

Approximate Time Frame: 6 - 8 weeks

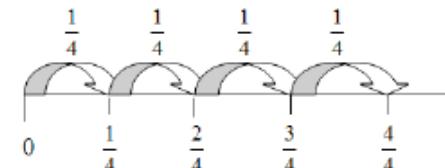
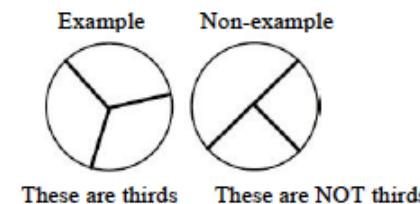
Connections to Previous Learning: Grade 2 students have partitioned circles and rectangles into two, three, or four equal shares. They have used fractional language such as *halves, thirds, half of, a third of*, etc., and described the whole as *two halves, three thirds, four fourths*. Students have begun to recognize that equal shares of identical wholes need not have the same shape, the basis of equivalency.

Focus of the Unit: In this unit, Grade 3 students begin their work on fractions in a more formal mathematical sense. This unit involves the sharing of a whole being partitioned, however models in Grade 3 include only area models and linear models (number lines).* Beginning with unit fractions students model the whole as the sum of fractional parts, and build an understanding that the size of the part is relative to the size of the whole. Students will also use fractions to represent numbers equal to, less than, and greater than one. Comparison of fractions is done using visual models (circles, rectangles, squares) and strategies based on observations of equal numerators or denominators.

*NOTE: Set models are not addressed in Grade 3.

Understanding that fractional parts must be equal-sized is an important concept for students to develop. (See diagram.) This development of *congruence* should be modeled with a variety of area models, such as tiles or pattern blocks, for students to gain this idea of *same size, same shape*. Students will need this understanding to solve word problems requiring them to create and reason about fair share.

Linear model of fractional parts begins for the first time in Grade 3 as students position fractions on number lines *between* whole numbers. Positioning fractions on the number line can model location, measurement of distance, and also serves as a precursor to addition. Furthermore, number lines will allow for modeling of equivalencies of fractions.



Finally, comparing fractions requires students in Grade 3 to reason about their size. Both area models and linear models can be used for this purpose.

Connections to Subsequent Learning: Grade 4 students will compare fractions with different numerators and different denominators, compose and decompose fractions and mixed numbers, and add and subtract fraction with like denominators. They will use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money. Finally Grade 4 students will understand decimal notation for fractions and compare decimal fractions.

From the 3-5 Number and Operations – Fractions progression document, pp. 2, 3, and 4

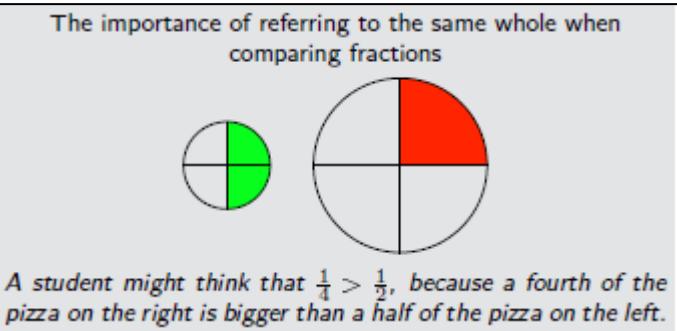
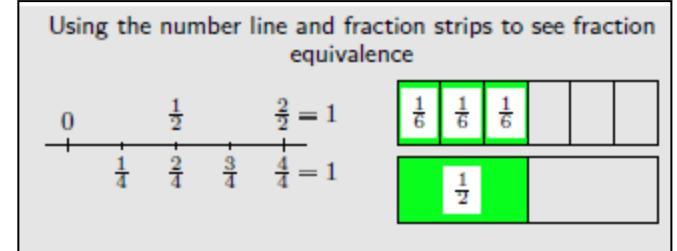
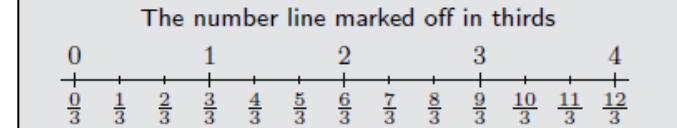
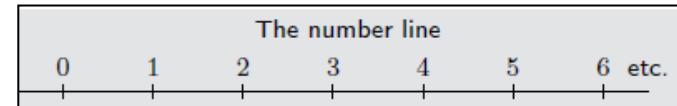
Grade 3 students start with unit fractions (fractions with numerator 1), which are formed by partitioning a whole into equal parts and taking one part, e.g., if a whole is partitioned into 4 equal parts then each part is $\frac{1}{4}$ of the whole, and 4 copies of that part make the whole. Next, students build fractions from unit fractions, seeing the numerator 3 of $\frac{3}{4}$ as saying that $\frac{3}{4}$ is the quantity you get by putting 3 of the $\frac{1}{4}$'s together.^{3.NF.1} They read any fraction this way, and in particular there is no need to introduce "proper fractions" and "improper fractions" initially; $\frac{5}{3}$ is the quantity you get by combining 5 parts together when the whole is divided into 3 equal parts.

The number line and number line diagrams On the number line, the whole is the *unit interval*, that is, the interval from 0 to 1, measured by length. Iterating this whole to the right marks off the whole numbers, so that the intervals between consecutive whole numbers, from 0 to 1, 1 to 2, 2 to 3, etc., are all of the same length, as shown. Students might think of the number line as an infinite ruler.

To construct a unit fraction on a number line diagram, e.g. $\frac{1}{3}$, students partition the unit interval into 3 intervals of equal length and recognize that each has length $\frac{1}{3}$. They locate the number $\frac{1}{3}$ on the number line by marking off this length from 0, and locate other fractions with denominator 3 by marking off the number of lengths indicated by the numerator.^{3.NF.2}

Students sometimes have difficulty perceiving the unit on a number line diagram. When locating a fraction on a number line diagram, they might use as the unit the entire portion of the number line that is shown on the diagram, for example indicating the number 3 when asked to show $\frac{3}{4}$ on a number line diagram marked from 0 to 4. Although number line diagrams are important representations for students as they develop an understanding of a fraction as a number, in the early stages of the NF Progression they use other representations such as area models, tape diagrams, and strips of paper. These, like number line diagrams, can be subdivided, representing an important aspect of fractions.

The number line reinforces the analogy between fractions and whole numbers. Just as 5 is the point on the number line reached by marking off 5 times the length of the unit interval from 0, so $\frac{5}{3}$ is the point obtained in the same way using a different interval as the basic unit of length, namely the interval from 0 to $\frac{1}{3}$.



Comparing fractions Previously, in Grade 2, students compared lengths using a standard measurement unit. 2.MD.3 In Grade 3 they build on this idea to compare fractions with the same denominator.

They see that for fractions that have the same denominator, the underlying unit fractions are the same size, so the fraction with the greater numerator is greater because it is made of more unit fractions. For example, segment from 0 to $\frac{3}{4}$ is shorter than the segment from 0 to $\frac{5}{4}$ because it measures 3 units of $\frac{1}{4}$ as opposed to 5 units of $\frac{1}{4}$. Therefore $\frac{3}{4} < \frac{5}{4}$.^{3.NF.3d} Students also see that for unit fractions, the one with the larger denominator is smaller, by reasoning, for example, that in order for more (identical)

pieces to make the same whole, the pieces must be smaller. From this they reason that for fractions that have the same numerator, the fraction with the smaller denominator is greater. For example $\frac{2}{5} > \frac{2}{7}$, because $\frac{1}{7} < \frac{1}{5}$, so 2 lengths of $\frac{1}{7}$ is less than 2 lengths of $\frac{1}{5}$.

As with equivalence of fractions, it is important in comparing fractions to make sure that each fraction refers to the same whole. As students move towards thinking of fractions as points on the number line, they develop an understanding of order in terms of position. Given two fractions—thus two points on the number line—the one to the left is said to be smaller and the one to right is said to be larger. This understanding of order as position will become important in Grade 6 when students start working with negative numbers.

Desired Outcomes

Standard(s):

Develop understanding of fractions as numbers

3.NF.1 Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size $1/b$.

3.NF.2 Understand fractions as number on number line. Represent fractions on number line diagram.

a. Represent a fraction $1/b$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $1/b$ and that the endpoint of the part based at 0 locates the number $1/b$ on the number line.

b. Represent a fraction a/b on a number line diagram by marking off a lengths $1/b$ from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line.

3.NF.3 Explain equivalence of fractions in special cases and compare fractions by reasoning about their size.

a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.

b. Recognize and generate simple equivalent fractions, e.g., $\frac{1}{2} = \frac{2}{4}$, $\frac{4}{6} = \frac{2}{3}$. Explain why the fractions are equivalent, e.g., by using a visual fraction model.

c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. Examples: Express 3 in the form $3 = \frac{3}{1}$; recognize that $\frac{6}{1} = 6$; locate $\frac{4}{4}$ and 1 at the same point of a number line diagram.

d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparison with the symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model.

Represent and interpret data.

3.MD.4 Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters.

Reason with shapes and their attributes.

3.G.2 Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole.

Transfer: Students will apply...

- Problem-solving skills to understand fractions as they relate to real-world problem situations, such as in measurement, cooking, pizza, money, music, etc.

Understandings: Students will understand that ...

- The size of the fractional part is relative to the size of the whole.
- Fractions represent quantities where a whole is divided into equal-sized parts using models, manipulatives, words, and/or number lines.
- Fractions can be used as a tool to understand and model quantities and relationships.
- Fractions are composed of unit fractions.
- Fractions that represent equal-sized quantities are equivalent.

Essential Questions:

- What do fractions represent?
- What makes fractions equivalent?

Highlighted Mathematical Practices: (Practices to be explicitly emphasized are indicated with an *.)

- 1. Make sense of problems and persevere in solving them.** Students demonstrate their ability to persevere and utilize reasoning to make sense of part-part-whole relationships.
- * **2. Reason abstractly and quantitatively.** Students will reason about the size of fractions. Students will make the connection between area models and linear models of fractions.
- 3. Construct viable arguments and critique the reasoning of others.** Students may construct arguments using concrete models of fractions to reason about the whole as they examine the fractional parts. They explain their thinking to others and respond to others' thinking.
- * **4. Model with mathematics.** In this unit, students representing fractions and wholes in multiple ways including numbers, words (mathematical language), drawing pictures, objects, etc. Both area models of fractions and linear models will be used.
- * **5. Use appropriate tools strategically.** Students will use concrete models to represent part-part-whole relationships.
- 6. Attend to precision.** Students represent and use clear and precise mathematical language in their descriptions of fractions as specifying the whole.
- * **7. Look for and make use of structure.** Students will recognize and utilize the structure of the part-part-whole relationships between various fractional pieces.
- 8. Look for express regularity in repeated reasoning.** Students will observe commonalities within the various models for fractional pieces and what they represent.

Prerequisite Skills/Concepts:*Students should already be able to...*

- Divide shapes (circles and rectangles) into no more than 4 equal sections and use vocabulary terminology to describe.
- Measure length and represent that data in a line plot.

Advanced Skills/Concepts:*Some students may be ready to...*

- Identify and work with more fifths, tenths, twelfths and/or fractions with unlike denominators.

Knowledge: Students will know...**Skills: Students will be able to ...**

- Divide shapes into parts with equal areas. (3.G.2)
- Represent the area of each part as a unit fraction. (3.G.2)
- Represent a whole using unit fractions. (3.NF.1)
- Use the term numerator to indicate the number of parts and denominator to represents the total number of parts a whole is partitioned into. (3.NF.1)
- Represent a fraction as the composition of unit fractions. (3.NF.1)
- Divide a number line diagram into equal segments and label the appropriate fractional parts. (3.NF.2)
- Model equivalent fractions using manipulatives, pictures, or number line diagrams and explain in words why the fractions are equivalent. (3.NF.3)
- Represent whole numbers as fractions using area models, number line diagrams, and numbers. (3.NF.3)
- Compare two fractions with the same numerator or same denominator using

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| | <ul style="list-style-type: none">visual models, symbols and words. (3.NF.3)Recognize that comparisons are valid only when the two fractions refer to identical wholes. (3.NF.3)Generate measurement data by measuring lengths to the $\frac{1}{4}$ and $\frac{1}{2}$ inch. (3.MD.4)Show data in a line plot given a scale in $\frac{1}{2}$, $\frac{1}{4}$, or whole numbers. (3.MD.4) |
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WIDA Standard:

English language learners communicate information, ideas and concepts necessary for academic success in the content area of Mathematics.

English language learners will benefit from:

- Concrete models of multiplication and division processes.
- Anchor Charts and visuals highlighting mathematical vocabulary specific to fractions.

Academic Vocabulary:

Critical Terms:

partition
equal parts
fraction
equal distance (intervals)
equivalent
equivalence
reasonable
denominator
numerator
justify
unit fraction
sixth
eighth

Supplemental Terms:

line plot
fraction
half
third
fourth
part – part - whole
comparison
linear measurement (using a unit fraction to show distance)

Assessment**Pre-Assessments****Formative Assessments****Summative Assessments****Self-Assessments****Sample Lesson Sequence**