27.

	$A \times B = \square$	$A \times \square = C$ and $C \div A = \square$	$\square \times B = C$ and $C \div B = \square$
Equal Groups of Objects	<p>Unknown Product</p> <p>There are A bags with B plums in each bag. How many plums are there in all?</p>	<p>Group Size Unknown</p> <p>If C plums are shared equally into A bags, then how many plums will be in each bag?</p>	<p>Number of Groups Unknown</p> <p>If C plums are to be packed B to a bag, then how many bags are needed?</p>
Arrays of Objects	<p>Unknown Product</p> <p>There are A rows of apples with B apples in each row. How many apples are there?</p>	<p>Equal groups language</p> <p>Unknown Factor</p> <p>If C apples are arranged into A equal rows, how many apples will be in each row?</p>	<p>Unknown Factor</p> <p>If C apples are arranged into equal rows of B apples, how many rows will there be?</p>
	<p>Unknown Product</p> <p>The apples in the grocery window are in A rows and B columns. How many apples are there?</p>	<p>Row and column language</p> <p>Unknown Factor</p> <p>If C apples are arranged into an array with A rows, how many columns of apples are there?</p>	<p>Unknown Factor</p> <p>If C apples are arranged into an array with B columns, how many rows are there?</p>
Compare	<p>Larger Unknown</p> <p>A blue hat costs $\\$B$. A red hat costs A times as much as the blue hat. How much does the red hat cost?</p>	<p>$A > 1$</p> <p>Smaller Unknown</p> <p>A red hat costs $\\$C$ and that is A times as much as a blue hat costs. How much does a blue hat cost?</p>	<p>Multiplier Unknown</p> <p>A red hat costs $\\$C$ and a blue hat costs $\\$B$. How many times as much does the red hat cost as the blue hat?</p>
	<p>Smaller Unknown</p> <p>A blue hat costs $\\$B$. A red hat costs A as much as the blue hat. How much does the red hat cost?</p>	<p>$A < 1$</p> <p>Larger Unknown</p> <p>A red hat costs $\\$C$ and that is A of the cost of a blue hat. How much does a blue hat cost?</p>	<p>Multiplier Unknown</p> <p>A red hat costs $\\$C$ and a blue hat costs $\\$B$. What fraction of the cost of the blue hat is the cost of the red hat?</p>

In Equal Groups, the roles of the factors differ. One factor is the number of objects in a group (like any quantity in addition and subtraction situations), and the other is a multiplier that indicates the number of groups. So, for example, 4 groups of 3 objects is arranged differently than 3 groups of 4 objects. Thus there are two kinds of division situations depending on which factor is the unknown (the number of objects in each group or the number of groups). In the Array situations, the roles of the factors do not differ. One factor tells the number of rows in the array, and the other factor tells the number of columns in the situation. But rows and columns depend on the orientation of the array. If an array is rotated 900, the rows become columns and the columns become rows.

This is useful for seeing the commutative property for multiplication 3.OA.5 in rectangular arrays and areas. This property can be seen to extend to Equal Group situations when Equal Group situations are related to arrays by arranging each group in a row and putting the groups under each other to form an array. Array situations can be seen as Equal Group situations if each row or column is considered as a group. Relating Equal Group situations to Arrays, and indicating rows or

columns within arrays, can help students see that a corner object in an array (or a corner square in an area model) is not double counted: at a given time, it is counted as part of a row or as a part of a column but not both. As noted in Table 3, row and column language can be difficult. The Array problems

given in the table are of the simplest form in which a row is a group and Equal Groups language is used (“with 6 apples in each row”). Such problems are a good transition between the Equal Groups and array situations and can support the generalization of the commutative property discussed above. Problems in terms of “rows” and “columns,” e.g., “The apples in the grocery window are in 3 rows and 6 columns,” are difficult because of the distinction between the number of things *in a* row and the number *of* rows. There are 3 rows but the number of columns (6) tells how many are in each row. There are 6 columns but the number of rows (3) tells how many are in each column. Students do need to be able to use and understand these words, but this understanding can grow over time while students also learn and use the language in the other multiplication and division situations. Grade 3 standards focus on area measurement.^{3.MD.5–7} Area problems where regions are partitioned by unit squares are foundational for Grade 3 standards because area is used as a model for single-digit multiplication and division strategies.^{3.MD.} Using a letter for the unknown quantity, the order of operations, and two-step word problems with all four operations Students in Grade 3 begin the step to formal algebraic language by using a letter for the unknown quantity in expressions or equations for one and two-step problems.3.OA.8 But the symbols of arithmetic, \times or \cdot for multiplication and \div or $\{$ for division, continue to be used in Grades 3, 4, and 5. Understanding and using the associative and distributive properties (as discussed above) requires students to know two conventions for reading an expression that has more than one operation:

1. Do the operation inside the parentheses before an operation outside the parentheses (the parentheses can be thought of as hands curved around the symbols and grouping them).

2. If a multiplication or division is written next to an addition or subtraction, imagine parentheses around the multiplication or division (it is done before these operations). At Grades 3 through 5, parentheses can usually be used for such cases so that fluency with this rule can wait until Grade 6. These conventions are often called the Order of Operations and can seem to be a central aspect of algebra. But actually they are just simple “rules of the road” that allow expressions involving more than one operation to be interpreted unambiguously and thus are connected with the mathematical practice of communicating precisely.MP6 Use of parentheses is important in

displaying structure and thus is connected with the mathematical practice of making use of structure.MP7 Parentheses are important in expressing the associative and especially the distributive properties. These properties are at the heart of Grades 3 to 5, the Grade 3 two-step word problems vary greatly in difficulty and ease of representation. More difficult problems may require two steps of representation and solution rather than one. Use of two-step problems involving easy or middle difficulty adding and subtracting within 1,000 or one such adding or subtracting with one step of multiplication or division can help to maintain fluency with addition and subtraction while giving the needed time to the major Grade 3 multiplication and division standards.

Table 1: Multiplication and division situations for measurement

	Unknown Product $A \times B = \square$	Group Size Unknown $A \times \square = C$ and $C \div A = \square$	Number of Groups Unknown $\square \times B = C$ and $C \div B = \square$
Grouped Objects (Units of Units)	You need A lengths of string, each B inches long. How much string will you need altogether?	You have C inches of string, which you will cut into A equal pieces. How long will each piece of string be?	You have C inches of string, which you will cut into pieces that are B inches long. How many pieces of string will you have?
Arrays of Objects (Spatial Structuring)	What is the area of a A cm by B cm rectangle?	A rectangle has area C square centimeters. If one side is A cm long, how long is a side next to it?	A rectangle has area C square centimeters. If one side is B cm long, how long is a side next to it?
Compare	A rubber band is B cm long. How long will the rubber band be when it is stretched to be A times as long?	A rubber band is stretched to be C cm long and that is A times as long as it was at first. How long was the rubber band at first?	A rubber band was B cm long at first. Now it is stretched to be C cm long. How many times as long is the rubber band now as it was at first?

Students in Grade 3 learn to solve a variety of problems involving measurement and such attributes as length and area, liquid volume, mass and time. ^{3.MD.1, 3.MD.2} Such work involve units of mass such as the kilogram.

A few words on volume are relevant. Compared to the work in area, volume introduces more complexity, not only in adding a third dimension and thus presenting a significant challenge to students' spatial structuring, but also in the materials whose volumes are measured. These materials may be solid or fluid, so their volumes are generally measured with one of two methods, e.g., "packing" a right rectangular prism with cubic units or "filling" a shape such as a right circular cylinder. Liquid measurement, for many third graders, may be limited to a one-dimensional unit structure (i.e., simple iterative counting of height that is not processed as three-dimensional). Thus, third graders can learn to measure with liquid volume and to solve problems requiring the use of the four arithmetic operations, when liquid volumes are given in the same units throughout each problem. Because liquid measurement can be represented with one-dimensional scales, problems may be presented with drawings or diagrams, such as measurements on a beaker with a measurement scale in milliliters.

Desired Outcomes

Standard(s):

Represent and solve problems involving multiplication and division

3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

3.MD.7 Relate area to the operations of multiplication and addition.

Solve problems involving the four operations, and identify and explain patterns in arithmetic.

3.OA.8 Solve two-step word problems using the four operations (+, -, x, ÷.) Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.

3.MD.2 Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.

Multiply and divide within 100.

3.OA.7 Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$) or properties of operations.

Use place value understanding and properties of operations to perform multi-digit arithmetic.

3.NBT.3 Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9×80 , 5×60) using strategies based on place value and properties of operations.

Transfer: *Students will apply...*

- The use of equal-sized groups, arrays, and area models to multiplication and division situations.
- Understanding of area models of multiplication.
- Problem-solving situations to multiply and divide to solve real-world problem situations.

Arrays of objects Unknown factor problem situation example: Twenty stickers have been arranged on a sheet into 5 rows. How many columns will there be?
 $5 \times ? = 20$

Understandings: *Students will understand that ...*

- Area is additive.
- Modeling multiplication and division problems based upon their problem-solving structure can help in finding solutions.
- There is a relationship between area and multiplication.
- Properties of Operations will assist in problem-solving situations.
- Metric measurement units are related to place value concepts/multiples of 10.

Essential Questions:

- How can modeling multiplication and divisions problems help in finding their solutions?
- What is the relationship between area and multiplication?
- What are the Properties of Operations?
- How does metric measurement connect to multiples of 10?

Highlighted Mathematical Practices: (Practices to be explicitly emphasized are indicated with an *.)

- * **1. Make sense of problems and persevere in solving them.** Students demonstrate their ability to persevere and utilize problem-solving structures to solve multiplication and division problems.
- 2. Reason abstractly and quantitatively.** Students will reason about the problem-solving structure and employ it to justify and explain their solution. Students will make the connection between quantity and area models of multiplication and division.
- * **3. Construct viable arguments and critique the reasoning of others.** Students may construct arguments using concrete models, such as objects, pictures, and drawings. They refine their mathematical communication skills as they participate in mathematical discussions that the teacher facilitates by asking questions such as “How did you get that?” and “Why is that true?” They explain their thinking to others and respond to others’ thinking.
- * **4. Model with mathematics.** In this unit, students experiment with representing multiplication and division problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, acting out, making a chart, list, or graph, creating equations, etc. Students will generate various mathematical representations to both equations and story problems, and explain the connections between representations.
- 5. Use appropriate tools strategically.** Students will use concrete models to represent multiplication and division situations.
- 6. Attend to precision.** Students represent and use clear and precise mathematical language in their discussions with others and in their own reasoning about multiplication and division problem solving.
- 7. Look for and make use of structure.** Students will recognize and utilize properties of operations to evaluate real-world problem-solving situations involving multiplication and division.
- 8. Look for express regularity in repeated reasoning.** Students will observe commonalities within multiplication and division, such as using the distributive property.

Prerequisite Skills/Concepts:*Students should already be able to...*

- Model with equal groups by partitioning rectangles.
- Solve basic problem-solving structures.
- Relate metric measurement to concepts and multiples of 10.

Advanced Skills/Concepts:*Some students may be ready to...*

- Identify and work with factors and multiples.
- Multiply and divide multi-digit whole numbers.
- Solve multi-step problems.

Knowledge: *Students will know...*

- Multiplication and division facts.
- Problem-solving structures for area/arrays and for equal groups.
- Metric measurements units for liquid volume and weight.

Skills: *Students will be able to ...*

- Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities. (3.OA.3)
- Use drawings and equations with a symbol for the unknown number to represent the problem. (3.OA.3)
- Relate area to the operations of multiplication and addition. (3.MD.7)
- Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a + b$ and $a + c$. (3.MD.7)
- Use area models to represent the distributive property in mathematical reasoning. (3.MD.7)

- Solve two-step word problems using the four operations. (3.OA.8)
- Represent these problems using equations with a letter standing for the unknown quantity. (3.OA.8)
- Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (3.OA.8)
- Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). (3.MD.2)
- Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem. (3.MD.2)
- Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$) or properties of operations. (3.OA.7)
- Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9×80 , 5×60) using strategies based on place value and properties of operations. (3.NBT.3)

WIDA Standard:

English language learners communicate information, ideas and concepts necessary for academic success in the content area of Mathematics.

English language learners will benefit from:

- Concrete models of multiplication and division processes.
- Anchor Charts highlighting mathematical vocabulary specific to unit.
- Repeated practice verbalizing solution pathways.

Academic Vocabulary:

Critical Terms:

multiplication
 division
 array
 area
 equal groups
 equal shares
 multiple
 product
 factor
 divisor
 dividend
 quotient
 remainder
 fact family
 unknown
 strategies
 reasonableness
 mental computation
 operation
 estimation
 patterns
 gram
 kilogram
 liter

Supplemental Terms:

inverse operation
 distributive property
 commutative property
 zero property
 identity
 equation
 milliliter

Assessment

Pre-Assessments	Formative Assessments	Summative Assessments	Self-Assessments

Sample Lesson Sequence

--