

CHAPTER 2

Transformations of Parent Graphs

In the first section of Chapter 2, you will learn how to change the equation of a parabola to make it fit a set of nonlinear data. After you learn how to stretch, compress, reflect, and shift the graph of $f(x) = x^2$, you will be able to create a variety of parabolic shapes and sizes.

You will learn that a graph's transformations are clearly recognizable when its equation is written in graphing form. Understanding this form will help you learn how to rewrite equations so that they are easier to graph. You will also use the quadratic family of functions to model physical situations, such as the arc of a jumping rabbit and the path of a soccer ball.

In Section 2.2, you will apply these same types of transformations to other parent functions.

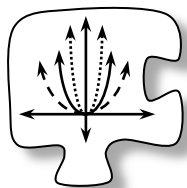
Guiding Question

Mathematically proficient students model with mathematics.

As you work through this chapter, ask yourself:

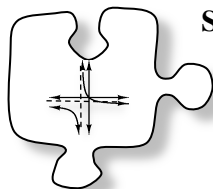
How can I model this everyday situation with mathematics?

Chapter Outline



Section 2.1

In this section, you will learn how to shift, stretch, compress, and flip the graph of $f(x) = x^2$. You will write a general equation for the family of quadratic functions. Then you will learn how to graph a quadratic function quickly when it is written in graphing form. You will model physical situations with quadratic functions.



Section 2.2

You will apply the concepts of transformation to other parent functions, and you will learn that transforming each parent function creates a whole family of functions. You will write a general equation for a family of functions. You will learn how the equation predicts the geometric transformations made to the graph of a function.

Chapter 2 Teacher Guide

Section	Lesson	Days	Lesson Title	Materials	Homework
Appendix A (Optional)		12	Sequences	Before teaching Chapter 2, it is highly recommended that you determine whether your students first need the lessons in Appendices A and B.	
Appendix B (Optional)		11	Exponential Functions		
2.1	2.1.1	1	Modeling Non-Linear Data	<ul style="list-style-type: none"> • Lesson 2.1.1 Res. Pg. • Heavy cardstock • Scales • Scissors and rulers • Compasses (opt.) • Poster-making materials 	2-4 to 2-10
	2.1.2	2	Parabola Investigation	<ul style="list-style-type: none"> • Lesson 2.1.2 Res. Pg. (opt.) • Colored pencils 	2-16 to 2-22 and 2-23 to 2-29
	2.1.3	1	Graphing a Parabola Without a Table	<ul style="list-style-type: none"> • Dynamic Tool • 2.1.3 Res. Pg. (opt.) 	2-35 to 2-41
	2.1.4	1-2	Rewriting in Graphing Form	<ul style="list-style-type: none"> • Lesson 2.1.4 Res. Pg. • Algebra tiles (opt.) • Tile storage (opt.) • Display tiles (opt.) 	2-50 to 2-56 and 2-57 to 2-63
	2.1.5	1	Mathematical Modeling with Parabolas	None	2-69 to 2-75
2.2	2.2.1	3	Transforming Other Parent Graphs	<ul style="list-style-type: none"> • Poster-making materials 	2-81 to 2-87 and 2-88 to 2-94 and 2-95 to 2-101
	2.2.2	2	Describing (h, k) for Each Family of Functions	<ul style="list-style-type: none"> • Lesson 2.2.2 Res. Pg. 	2-107 to 2-113 and 2-114 to 2-120
	2.2.3	1	Transformations of Functions	None	2-125 to 2-131
	2.2.4	2	Transforming Non-Functions	None	2-139 to 2-145 and 2-146 to 2-152
	2.2.5	1	Transforming Piecewise-Defined Functions	<ul style="list-style-type: none"> • Lesson 2.2.5A-B Res Pgs. • Poster-making materials, optional 	2-162 to 2-169
Chapter Closure		Various Options			

Total: 15-16 days plus optional time for Appendices, Closure, and Assessment

Overview of the Chapter

Before teaching Chapter 2, it is highly recommended that you determine whether your students first need the additional lessons in Appendices A and B. See the “Ideas for Individual Test” in the Suggested Assessment section at the beginning of Chapter 1 for guidance in making this determination.

An important theme of this chapter is generalizing. Students learn how to generate families of functions from parent functions starting with the parent function $f(x) = x^2$ in Section 2.1. Motivated by trying to write an equation to model specific nonlinear data, students begin their work to transform graphs by focusing on vertically stretching (or compressing) the graph of $y = x^2$. Next, horizontal and vertical translations are added, and students develop a general equation $f(x) = a(x - h)^2 + k$ for the family of quadratic functions. Students learn to graph a parabola quickly by identifying its orientation, vertical stretch (or compression), and vertex. This motivates the need to convert the equation of a quadratic function from the standard form $f(x) = ax^2 + bx + c$ into the graphing form $f(x) = a(x - h)^2 + k$ by completing the square. At the end of the section, students use quadratic functions to model physical situations.

In Section 2.2, students continue to generalize about functions by applying the same kinds of transformations they learned about in the first section to several other parent functions, including $f(x) = \frac{1}{x}$, $f(x) = \sqrt{x}$, $f(x) = b^x$, $f(x) = |x|$, and $f(x) = x^3$. Students then spend two days creating a Parent Graph Toolkit and describing the role of the point (h, k) for each family of functions. Then they investigate geometric transformations using the terms translation, reflection, and dilation. In the last lesson of the section, students learn about piecewise functions, and they consolidate their understanding of transformations by transforming piecewise functions.

Common Core State Standards for Mathematical Practice

Mathematically proficient students model with mathematics. As students work through this chapter a guiding question they should be asking themselves is: How can I model this everyday situation with mathematics?

Where Is This Going?

The idea of families of functions will be revisited several times in this course. Each time students are introduced to a new parent function (inverse and logarithmic functions in Chapter 5, trigonometric functions in Chapter 7), they will be asked to graph members of its family and write a general equation for the family. The members of a family of functions (as the idea is used here) are all related to a parent function and to each other by a sequence of transformations.

Suggested Assessment Plan for Chapter 2

In the *Core Connections* courses, a broad range of skills and abilities is valued. It is therefore crucially important that a broad range of assessment strategies are used: Individual Tests, Team Tests, Participation Quizzes, student presentations, class observations, and portfolios.

For complete discussion and recommendations about assessment strategies and grading, refer to the Assessment section of this Teacher Edition.

Suggestion for Participation Quiz

Lessons 2.2.1 (Day 1) and/or 2.2.2 (Day 1)

Suggestion for Students Presentations

Lessons 2.1.1, 2.2.1, and/or 2.2.5

Ideas for Individual Test

As you choose topics and problems to put on individual tests, consider that mastery takes time. Ask yourself, “What do I expect students to have mastered *at this point*? Which topics are so new that it might be best to save them for the next test?” There is clear potential to set your students up for failure (and demoralize them) by giving them the perception that you expect mastery of an idea that they are still coming to understand. Pushing for mastery too soon may also lead students to memorize procedures (often incorrectly) without conceptual understanding.

It is strongly recommended that **more than half** of each test be made up of material from previous chapters. Along with any previous material, it is now appropriate to test students’ ability to:

- Write equations for shifting parabolas, as in problems 2-7, 2-50, 2-83, 2-117, 2-146, and CL 2-170.
- Graph sketches of parabolas given in standard or graphing form, as in problems 2-23, 2-24, 2-25, 2-70, 2-82, 2-93, 2-147, and CL 2-177. (Includes finding the vertex of a parabola from its x -intercepts.)
- Parabola applications as in problems 2-69, 2-81, 2-22, and CL 2-173.
- Modify the equation of a function to transform its graph, as in problems 2-25, 2-52, 2-95, 2-96, 2-109, 2-118, 2-143, and CL 2-174.
- Write a quadratic in graphing form (complete the square), as in problems 2-36, 2-37, 2-50, 2-88, 2-119, 2-128, and 2-166.
- Find the x -intercepts of parabolas with equations given in graphing form as in problems 2-73, 2-118, 2-128, 2-151, 2-166, and CL 2-177.
- Write equations and graph sketches of other parent graphs, as in problems 2-52, 2-107, 2-108, 2-109, 2-126, 2-164, CL 2-171, and CL 2-172.
- Show mastery of Checkpoint 2A: Finding the Distance Between Two Points and the Equation of the Line as in problems 2-10, 2-21, 2-26, and 2-53.
- Show mastery of Checkpoint 2B: Solving Linear Systems in Two Variables as in problems 2-20, 2-60, 2-87, 2-96, 2-114, 2-117, and 2-129.

You may also consider testing these concepts from the previous chapter:

- From Chapter 1, describe the graph of a function by using shape, intercepts, symmetry, special points, asymptotes, domain, and range as in problems 2-118, 2-128, and 2-131.

Students need time to practice before they can master new topics. Consider *waiting* to assess the following new concepts until a future chapter:

- Use geometric terms (translation, reflection, dilation), and notation such as $f(x+k)$, $k \cdot f(x)$, etc., when describing a transformation, as in problems 2-109, 2-121, and 2-122.
- Distinguish between odd and even functions as in problems 2-125, 2-126, 2-142, and 2-163.
- Write the equations for transformed circles as in Lesson 2.2.4.
- Write equations for a piecewise function, as in Lesson 2.2.5.
- Transform a piecewise function, as in Lesson 2.2.5.

If students did not complete Appendices A and B during the current course, consider waiting to assess all sequence and exponential function topics until students have had another chapter of homework problems for review. Otherwise, if your student did complete the Appendices, then some of these topics could be assessed at the end of this chapter:

- From Appendix B, distinguish exponential growth and decay by whether the multiplier is greater than one or less than one, as in problems 2-29, 2-39, 2-63, and 2-85.
- From Appendix B, simplifying fractional exponents, as in problems 2-40, 2-86, and 2-111.
- From Appendix B, algebraically solve a system of exponential equations as in problem 2-145.
- From Appendix B, curve-fitting an exponential equation to two points, as in problems 2-72 and 2-110.
- From Appendix B, write the equation for a sequence in $t(n) = m(n-1) + b$ or $t(n) = ab^{n-1}$ (first term) form as in problems 2-8, 2-100, 2-120, 2-130, and CL 2-176.
- From Appendix A, distinguish between the domain of a sequence and its corresponding function, as in problems 2-8 (d), and 2-89 (d).

Lesson 2.1.1 How can an equation help me predict?

Modeling Non-Linear Data

Lesson Objective: Students will collect non-linear data, fit an equation to their data, and use their equation to make predictions.

Mathematical Practices: reason abstractly and quantitatively, model with mathematics, use appropriate tools strategically, look for and make use of structure

Length of Activity: One day (approximately 50 minutes)

Core Problems: Problems 2-1 through 2-3

Materials: Lesson 2.1.1 Resource Page (Shrinking Targets Lab), copied onto heavy cardstock, one per team. (The lighter the cardstock, the more precision the scale will need.)

Scales, digital measuring to 0.1 g or triple-beam balance

Scissors, one per team

Rulers, one per team

Compasses, one per team (optional)

Poster-making materials (for class closure)

Technology Notes: In problem 2-1 part (b) students enter data into their graphing calculators to create a scatter plot.

If students have not entered data and made scatterplots on their graphing calculators in previous courses, they will be heavily reliant on you to demonstrate the use of the calculator in this lesson.

Make sure you are well prepared and have practiced the lesson thoroughly before attempting to demonstrate to students.

The TI-83+/TI-84+ Calculator Instructions in the front matter of this Teacher Edition will help you master the statistical functions on the calculator. Specifically, these sections of the instructions may be helpful:

- 2a. Lists of Data
- 2b. Clearing Lists of Data
- 2c. Editing Lists of Data
- 4a. Setting Up a Scatterplot
- 4b. Using Window Settings
- 4c. Viewing the Scatterplot

Suggested Lesson Activity: Start the lesson by introducing the big ideas of this chapter: developing a way to manipulate the equation of a function to intentionally alter its graph and learning how to write an equation for a function by looking at its graph. You might show students that by the end of this chapter they will be able to sketch reasonable graphs of equations such as

$y = \frac{1}{2}(x - 3)^2 - 10$ or $y = -\frac{1}{x+2} - 6$ just by looking at the equations – without using an $x \rightarrow y$ table or their graphing calculators. Show the class how you can quickly sketch the graphs, but avoid including any numbers or clues that would begin to give away the strategies students will soon discover.

Today’s lesson focuses on recording data and transforming a function to make it a good model that is useful for making predictions beyond the range of measurable data. This also lays a foundation for a more in-depth study of transformations in Lesson 2.1.2.

Distribute materials and start teams on problem 2-1 using a Teammates Consult. Two digital scales should be sufficient for the entire class, though you may need one per team if you use a triple-beam balance since they require more time to use when weighing each disk. You may choose to have students draw their own circles on heavy cardstock. If so, you will need to distribute compasses as well. You can have the Resource Managers collect all of the supplies if you use Team Roles. Tell teams to use at least eight different radii for the different circles and to include some circles with very small radii as well. As teams collect data, encourage them to predict the shape of the graph of the data by asking questions, such as “*Is your data linear? If not, what do you think the graph will look like?*”

Once students have collected their data, remind them to answer questions (a) through (d) of problem 2-1. If this is the first time your students will be entering data into their graphing calculators, they will need guidance from you. Students should enter their data, plot it using a viewing window of their choice, and graph on paper what they see in their viewing window.

Part (d) of problem 2-1 asks teams to decide what kind of equation would best fit their data. The model they choose should make physical sense. Students should reason that the function must be quadratic because the mass of the disk depends on the area (πr^2 is quadratic). (Note that the function is not cubic because the thickness that creates volume is held constant—thickness is not a variable in this situation.)

Teams may realize that the data forms a curve without knowing what equation to try. Ask, “*Besides exponentials, what other kinds of curves can you think of?*” “*Can you give an example of an equation for that kind of curve?*” “*Test it in your graphing calculator – does it fit the data?*” “*How can you modify the equation so it will fit better?*” Teams should test their equation by graphing it along with the scatter plot of their data on a graphing calculator. Expect most teams to use guessing and checking to approximate the value of a in $y = ax^2$. Some may decide to use a data point to solve for the value of a . Students will be specifically asked to do this in Lesson 2.1.5, so it is not necessary to suggest this approach now.

As you circulate, ask questions such as: “*When you make the value of a larger, what happens to the shape of your graph?*” “*Could you change*

your equation to match your data more closely?” “Can you find another way to figure out your equation?”

Some teams may recall from a previous course how to do a quadratic regression on the calculator. Direct these teams to use the quadratic regression as a starting point, but encourage them to find a model that goes through the x - and y -intercept they predicted. Encourage the team to explore different values for a , as exploring the effect of values of a is also an objective of this lesson.

It is possible that some teams may decide that this data is exponential and they may be able to compensate for the y -intercept of 0 by shifting an exponential curve down. If this happens, be sure to address it in closure. Asking students to think about the difference between quadratic and exponential growth could lead to a great closure discussion. They might even say that exponential patterns grow by multiplying a fixed quantity, while quadratic patterns grow by adding amounts that increase linearly. (For example, the pattern $t(n) = n^2$ grows by differences of 1, then 3, then 5, etc.) Point students back to the physical situation: a quadratic model makes physical sense.

After teams have decided on equations, they can use them to predict the mass of a disc with double the radius of their original disk for part (f) of problem 2-1. Expect the result to be surprising for many students. Based on what they learned in previous courses, some students may question the validity of a prediction based on extrapolating so far from the collected data. By all means, support that concern: models are often not valid far from the measured data. More measurements would have to be collected in order to verify the validity of the model. In the case of shrinking targets, it is expected that a quadratic model would be valid for all diameters.

Problem 2-2 asks students to consider the domain and range of their function and how they might relate to the model they are using. As you circulate, you can ask questions that will illustrate that only the positive inputs of the function are needed to model the lab and that these will only generate positive outputs.

Closure:
(10 minutes)

Problem 2-3 asks students to think about the changes they made to their equations and how those changes affected the graphs of these equations. Give them a few minutes to discuss their ideas in teams and then pull the class together for a discussion. Alternately, you could have the teams do a Walk and Talk or find proximity partners to have a short discussion before the whole-class discussion. In a Walk and Talk students find a partner (usually not a teammate) and walk around the edge of the classroom (or outdoors, if possible) for 1-2 minutes discussing a math topic. After the time is up, bring everyone back to their teams.

If there is not enough room to have the pairs walk around, another idea is to have the students find proximity partners. They move around the room

to find a partner and then stand there and talk about the topic. When they are finished, they say thank you and return to their team.

As a class, create a poster or display a summary page entitled “What We Know About Transforming $y = x^2$.” Explain that, in this lesson, students are beginning to learn how to transform the graph of a parabola by altering its equation. Ask students how much they have learned about transformations so far. As teams contribute their ideas, add these ideas to the poster, making sure to leave lots of room to add new ideas later. At this point, students may only talk about what happens to the shape of the parabola as the a -value changes in the equation $y = ax^2$. Explain that the class will keep adding to this poster (or summary) over the next few lessons and that by the end of this section students will know how to apply a variety of transformations to the parent function $y = x^2$.

Universal Access: Problems 2-1 and 2-3:

EL and/or special needs students may not recognize the phrase “Shrinking Targets.” You could make reference to a dart board and how the outer circles shrink as you get closer to the target (bulls-eye).

Some students may not know how to use the digital measuring scales, so a quick demonstration could be beneficial. This is a meaty lesson with many tasks to perform. Be sure that instructions for procedures are clear to everyone before students start to collect data. Utilize the Technology Notes, since some students may be unfamiliar with statistical functions and calculating regression on the calculator.

Some students may struggle with the statement “adjust your equation.” Instead ask, “How does changing a constant move the graph?” Once students see that the graph changes, it reinforces the upcoming term “transformation” for when the graph raises or lowers, stretches or compresses, or slides. This will provide both contextual and visual aids.

Problem 2-2:

If EL and/or special needs students have difficulty locating the domain and range, consider “framing.”

Framing is a strategy in which the student can trace a picture frame around the graph. This will provide a visual aid that will assist struggling students with identifying the domain and range.

Math Notes – Methods and Meanings:

There are important vocabulary words bolded in this section. Consider a way for students to organize and access meanings of main concepts, ideas, and vocabulary terms in this chapter and then later for each subsequent chapter. Suggestions include: chart in the classroom, word-wall with diagrams, section in the students’ binders for vocabulary, Algebra Dictionary, etc.

Team Strategies: If you have not changed teams recently, consider rearranging your seating chart so that students are in new teams and are assigned new Team Roles as well.

Throughout this section, teams will be faced with investigations and activities that will depend on good communication and listening skills. Often, if a team is not working well together, one of the following issues is the root of the problem:

- The thinking/discussion is dominated by one student; or,
- Students do not fully understand what they are investigating or what they are expected to do.

This means that the team should learn to rely more on the Facilitator to be sure that everyone understands the task at hand, all ideas are heard, and that the team collectively decides on a strategy.

Remind the class periodically that it is *everyone's* job to ask mathematical questions that help propel the investigation. These questions often start with, “What about...?” or “What if...?”

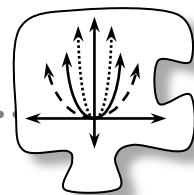
Homework: Problems 2-4 through 2-10

Note: Problem 2-5 is a pre-problem for Lesson 2.1.2. Students can answer it using a table of values, mental math, or a graphing calculator.

Notes to Self:

2.1.1 How can an equation help me predict?

Modeling Non-Linear Data



This chapter will help you develop the power to manipulate functions so that they are useful in a wide variety of situations. Today's lesson focuses on collecting data and finding a function to model the trend in that data. You will then generalize your results and make predictions beyond the range of data you can measure. Discuss the following focus questions with your team while you work:

What will the graph look like?

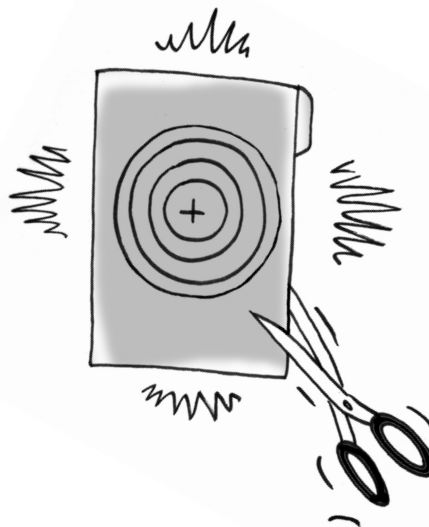
Should we connect the data points?

How can we find an equation that fits the data?

2-1. SHRINKING TARGETS LAB

What is the relationship between the radius of a disk and its mass? If you double the radius of the disk, does the mass also double?

To answer these questions, your team will use scissors, a scale, and a Lesson 2.1.1 Resource Page. You will measure the weight of at least 8 different circular disks of varying radii (the plural of “radius”). Find your first data point by cutting out the large circle, measuring its radius, and using the scale to weigh it carefully. Repeat this process for circles of different radii.



After your team has collected its data, answer the questions below. [See the “Suggested Lesson Activity” for expected student responses.]

- Look at your data with your team and predict what you think the graph will look like. Justify your prediction.
- Enter your data in the graphing calculator and plot it. Sketch the graph of your data on your paper.
- Consider the shrinking targets situation, what do you predict the x - and y -intercepts should be? What do they represent? Does the graph of your equation have these same intercept(s)? If not, explain completely why not. [Students should mention that when the length of the radius is zero, the mass is zero.]
- What kind of equation do you think will model your data? Will your model predict the intercepts correctly? [A quadratic equation of the form $y = ax^2$ will go through the origin and makes physical sense.]
- Work with your team to find an equation that fits your data. Test the accuracy of your team’s equation by entering it into your graphing calculator. If necessary, adjust your equation to make its graph fit your data and the x - and y -intercepts better. Once you are satisfied with your model, sketch the graph of your equation on your graph of data points from part (b). [Equations will vary.]
- What would be the mass of a target with a radius twice as large as the largest one you measured? How do you know? [Answers will vary, but students should use their equation from part (e) to predict the mass.]

- 2-2. What more can be said about the equation you used to model your data from the Shrinking Targets Lab? Consider this as you answer the questions below.
- What are all of the acceptable input and output values (domain and range) for the activity in Shrinking Targets Lab? Do they match the domain and range of the function you used to model your data? If not, why are they different?
[For the data, the input cannot be negative, but for the function it can. The range for both is the same, but is limited by the domain in the case of the lab.]
 - In part (a), you may have noticed that your equation only makes sense as a model for your data for part of its domain. Therefore, to accurately describe your model, you can add a condition to your equation, such as, “This equation is a good model when _____.”

What condition can you add to describe when your model is valid?
[The equation is a valid model only when x (the radius) is greater than or equal to zero.]
- 2-3. Look back at the adjustments you made to your equation in problem 2-1 in order to make it fit your data. What did you change in your equation, and what effects did your changes have on its graph? Discuss these questions with your team and be prepared to share your ideas with the class.



METHODS AND MEANINGS

Exponential Functions

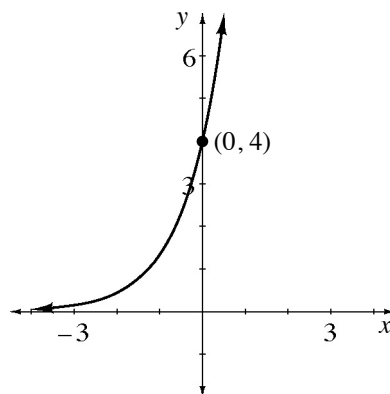
An **exponential function** has the general form $y = a \cdot b^x$, where a is the **initial value** (the y-intercept) and b is the **multiplier** (the growth). Be careful: The independent variable x has to be in the exponent. For example, $y = x^2$ is *not* an exponential equation, even though it has an exponent.

For example, in the multiple representations below, the y-intercept is $(0, 4)$ and the growth factor is 3 because the y-value is increasing by multiplying by 3.

$$y = 4 \cdot 3^x$$

x	y
-3	$\frac{4}{3^3}$ or $\frac{4}{27}$
-2	$\frac{4}{3^2}$ or $\frac{4}{9}$
-1	$\frac{4}{3}$
0	4
1	12
2	36
3	108

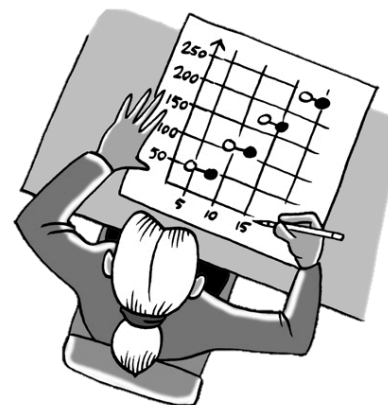
$\times 3$
 $\times 3$



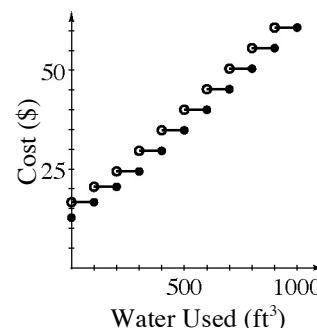
To increase or decrease a quantity by a percentage, use the multiplier for that percentage. For example, the multiplier for an increase of 7% is $100\% + 7\% = 1.07$. The multiplier for a decrease of 7% is $100\% - 7\% = 0.93$.

- 2-4. Jamilla was moving to a new city. She researched the rates charged by the local utility company for water. She found the listing of charges below. She expects that her family may use up to 1,000 cubic feet of water each month.

- \$12.70 monthly service fee
- First 300 cubic feet of water used:
\$3.90 per 100 cubic feet, or fraction thereof
- After the first 300 cubic feet:
\$5.20 per 100 cubic feet, or fraction thereof



- a. Sketch a graph of the cost of Jamilla's possible water usage in one month. Be sure to consider what the cost would be for partial units such as 220 or 675 cubic feet of water. [See graph at right.]
- b. Is this graph a function? Why or why not? [**Yes, for every possible amount of water usage, there is only one possible cost.**]
- c. What are the domain and range of this graph? [**Domain: 0 to 1,000 cubic feet; range: discrete values including: \$12.70, \$16.60, \$20.50, \$24.40, \$29.60, \$34.80, \$40, \$45.20, \$50.40, \$55.60, \$60.80**]



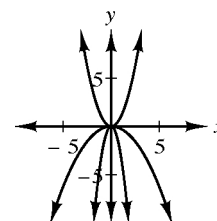
- 2-5. For each equation in parts (a) through (d) below, find the input value that gives the *smallest* possible output. In other words, find the x -value of the *lowest* point on the graph. Then find the input value that gives the *largest* possible output (or the x -value of the *highest* point on the graph). [**Smallest: a: 2; b: 0; c: -3; d: none.**

Largest: a: none. b: none. c: none. d: 0. e: At the vertex.]

a. $y = (x - 2)^2$ b. $y = x^2 + 2$ c. $y = (x + 3)^2$ d. $y = -x^2 + 5$

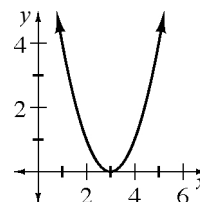
- e. Where on the graphs of each of the above equations would you find the points with the smallest or largest y -values?

- 2-6. Sketch $y = x^2$, $y = -3x^2$, and $y = -0.25x^2$ on the same set of axes. What does a negative coefficient do to the graph? [**The negative coefficient causes parabolas to open downward, without changing the vertex. See graph at right.]**



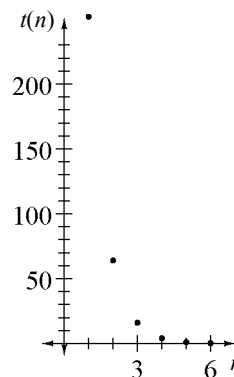
2-7. Your results from this problem will be useful in the parabola investigation that you will do in Lesson 2.1.2.

- Draw the graph of $y = (x - 3)^2$. If you are drawing the graph by hand be sure to use the domain $0 \leq x \leq 6$.
[**Parabola with vertex (3, 0), see graph at right.**]
- How is this graph different from the graph of $y = x^2$?
[**Shifted to the right three units.**]



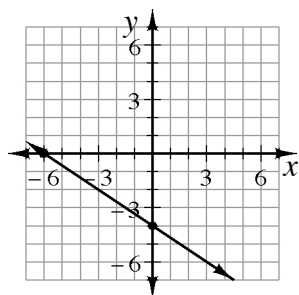
2-8. Consider the sequence with the initial value 256, followed by 64, 16, ...

- Write the next three terms of this sequence, then find an equation for the sequence. [**4, 1, 0.25; $t(n) = 256(0.25)^n$**]
- If you were to keep writing out more and more terms of the sequence, what would happen to the terms? [**They get smaller, but are never negative.**]
- Sketch a graph of the sequence. What happens to the points as you go farther to the right? [**See graph at right. They get very close to zero.**]
- What is the domain of the sequence? What is the domain of the function with the same equation as this sequence? [**The domain is n integers greater than or equal to zero. The domain of the function is all real numbers.**]

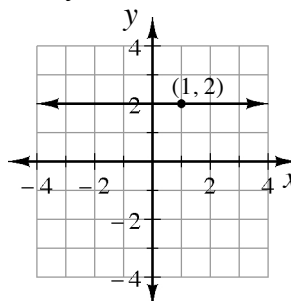


2-9. Write the equation for each graph.

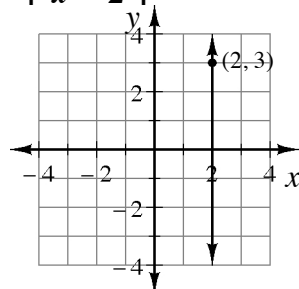
- [$y = -\frac{2}{3}x - 4$]



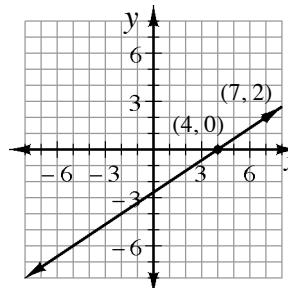
- [$y = 2$]



- [$x = 2$]



- [$y = \frac{2}{3}x - \frac{8}{3}$]



- 2-10. The slope of \overline{AB} is 5, with points $A(-3, -1)$ and $B(2, n)$. Find the value of n and the distance between points A and B . [$n = 24$; $\sqrt{650} = 5\sqrt{26}$]

Lesson 2.1.2 How can I shift a parabola?

Parabola Investigation

Lesson Objective: Students will connect transformations of parabolas with their equations in graphing form.

Mathematical Practices: make sense of problems and persevere in solving them, model with mathematics, attend to precision, look for and make use of structure

Length of Activity: Two days (approximately 100 minutes)

Core Problems: Day 1: Problem 2-11
Day 2: Problems 2-12, 2-14, and 2-15 (and problem 2-13 if using “Further Guidance”)

Materials: Lesson 2.1.2 Resource Page (“Transforming Parabolas”), one double-sided copy per student (optional)

Colored pencils

Suggested Lesson Activity: **Lesson Overview:** This lesson consists of two lab investigations that are designed to span two days. By the end of the first investigation, teams should have preliminary ideas about how to stretch and compress parabolas vertically and how to flip them. They will also begin to develop an idea of how to shift a parabola left or right. By the end of the second investigation, teams should be able to write the general equation for parabolas and use it to make any transformation. Be careful not to expect mastery of these ideas at this point. Students will have multiple opportunities throughout this chapter and beyond to practice and consolidate their techniques.

Day 1: Start the day with a quick review of the class summary poster created during the Closure of Lesson 2.1.1. This can be done as Reciprocal Teaching where the first person tells everything they remember from Lesson 2.1.1 and then second person adds anything that was left out. Explain that the objective for this lesson is to add to the class’s knowledge of parabolas, specifically looking at how the equations and graphs of parabolas are related.

Have a volunteer read the lesson introduction. Then ask teams (Facilitators if using Team Roles) to read problem 2-11 aloud and begin work. You might warn teams that part (b) may take a long time, but to keep discussing and experimenting because once they get it, the rest of problem 2-11 will go much more quickly. Students will be alternating between making changes in equations and observing the impact of these

changes on graphs. If a team is stuck on part (b) for too long, you may need to help them summarize what they have tried to help them see what else they might do to the equation. Or allow teams to use I Spy. An I Spy can be used at any time when a team is stuck. The teacher chooses one person from a team to quietly spy on another team. That team should not know that the spy is even there. Then the spy goes back to his/her team to report what they learned. Sometimes it is a good idea to let a student who has a perceived low status to be the person chosen to be the spy. That way they go back to their team with important information that the rest of the team needs.

You also might ask teams, “*Why does $y = (x - 2)(x - 2)$ only touch the x -axis at $x = 2$? What does that tell you about what you should and should not change about the equation?*” Some students may come up with ideas such as $y = (2x - 4)(2x - 4)$. In such cases, you could ask questions like, “*How can you change your equation so that the quantity inside the parentheses looks like the equation in part (a)?*” The idea is for students to generate the factored form so they can easily see its connection to the general equation. If the equation is not in factored form, students will struggle to determine how to shift the parabola horizontally. The factored form also helps with recognizing the stretch factor of the general equation.

End this day by bringing teams together to share some of the equations they have developed. This can be done with a Swapmeet. At this point, do not link specific equations to specific transformations; students will continue to build these ideas in the second day of the lesson, so summarizing at this point will narrow the subsequent investigation. Instead, use the class’s collection of equations to launch a discussion about the many different ways the equations of parabolas may be written. Expect to see equations of the form $y = (x - h)(x - h)$, $y = a(x - h)(x - h)$, and possibly $y = (bx - d)(bx - d)$, although you should expect students to share equations with numbers rather than variables at this point. Some students may also share equations in standard form $y = ax^2 + bx + c$ or in the form $y = a(x - h)^2$. Ask the class how they could decide if these all represent parabolas and how they justify their conclusions. If no one initially shares an equation of the form $y = a(x - h)^2$, ask students how they might be able to write their equations more simply. The class should be able to see that they could rewrite each equation into the form $y = a(x - h)^2$. Do not expect a general form at this point, because this will be developed at the end of Day 2. Equations such as $y = (x + 4)^2$ are sufficient.

Day 2: Start by having a student read problem 2-12. Before prompting students to begin the investigation in their teams, brainstorm as a class about some of the different ways that a parabolic graph can be transformed. This can be done as a Think-(Ink)-Pair-Share. As students volunteer ideas, make a list on the board. Your list might contain ideas

such as:

Slide it right or left.	Slide it up or down.
Slide it diagonally.	Reflect it.
Flip it.	Make it wider.
Make it taller.	Squish it.
Rotate it.	

This can also be done as a Whiparound.

Ask each team to reread the task statement and Discussion Points out loud before they begin to work. If you prefer a more structured approach, direct teams to the “Further Guidance” questions in problem 2-13.

Students will solidify what they learned in the first investigation about controlling a parabola’s orientation, as well as stretching and compressing the graph. Today they will also look closely at shifting the graph (vertical and horizontal translations). Teams may need some guiding questions such as, “*Have you tried a negative value?*” but be careful not to intervene too soon. Often, students will come to these ideas themselves and their success will become infectious, stimulating other discoveries in the class.

Students sometimes have trouble with the ideas of vertical stretch and vertical compression. A rubber band makes a good visual aid. Also effective is a piece of inner tube (it is easily stretched in both directions) with a parabola drawn on it in chalk. Students often use terms such as “wider” or “narrower” to describe the shape of a vertically compressed or stretched parabola. It is interesting to discuss with students what the effect of a actually is on the y -values of the graph. This will also be addressed in Lesson 2.1.3.

Students often have trouble understanding why an equation like $y = (x + 2)^2$ shifts the graph to the left (the negative direction) instead of to the right (the positive direction). Ask students questions about this shift to help them understand what is taking place. One way to do this is to discuss how they find x -intercepts of a parabola when the vertex is not on the x -axis. Students are familiar with equations of parabolas in the form $y = (x - 3)(x - 4)$ and are probably comfortable using the Zero Product Property to find the x -intercepts as $(3, 0)$ and $(4, 0)$. Alternately, you can ask teams what the y -value is for the vertex in the equation $y = x^2$. Ask them what x equaled in order to get this value, and then ask what x must equal in order to make $(x + 2)^2 = 0$.

Problem 2-14 combines several moves, in preparation for the general equation students are asked find in today’s closure.

Be sure to leave time for the closure activity today.

Closure:

Save some time for closure to help students consolidate their learning.

(10 minutes)

The final question of classwork in this lesson, problem 2-15, prompts teams to try to write a general equation for a parabola. The idea is to generate a general equation like $y = a(x - h)^2 + k$. Leave open the possibility of students coming up with their own formulation – ideally, students will create a general, abstract way to write the equation of a parabola as a general equation. Check for understanding by asking students to make up a few examples. You could do a Traveling Salesman where each team writes the letter to Polly Parabola and then prepares a mini-presentation. One person is chosen from the team to “travel” and make the presentation to another team. The other team can then offer suggestions.

Universal Access:

Problem 2-11:

Generating equations and sketching graphs could take EL and/or special needs students a long time. Since some students spend an inordinate amount of time drawing and labeling axes, it is a good idea to provide ready-made graphs (only label the x - and y -axes) for these students. Avoid numbering the graphs; this is an important concept students should learn.

If students get bogged down in parts (b) and (c), consider allowing them to generate one equation for each section. If time permits, they can return to generate additional equations.

Problems 2-12 and 2-13:

Do not assume that students will be able to understand problem 2-12 just by reading or having it read. Problem 2-13 (Further Guidance) will provide the structure and scaffolding that EL and/or special needs students may need in order to attack the parabola investigation (problem 2-12).

Resource Page 2.1.2 is as a useful graphic organizer for students to record and organize important information from the parabola investigation and should not be optional for EL and special needs students.

Team Strategies:

Suggested Study Team Collaboration Strategies: For problem 2-11, consider using a Swapmeet. For problem 2-12 you could use a Think-Ink-Pair-Share.

If students are using their own graphing calculators (or other graphing technology) in this lesson you will need to take particular care to structure team conversation. Because students may be working with their own ideas and entering their conjectures into their own calculators, it may be tempting for teams to shift into individual work rather than students sharing their ideas with each other. Remind students of the importance of discussing what they see on their calculators. Before students start investigating, ask Recorder/Reporters to remind others to “*show us what you see on the calculator.*” Facilitators can keep the

group together by regularly asking, “*What have we figured out so far?*” or “*Who has an idea to share?*” Consider reminding students of their roles before the investigations begin and highlighting the importance of active team conversation throughout the lesson.

Homework:

Day 1: Problems 2-16 through 2-22

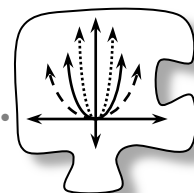
Day 2: Problems 2-23 through 2-29

Problem 2-25 asks students to connect their investigations from this lesson to ideas they will work with in Lesson 2.1.3. Students may want to refer to this problem at the beginning of the next lesson.

2.1.2 How can I shift a parabola?

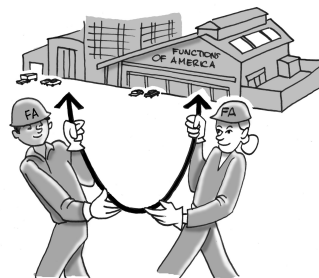
Parabola Investigation

In Algebra 1 you learned about slope and y-intercept, ideas that allow you to write equations and sketch graphs of any line. During this lesson you will work on developing similar tools for parabolas.

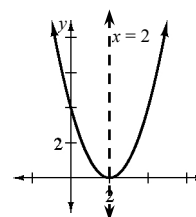


2-11. PARABOLA LAB, Part One

What happens to a parabola’s graph when you change the numbers in the equation? To get a better sense of the different ways to transform the graph of a parabola, as a team complete the investigation outlined below. As you work, be sure to sketch the graphs you see in your graphing calculator carefully and record the equations you enter.



- a. On graph paper, graph the equation $y = (x - 2)(x - 2)$. Be sure to label any important points on your graph, including the lowest point on the graph, called the **vertex**. (If the graph were to open downward, the vertex would be the highest point on the graph.) Also sketch and write the equation of the line of symmetry of your graph. [**a: See graph at right. Line of symmetry is $x = 2$.**]



- b. Use your graphing calculator to find the equations of two parabolas with *different* graphs that also open upward and still have a vertex at (2, 0). Add sketches of these two new graphs to your graph from part (a), along with their equations. As you work, keep track of any ideas you try along with their results, even if they do not answer this question, as they may help you later. [**Examples are $y = 2(x - 2)(x - 2)$ and $y = \frac{1}{2}(x - 2)(x - 2)$.**]

Problem continues on next page →

2-11. *Problem continued from previous page.*

- c. Use your graphing calculator to find the equations of two different parabolas that open *downward*, each with its vertex on the x -axis at $x = 2$. How did you change the equation so that the parabola would open downward? Add sketches of these graphs and their equations to your axes. What are their lines of symmetry? [**Examples are $y = -(x - 2)(x - 2)$ and $y = -2(x - 2)(x - 2)$. Line of symmetry is still $x = 2$.**]
- d. Use your graphing calculator to find the equation of a parabola that opens downward with a vertex at $(-4, 0)$. What is the equation of your parabola's line of symmetry? [**One example is $y = -(x + 4)(x + 4)$. Line of symmetry is $x = -4$.**]
- e. Choose a new point on the x -axis and find at least three equations of parabolas that touch the x -axis only at that one point. [**Answers vary.**]

2-12. PARABOLA LAB, Part Two

Polly Parabola had been the manager of the Parabola Department of Functions of America, but she has decided to start her own company called "Professional Parabola Productions." She needs your help. See her memo below.

MEMO

To: Your Study Team
From: Ms. Polly Parabola, CEO
Re: New Parabola Possibilities

I am starting a new company specializing only in parabolas. To win over new customers, I need to be able to show them that we know more about parabolas than any of the other function factories around, especially since every company already sells $y = x^2$.

My customers will need all sorts of parabolas, and we need the knowledge to make them happy. I would love to offer parabolas that are completely new to them.

Please investigate all different kinds of parabolas. Determine all the ways that you can change the equation $y = x^2$ to change the shape, direction, and location of a parabola on a graph.

Remember that I'm counting on you! I need you to uncover the parabola secrets that our competitors do not know.

Sincerely,
Ms. Polly Parabola

Your Task: Work with your team to determine all of the ways you can change the graph of a parabola by changing its equation. Be prepared to share your ideas with the class. As other teams contribute ideas to a class discussion, write down any new ideas.

Start by choosing one transformation from the list generated by the class; then find a way to change the equation $y = x^2$ to create this transformation. Whenever you figure out a new transformation, record a clear summary statement before moving on to the next transformation. Be prepared to explain your summary statement to Ms. Polly Parabola.

Discussion Points

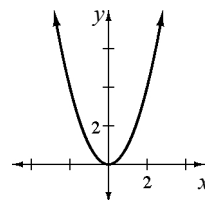
What changes can we make to a parabola's graph?

What changes can we make to the equation $y = x^2$?

How do changes in the equation relate to changes in the graph?

Further Guidance

- 2-13. Graph the parabola $y = x^2$. Be sure to label any important points. When you are sure that your graph is complete and accurate, trace over it in colored pencil. [**See graph at right.**]



- a. Find a way to change the equation to make the $y = x^2$ parabola *stretch vertically*. That is, to make the graph look narrower, so the points in the parabola seem to rise away from the vertex more quickly. The new parabola should have the same vertex and orientation (i.e., open up) as $y = x^2$. Record the equations you try, along with their results. Write down the results even when they are wrong – they may come in handy later on. [**$y = 2x^2$ is the most common answer.**]
- b. Find a way to change the equation to make the $y = x^2$ parabola *compress vertically*. That is, to make the graph look flatter, so that the points seem to rise away from the vertex less quickly. Record the equations you try, along with their results and your observations. [**$y = 0.5x^2$ is one answer.**]
- c. Find a way to change the equation to make the same parabola *open downward*. The new parabola should be congruent (the same shape and size) to $y = x^2$, with the same vertex, except it should open downward so its vertex will be its highest point. Record the equations you try, their results, and your observations. [**$y = -x^2$**]



Problem continues on next page →

2-13. *Problem continued from previous page.*

- d. Find a way to change the equation to make the $y = x^2$ parabola *move 5 units down*. Your new parabola should look exactly like $y = x^2$, but the vertex should be at $(0, -5)$. Record the equations you try, along with their results. Include a comment about moving the graph up as well as down. [$y = x^2 - 5$]
- e. Find a way to change the equation to make the $y = x^2$ parabola *move 3 units to the right*. Your new parabola should look exactly like $y = x^2$, except that the vertex should be at the point $(3, 0)$. If you need an idea to get started, review your work on problem 2-11. Record the equations you try, along with their results. Include a comment about how to move the parabola to the left as well as how to move it to the right. [$y = (x - 3)^2$]
- f. Find a way to change the equation to make the $y = x^2$ parabola *move 3 units to the left*, as in part (e), AND *stretch vertically*, as in part (a). Record the equations you try, along with their results. [e.g. $y = 4(x + 3)^2$]

===== *Further Guidance* =====
section ends here.

2-14. Find a way to change the equation to make the $y = x^2$ parabola *vertically compressed, open down, move six units up, and move two units to the left*. Where is the vertex of your new parabola? [**One example is $y = -0.5(x + 2)^2 + 6$, $(-2, 6)$.**]

2-15. Now that you are a parabola expert, you can impress Ms. Polly Parabola!

- a. Make up your own fancy transformation and show her how you can change your equation to create it.
- b. Write a general equation for a parabola that could be shifted or stretched in any direction by any amount. Be prepared to share your ideas with the class.
[$y = a(x - h)^2 + k$]



2-16. Explain the differences between an *accurate sketch* and a *careful graph*.
[**Explanations vary, but a careful graph is to scale, done on graph paper, and with key points clearly labeled.**]

- 2-17. If $p(x) = x^2 + 5x - 6$, find:
- Where $p(x)$ intersects the y -axis. [**(0, -6)**]
 - Where $p(x)$ intersects the x -axis. [**(-6, 0) and (1, 0)**]
 - If $q(x) = x^2 + 5x$, find the intercepts of $q(x)$ and compare the graphs of $p(x)$ and $q(x)$. [**x -intercepts at (0, 0) and (-5, 0) and y -intercept at (0, 0); the graph of $p(x)$ is 6 units lower than $q(x)$**]
 - Find $p(x) - q(x)$. [**-6**]

2-18. Solve for z in each equation below.

- $4^z = 8$ [**$z = 1.5$**]
- $4^{2z/3} = 8^{(z+2)}$ [**$z = -\frac{18}{5}$**]
- $3^z = 81^2$ [**$z = 8$**]
- $5^{(z+1)/3} = 25^{1/z}$ [**$z = -3, 2$**]


2-19. Simplify each of the following expressions. Be sure that your answer has no negative or fractional exponents.

- $(\frac{1}{81})^{-1/4}$
[**3**]
- $x^{-2}y^{-4}$
[**$\frac{1}{x^2y^4}$**]
- $(2x)^{-2}(16x^2y)^{1/2}$
[**$\frac{\sqrt{y}}{x}$**]

- 2-20. Daniela, Kieu, and Duyen decide to go to the movies one hot summer afternoon. The theater is having a summer special called Three Go Free. They will get free movie tickets if they each buy a large popcorn and a large soft drink. They take the deal and spend \$22.50 on food, drinks and movie tickets. The next week, they go back again, only this time, they each pay \$8.00 for their ticket, they each get a large soft drink, but they share one large bucket of popcorn. This return trip costs them a total of \$37.50.



- Find the price of a large soft drink and the price of a large bucket of popcorn.
[**$3p + 3d = 22.50$ and $p + 3d + 3(8) = 37.5$, so popcorn costs \$4.50 and a soft drink costs \$3.00.**]
- Did you write two equations or did you use another method? If you used another method, write two equations now and solve them. If you already used a system of equations, skip this part. [**Answers vary.**]

- 2-21. Plot each pair of points and find the distance between them. Give answers in both square-root form and as decimal approximations.
- $(3, -6)$ and $(-2, 5)$ [$\sqrt{146} \approx 12.1$]
 - $(5, -8)$ and $(-3, 1)$ [$\sqrt{145} \approx 12.0$]
 - $(0, 5)$ and $(5, 0)$ [$\sqrt{50} \approx 7.1$]
 - Write the distance you found in part (c) in simplified square-root form. [$5\sqrt{2}$]
- 2-22. The amount of profit (in millions) made by Scandal Math, a company that writes math problems based on tabloid articles, can be found by the equation $P(n) = -n^2 + 10n$, where n is the number of textbooks sold (also in millions). Find the maximum profit and the number of textbooks that Scandal Math must sell to realize this maximum profit. [**Maximum profit is \$25 million when $n = 5$ million.**]
- 2-23. Your friend is taking an algebra class at a different school where she is not allowed to use a graphing calculator. 
- Explain to her how she can get a good sketch of the graph of the function $y = 2(x + 3)^2 - 8$ without using a calculator *and* without having to make an $x \rightarrow y$ table. Be sure to explain how to locate the vertex, whether the parabola should open up or down, and how its shape is related to the shape of the graph of $y = x^2$. [**vertex at $(-3, -8)$, opens up, vertically stretched.**]
 - Your friend also needs to know the x - and y -intercepts. Show her how to find them without having to draw an accurate graph or use a graphing calculator. [**x -intercepts $(-5, 0)$ and $(-1, 0)$; y -intercept $(0, 10)$]**
- 2-24. Consider the equations $y = 3(x - 1)^2 - 5$ and $y = 3x^2 - 6x - 2$.
- Verify that they are equivalent by creating a table or graph for each equation. [**Tables or graphs should be the same.**]
 - Show algebraically that these two equations are equivalent by starting with one form and showing how to get the other. [**See sample student work at right.**]
 - Notice that the value for a is 3 in both forms of the equation, but that the numbers for b and c are different from the numbers for h and k . Why do you think the value for a would be the same number in both forms of the equation? [**Students could point out that the a ends up being the coefficient of x^2 after the binomial is squared.**]

$$y = 3(x - 1)^2 - 5$$

$$y = 3(x^2 - 2x + 1) - 5$$

$$y = 3x^2 - 6x + 3 - 5$$

$$y = 3x^2 - 6x - 2$$

2-25. Use what you learned in the parabola investigation to write an equation for each of the parabolas described below.

- A parabola opening upward, shifted 8 units right, and 5 units down.
[$y = (x - 8)^2 - 5$]
- A parabola with a stretch factor of 10, sitting with its vertex on the x -axis at $x = -6$. [$y = 10(x + 6)^2$]
- A downward-opening parabola with vertex $(-7, -2)$ and a vertical compression of 0.6. [$y = -0.6(x + 7)^2 - 2$]

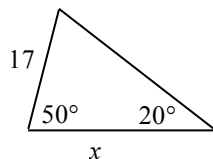
2-26. The point $(3, -7)$ is on a line with a slope of $\frac{2}{3}$. Find another point on the line.
[**Some possibilities are $(0, -9)$ and $(6, -5)$.**]

2-27. Simplify each expression without using a calculator. Remember that to simplify expressions with radicals, you can remove perfect square factors such as in this example: $\sqrt{18} = \sqrt{9 \cdot 2} = \sqrt{9} \cdot \sqrt{2} = 3\sqrt{2}$

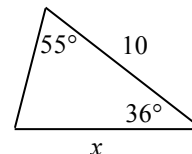
- $\sqrt{50}$
[$5\sqrt{2}$]
- $\sqrt{72}$
[$6\sqrt{2}$]
- $\sqrt{45}$
[$3\sqrt{5}$]

2-28. Find the value of x .

- [$x = 46.71$]



- [$x = 8.19$]



2-29. Suppose your parents spend an average of \$300 each month for your food.

- In five years, when you are living on your own, how much will you be spending on food each month if you are eating about the same amount and inflation averages about 4% per year? [**About \$365.00.**]
- Write an equation that represents your monthly food bill x years from now if both the rate of inflation and your eating habits stay the same.
[$y = 300(1.04)^x$]

Lesson 2.1.3 How can I graph it quickly?

Graphing a Parabola Without a Table

- Lesson Objective:** Students will graph quadratic equations without making tables. Students will rewrite quadratic equations from standard form into graphing form.
- Mathematical Practices:** use appropriate tools strategically, look for and make use of structure, look for and express regularity in repeated reasoning
- Length of Activity:** One day (approximately 50 minutes)
- Core Problems:** Problems 2-30 (or demonstration) and 2-31 through 2-33
- Materials:** Dynamic Tool: TI Application: *Transfrm* Transformation Graphing v1.03 or computer(s) with *Transforming Parabolas* applet
Lesson 2.1.3 Resource Page (“*Transfrm* Instructions”), optional
- Technology Notes:** If technology is available, you can use a dynamic tool to emphasize the effect of the parameters on the graph of the parabola. Specifically, students will be able to dynamically see the impact of changing the values of a , h , and k on a parabola’s graph, and determine the reason that changes in those values impact the graph in specific ways. There are two possible ways to incorporate the technology: with the graphing calculator or with a web-based dynamic graphing tool on individual computers or as a demonstration. For some students, seeing this demonstration to illustrate the effect of a on the graph of $y = ax^2$ is enough. For other students you may choose to also have them explore changes in the other parameters.

Graphing calculator:

The TI-83+/84+ calculators have a free application called **Transfrm**, Transformation Graphing v1.03, which can be used on any function entered in the $Y=$ screen. If you press the $\boxed{\text{APPS}}$ button and scroll or press $\boxed{\text{ALPHA}}\boxed{4}$, T, you can check to see if **Transfrm** is loaded on the calculator. You can transfer the program from calculator to calculator with a link cord or download it from www.education.ti.com using a computer link cord. Note: This application uses the names A , B , C for the parameters. For parabolas $A = a$, $B = h$ and $C = k$. An optional Lesson 2.1.3 Resource Page is provided with instructions for using **Transfrm**.

Dynamic Tool: “Transforming Parabolas.”

This tool allows students to manipulate the parameters a , h , and k in the graphing form of the equation of a parabola and see the impact of those parameters on a graph. Following is a description of a recommended demonstration.

When you start, point out to students that the parabola that already exists on the axes is a graph of $y = x^2$. This is the **parent graph**, the most basic

parabola from which other transformations will be generated. This terminology is formally introduced to students in Lesson 2.2.1.

Notice that the equation $y = ax^2$ appears on the screen with a slider of a -values ranging from -5 to 5 . Ask students how you should move the slider (centered on one) to make the parabola open wider or compress horizontally. Follow students' directions and a red parabola will appear on the axes (the blue parent graph will remain constant). As you slide the a -value, the parabola will adjust. Follow up by asking how to stretch the parabola vertically, then follow the students' directions. Ask, "*What will happen to the graph if I adjust the value of a to be negative?*" Follow students' directions and discuss the results.

When students are comfortable with the idea that a -values greater than 1 stretch the parabola, a -values between 0 and 1 compress the parabola, and negative a -values change the parabola's vertical orientation, prompt students to think about why a has this affect. Consider looking at a specific case, for example, asking, "*Why does $a = 2$ stretch the graph?*" Set the slider so that the equation is $y = 2x^2$. Double-click on the point $(1, 0)$ to see a vertical segment extend up through $y = x^2$ and intersect with $y = 2x^2$. You will be able to see that the length of the segment to the second parabola is twice the length of the segment to the first parabola.

Reset the parameters and click on the button for h . Another slider and equation appear showing $y = (x - 0)^2$. Ask students to share some of their observations about h from the previous lesson and test their conjectures about the impact of h using the sliders. Do the same for k , following the questions in the student text.

Suggested Lesson Activity:

Lesson Overview: In this lesson students will see the power of the work they have done in the parabola lab over the last two days. The lesson starts with a class discussion using a dynamic algebra tool to illustrate how changes in the parameters of an equation impact the graph of a parabola. The lesson continues with students using what they have learned about parabolas to make graphs of various parabolic equations quickly. Then students rewrite quadratic equations from standard into graphing form to be able to make a graph quickly. Students will do the rewriting using the x -intercepts to find the vertex.

Note: Rewriting equations by completing the square will be introduced in Lesson 2.1.4, so the equations that students convert from standard to graphing form will not yet require that skill.

Lesson Activity: Begin the day by explaining that students will be using the connections between quadratic equations and their graphs they found in Lesson 2.1.2 to now find shortcuts for drawing accurate parabolic graphs without having to make tables. You can draw students' attention to the knowledge they have gained in previous math classes, specifically the connections between equations of lines and their graphs, which allow them to sketch the graph of a line from an equation in $y = mx + b$ form quickly. Today they will be doing something similar with a parabola.

Open the *Transforming Parabolas* tool or the **Transfrm** program and introduce it to your students. If you are using the **Transfrm** program, you might distribute the Lesson 2.1.3 Resource Page so that teams can use the program independently or you might do this as a demonstration as described above. Use the dynamic tool to further investigate the impact of a , h and k on the graph of a parabola expressed by an equation in the form $y = a(x - h)^2 + k$ as suggested in problem 2-30. Ask questions that prompt students to look beyond the impact of the parameters to try to identify why changes in the parameters have those specific effects. See the questions listed in problem 2-30 and the Technology Notes section above.

After a discussion using the dynamic tool, assign teams problem 2-31. This problem can be done as a Pairs Check. One person explains to their partner how to do the problem, while the partner writes each step. Then they change roles for the second problem. After the two problems are finished, the pair finds another pair to check answers. If they all four agree, then the pairs can go on to the next two problems and repeat the process. The teacher should circulate and listen to the explanations, stepping in when necessary to ask clarifying questions. This strategy forces students to work together and to explain aloud their thinking. Give teams about 10 minutes to figure out how to draw the graphs, and then ask for student volunteers to show the strategies they have developed.

Encourage different students to share so that the class builds a varied set of strategies. Ask questions such as: “*How do you apply this quick graph idea to parabolas?*” “*What do we need to know about a parabola to graph it quickly?*” Teams should state that they need to know the vertex and the stretch factor (the value of a). Once teams have figured out where the vertex is, ask them to think about how the stretch factor can help them to make an accurate graph. Look for teams to decide that they can compare the shape to that of $y = x^2$. If the stretch factor is 1, they can plot two points that are one unit left and one unit right of the vertex and one unit up (corresponding with the points $(1, 1)$ and $(-1, 1)$ on the graph of $y = x^2$). They can also plot two points that are two units left and right of the vertex and four units up (corresponding with the points $(2, 4)$ and $(-2, 4)$ on the graph of $y = x^2$). If the stretch factor is 2, they can stretch each of these vertical distances by a factor of two, etc.

Ask teams to use these techniques as they work on problems 2-32, 2-33, and, if time permits, problem 2-34. Once teams realize that they can find the vertex by averaging the x -intercepts to find h and substituting to find k , they should be able to use this information to rewrite the equation into vertex form. Have a brief class discussion after teams have worked on problem 2-33 to make sure all the students understand this method. Teams may state that the vertex is between the x -intercepts instead of stating that it is the average of the intercepts. Ask students, “*How can you find the middle?*” You can use an example where the intercepts are something like $(-8, 0)$ and $(3, 0)$. Some students have little difficulty finding the x -value of the vertex, but then will not think of substituting

back into the equation to find the y -value. If teams are stuck, you might remind them that the vertex is a point on the graph and ask, “*What has to be true of each point that is on the graph?*” or “*How do you generate input/output tables?*”

Some teams may use completing the square that they learned in a previous course. If this happens be sure to have those teams share briefly, but do not dwell on it. Students will review (or learn) the technique of completing the square in Lesson 2.1.4.

If time permits, move teams on to problem 2-34, where parts (c) and (d) may be challenging. They will need to realize that they can use the Quadratic Formula to find the x -intercepts in part (c) and then though averaging them may appear daunting, it turns out to be quite easy. You can help support teams in need of help by asking, “*If you can’t factor, what other tools do you have?*” This can also be done using a Hot Potato, where one paper with the problems is placed in the middle of the team. The first team member does a step, talking aloud through their work. Then the second team member does the next step, also speaking out loud to explain what they are doing. The team repeats this until the problem is finished. Another way of doing this is with a timer. The first person works on the problem doing as much as they can in 30 seconds, explaining aloud everything that they are doing. Then the next person picks up where the first person left off. They work for another 30 seconds and so on. The team continues until all of the problems are finished.

Closure:
(5 minutes)

To end the lesson, have teams quickly share their strategies for graphing equations in standard form. They could demonstrate their work in problem 2-33 or their solutions to problem 2-34, if they did this problem. If time permits, investigate part (c) of problem 2-34 further. You could suggest that students graph the equations $y = x^2 - 4x - 2$ and $y = -2$. Ask them how they might be able to use these equations to make finding the vertex easier. They could then solve the equation $-2 = x^2 - 4x - 2$ or use their graphs to find the points of intersection. The vertex will occur in the middle of the x -values of the solution.

Universal Access: Problem 2-30:

Utilize the Technology Notes! The dynamic graphing tool and the graphing calculator application will provide visual representations that EL and/or special needs students need to make connections between changing parameters and graph orientation.

Resource Page 2.1.3 provides an easy-to-follow set of instructions for using the graphing calculator application and should not be optional for EL and/or special needs students. Be sure students understand that the A, B, and C on the calculator corresponds with the a , h , and k in the equation.

Problem 2-31:

Since some students spend an inordinate amount of time drawing and

labeling axes, it is a good idea to provide ready-made graphs for these students. Avoid numbering the graphs. This is an important concept students should learn.

If EL and/or special needs students are struggling to make predictions, you may consider allowing them to use the calculator. However, encourage them to attempt (a) and (b); they may surprise themselves and gain the confidence to try more without the calculator.

Problem 2-32:

If EL and/or special needs students are struggling to make the graphs, you may consider allowing them to use the calculator OR you could pair up a stronger student with a weaker student.

Math Notes – Methods and Meanings:

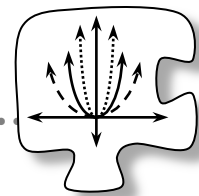
This provides a good opportunity to review the three forms of quadratic equations. Consider a way for students to organize and access this information.

Team Strategies: One way to encourage broad participation in a class discussion in which teams are sharing their strategies is to assign a particular student to report their team's ideas. While this is commonly the Recorder/Reporter, it is not necessary to have only that role share the team's thinking. If you notice that students who happen to be Task Managers are not vocal in whole class discussions, announce that you will be asking Task Managers (or whatever role you choose) to share the ideas the team has generated. You can announce this before teams start work or interrupt teamwork a few minutes before the class discussion so that students have time to prepare.

Homework: Problems 2-35 through 2-41

2.1.3 How can I graph it quickly?

Graphing a Parabola Without a Table



You have developed several tools that enable you to transform graphs of parabolas by altering their equations. In the next few lessons, you will use this knowledge to do more with the equations and graphs of parabolic functions than ever before. In this lesson, you will figure out how to use your growing knowledge of transforming graphs to make a quick and fairly accurate graph of any parabolic function.

2-30. TRANSFORMING GRAPHS

Use your dynamic graphing tool to support a class discussion about the equation $y = a(x - h)^2 + k$. Refer to the bulleted points below.



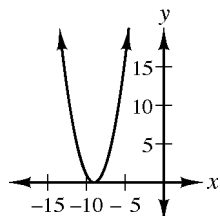
- Identify which **parameter** (a , h , or k) affects the orientation, vertical shift, horizontal shift, vertical stretch, and vertical compression of the graph compared to the graph of the parent function $y = x^2$.
- What values stretch the graph vertically? Compress the graph horizontally? Why do those values have these impacts?
- What values cause the graph to flip vertically?
- What values cause the graph to shift to the left? To the right? Why?
- What values cause the graph to shift up or down? Why?
- Are there points on your graph that connect to specific parameters in the equation? Explain.

2-31.

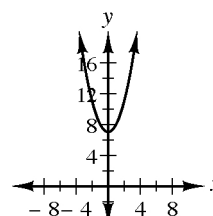
For each equation below, predict the coordinates of the vertex, the orientation (whether it opens up or down), and whether the graph will be a vertical stretch or a compression of $y = x^2$. Do not use a graphing calculator. Quickly make an accurate graph based on your predictions. How can you make the shape of your graph accurate without using a table? Be prepared to share your strategies with the class. [**a: $(-9, 0)$, opens up, none; b: $(0, 7)$, opens up, none; c: $(0, 0)$, opens up, stretch; d: $(1, 0)$, opens up, compression; e: $(7, 6)$, opens down, none; f: $(-3, -8)$, opens up, stretch. See Suggested Lesson Activity section for strategies for making accurate graphs.]**



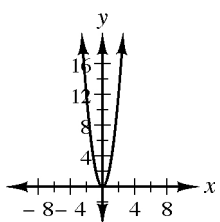
a. $y = (x + 9)^2$



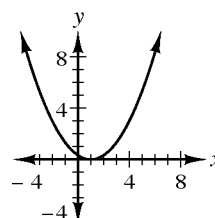
b. $y = x^2 + 7$



c. $y = 3x^2$



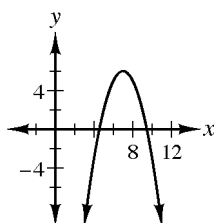
d. $y = \frac{1}{3}(x - 1)^2$



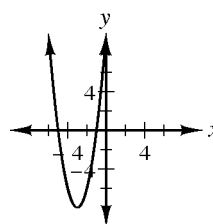
Problem continues on next page →

2-31. *Problem continued from previous page.*

e. $y = -(x - 7)^2 + 6$



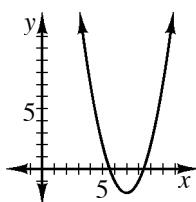
f. $y = 2(x + 3)^2 - 8$



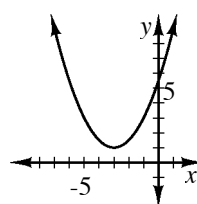
- g. Now take out your graphing calculator and check your predictions for the equations in parts (a) through (f). Did you make any mistakes? If so, describe the mistake and what you need to do in order to correct it.

2-32. Graph each equation below without making a table or using your graphing calculator. Look for ways to go directly from the equation to the graph. What information did you need to make a graph without using a table? How did you find that information from the equation? Be ready to share your strategies with the class. [**Students should answer that they need the vertex and stretch factor (or a -value). a: vertex at $(7, -2)$, opens up; b: vertex at $(-3, 1)$, opens up and compressed by one half.]**

a. $y = (x - 7)^2 - 2$

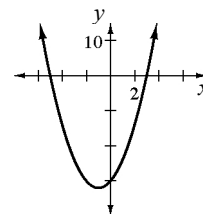


b. $y = 0.5(x + 3)^2 + 1$



2-33. In problem 2-32, you figured out that having an equation for a parabola in **graphing form** ($y = a(x - h)^2 + k$) allows you to know the vertex, the orientation, and the stretch factor, and that knowing these attributes allows you to graph without having to make a table. How can you make a graph without a table when the equation is given in **standard form** ($y = ax^2 + bx + c$)? Consider the equation $y = 2x^2 + 4x - 30$.

- What is the orientation of $y = 2x^2 + 4x - 30$? That is, does it open upward or open downward? How could you change the equation to make the graph open the opposite way? [**upward, $a > 0$, change 2 to -2**]
- What is the stretch factor of $y = 2x^2 + 4x - 30$? Justify your answer. [**2, it is the coefficient of x^2**]
- Can you identify the vertex of $y = 2x^2 + 4x - 30$ by looking at the equation? If not, talk with your team about strategies you could use to find the vertex without using a table or graphing calculator and then apply your new strategy to the problem. If your team is stuck consider doing parts (i) through (iii) below. [**The vertex is $(-1, -32)$.**]
 - What are the x -intercepts of the parabola? [**$(-5, 0)$ and $(3, 0)$**]
 - Where is the vertex located in relation to the x -intercepts? Can you use this relationship to find the x coordinate of the vertex? [**The vertex is directly between the x -intercepts. Its x coordinate is -1 .**]
 - Use the x -coordinate of the vertex to find its y -coordinate. [**-32**]
- Make a quick graph of $y = 2x^2 + 4x - 30$ and write its equation in graphing form. [**See graph at right. $y = 2(x + 1)^2 - 32$**]



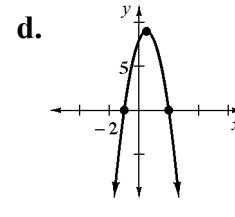
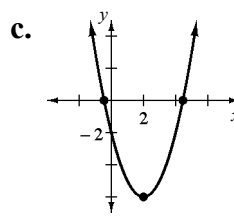
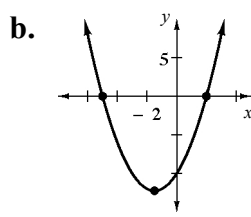
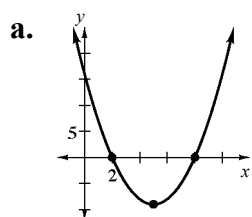
2-34. Rewrite each equation in graphing form and then sketch a graph. Label each sketch so that it is possible to connect it to the equation. [**a: $(8, 0), (2, 0), x\text{-avg.} = 5, f(5) = -9$, vertex $(5, -9)$, $p(x) = (x - 5)^2 - 9$; b: $(-5, 0), (2, 0), x\text{-avg.} = -\frac{3}{2}, f(-\frac{3}{2}) = -\frac{49}{4}$, vertex $(-1.5, -12.25)$, $f(x) = (x + \frac{3}{2})^2 - \frac{49}{4}$; c: $(2 \pm \sqrt{6}, 0)$, $x\text{-avg.} = 2, g(2) = -6, (2, -6)$, $g(x) = (x - 2)^2 - 6$; d: $(2, 0), (-1, 0), x\text{-avg.} = \frac{1}{2}, h(\frac{1}{2}) = 9$, vertex $(\frac{1}{2}, 9)$, $h(x) = -4(x - \frac{1}{2})^2 + 9$; See graphs below.]**

a. $p(x) = x^2 - 10x + 16$

b. $f(x) = x^2 + 3x - 10$

c. $g(x) = x^2 - 4x - 2$

d. $h(x) = -4x^2 + 4x + 8$





METHODS AND MEANINGS

Forms of Quadratics

There are three main forms of a quadratic function: standard form, factored form, and graphing form. Study the examples below. Assume that $a \neq 0$ and that the meaning of a , b , and c are different for each form below.

Standard form: $f(x) = ax^2 + bx + c$. The y -intercept is $(0, c)$.

Factored form: $f(x) = a(x + b)(x + c)$. The x -intercepts are $(-b, 0)$ and $(-c, 0)$.

Graphing form (vertex form): $f(x) = a(x - h)^2 + k$. The vertex is (h, k) .

Similarly, there are three forms of a single-variable quadratic equation.

Standard form: Any quadratic equation written in the form $ax^2 + bx + c = 0$.

Factored form: Any quadratic equation written in the form $a(x + b)(x + c) = 0$.

Perfect Square form: Any quadratic equation written in the form $(ax - b)^2 = c^2$.

Solutions to a quadratic equation can be written in **exact form (radical form)** as in:

$$x = \frac{-3 + \sqrt{5}}{2} \quad \text{or} \quad x = \frac{-3 - \sqrt{5}}{2}$$

Solutions can also be estimated and written in **approximate decimal form**:

$$x = -0.38 \quad \text{or} \quad x = -2.62$$



2-35. Solve each of the following equations *without using the Quadratic Formula*.

a. $y^2 - 6y = 0$
[$y = 0$ or 6]

b. $n^2 + 5n + 7 = 7$
[$n = 0$ or -5]

c. $2t^2 - 14t + 3 = 3$
[$t = 0$ or 7]

d. $\frac{1}{3}x^2 + 3x - 4 = -4$
[$x = 0$ or -9]

- e. Zero is one of the solutions of each of the above equations. What do all of the above equations have in common that causes them to have zero as a solution?
[**There is no constant term when each equation is set equal to zero, so the variable is a common factor after like terms are collected.**]

- 2-36. Find the vertex of each of the following parabolas by averaging the x -intercepts. Then write each equation in graphing form.
- | | |
|--|---|
| a. $y = (x - 3)(x - 11)$
[(7, -16), $y = (x - 7)^2 - 16$] | b. $y = (x + 2)(x - 6)$
[(2, -16), $y = (x - 2)^2 - 16$] |
| c. $y = x^2 - 14x + 40$
[(7, -9), $y = (x - 7)^2 - 9$] | d. $y = (x - 2)^2 - 1$
[(2, -1)] |
- 2-37. Did you need to average the x -intercepts to find the vertex in part (d) of the preceding problem?
- What are the coordinates of the vertex for part (d)? [(2, -1)]
 - How do these coordinates relate to the equation? [**When $x = 2$, $(x - 2)^2$ will equal zero and $y = -1$, the smallest possible value for y in the equation. So the y -value of the vertex is the minimum value in the range of the function.**]
- 2-38. Scientists can estimate the increase in carbon dioxide in the atmosphere by measuring increases in carbon emissions. In 1998 the annual carbon emission was about eight gigatons (a gigaton is a billion metric tons). Over the last several years, annual carbon emission has been increasing by about one percent.
- At this rate, how much carbon will be emitted in 2010? [**9.015 gigatons**]
 - Write a function, $C(x)$, to represent the amount of carbon emitted in any year starting with the year 2000. [**$C(x) = 8(1.01)^{(x+2)}$ if x represents years since 2000 or $8.16(1.01)^x$.**]
- 2-39. Make predictions about how many places the graph of each equation below will touch the x -axis. You may first want to rewrite some of the equations in a more useful form. [**Let students figure out what form is more useful.**]
- | | |
|--------------------------------------|-------------------------------------|
| a. $y = (x - 2)(x - 3)$ [2] | b. $y = (x + 1)^2$ [1] |
| c. $y = x^2 + 6x + 9$ [1] | d. $y = x^2 + 7x + 10$ [2] |
| e. $y = x^2 + 6x + 8$ [2] | f. $y = -x^2 - 4x - 4$ [1] |
- Check your predictions with your calculator.
 - Write a clear explanation describing how you can tell whether the equation of a parabola will touch the x -axis at only one point. [**If the factored version includes a perfect-square binomial factor, the parabola will touch at one point only.**]

2-40. Simplify each of the following expressions. Be sure that your answer has no negative or fractional exponents.

a. $64^{1/3}$
[4]

b. $(4x^2y^5)^{-2}$
[$\frac{1}{16x^4y^{10}}$]

c. $(2x^2 \cdot y^{-3})(3x^{-1}y^5)$
[$6xy^2$]

2-41. Suppose you have a 3 by 3 by 3 cube. It is painted on all six faces and then cut apart into 27 pieces, each a 1 by 1 by 1 cube. If one of the cubes is chosen at random, what is the probability that:

a. Three sides are painted? [$\frac{8}{27}$]

b. Two sides are painted? [$\frac{12}{27}$]

c. One side is painted? [$\frac{6}{27}$]

d. No sides are painted? [$\frac{1}{27}$]

Lesson 2.1.4 How can I rewrite it in graphing form?

Rewriting in Graphing Form

Lesson Objective: Students will review, use, and compare two methods for rewriting quadratic equations from standard form to graphing form, averaging the intercepts and completing the square.

Mathematical Practices: construct viable arguments and critique the reasoning of others, use appropriate tools strategically, look for and make use of structure

Length of Activity: One to two days (approximately 50–100 minutes). See the Note in Core Problems below.

Core Problems: Problem 2-42, 2-46, and 2-47

Note: Classes that need to review completing the square will need to take an extra day on this lesson. First complete the “Further Guidance” problems 2-43 through 2-45, and then move on to problems 2-42, 2-46, and 2-47. Otherwise, skip over the “Further Guidance” problems and do the Core Problems above in one day.

Materials: Lesson 2.1.4 Resource Page, one per student

Algebra Tiles, optional, for the Further Guidance. The tiles will not be used again for this course, so it may be best if they are borrowed from an Algebra I teacher rather than purchased.

The Further Guidance uses the same types and number of tiles provided in a set of Algebra Models™ (made by Classroom Products™). Classroom sets of the tiles may be ordered from CPM. Each student (or pair of students) should have at least 20 unit tiles, 12 x -tiles, and $5x^2$ -tiles. Two sets of Algebra Models™

contain enough tiles for a team of four students.

Tile storage, optional. Some teachers use zipper-seal bags or envelopes. Others use kitchen storage containers.

Display tiles that match students' manipulatives, optional. Also available from Classroom Products™ and included with a classroom set. Alternatively tiles can be placed on a document camera. Some teachers have made oversized tiles out of cardboard and attached adhesive magnets to the back to demonstrate tiles on a steel whiteboard.

Suggested Lesson Activity:

Consider starting the class with Reciprocal Teaching. The first person explains what the standard form of a quadratic equation is and the second person explains what the graphing form looks like. You can also have students review the ways to find the vertex by the averaging intercepts, quadratic formula and the completing the square methods. Direct teams to problem 2-42, in which they will rewrite quadratic equations given in standard form into vertex form using both the strategy of averaging intercepts and by completing the square. To average the intercepts to locate the vertex, some students may need reminding that they can use the Quadratic Formula to identify the x -intercepts, but once they have found them they should see that it is easy to find the point halfway between.

If students do not remember the method of completing the square as they attempt part (a) of problem 2-42, direct them to the Further Guidance problems 2-43 through 2-45 before they go back to complete problems 2-42 and 2-46. This lesson will take two days if you direct students to the Further Guidance.

Teams doing the Further Guidance should work through problem 2-43. If each team has algebra tiles they can make the same manipulations as stated in the problem. As they complete the square, students will have too few tiles to make the square or extra tiles left over. Teams can verify the results by expanding $(x + 4)^2 - 6$ and by graphing as suggested in the problem.

When teams try to construct a square in problem 2-44, they will have extra tiles left over. They can use tiles to work through the problem or use a diagram. Problem 2-45 looks at what happens when the linear term, that is, bx , is not even. If teams are stuck on how much is missing, ask them to think about the area of the square that would be formed by the unit pieces. Once students realize that the area is 2.5^2 , they can figure out how many unit pieces are missing. Teams should then be able to return to problems 2-42 and then complete problem 2-46.

After teams complete problem 2-42, with or without the use of the Further Guidance, direct them to problem 2-46, where they will use the method of their choice to write each equation in graphing form. You could do this as a Pairs Check. Then have teams move on to problem 2-47.

In part (b) of problem 2-47, students will need to solve the quadratic equation for x when $y = 0$. Let them wrangle with this in their teams for awhile. Then, if it appears the class needs more guidance and structure, move them on to part (c) and pass out the Lesson 2.1.4 Resource Page, which has teams justify each step in the solution. Regardless of whether you use the structure of the Resource Page or not, you will need to discuss with the whole class the affect of the absolute value operation when moving from step (e) to step (f) of the Resource Page. See the Mathematical Background section for more information. This important concept will come up again when students graph non-functions in Lesson 2.2.4.

If your students were able to complete problems 2-42 and 2-46 without the review in Further Guidance, they should have time to continue to problems 2-48 and 2-49, where they will find the general vertex form directly from the general standard form of a quadratic function and then compare their results to verify that this method produces an equivalent equation. You could split the class into two sets of teams, half of them averaging the intercepts and half of them completing the square. Then lead a discussion to compare and verify results. Problem 2-46 can be done as a Pairs Check.

In general, teams that work on problems 2-48 and 2-49 may have difficulty with simplifying some of the results they get. If that is the case, you might lead a class discussion based on what they have done and, as a class, reach agreement on a way to write the equations and the vertex.

Note that while knowing the x -value of the vertex is useful because it gives you the axis of symmetry, $x = -\frac{b}{2a}$, the general form of the vertex is not really useful as a formula because the y -coordinate of the vertex is difficult to remember or use. Teams should realize that it makes more sense to go through the process of averaging intercepts or completing the square rather than to substitute the values for a , b , and c in a memorized general form of the vertex.

Closure:
(10 minutes)

With 10 minutes left in class, have several teams present their results for problem 2-46, or for problems 2-48 and 2-49. Encourage students to ask the presenters questions to make sure they understand each other's thinking.

Then ask teams to write the graphing form for $y = x^2 - 12x + 4$ and ask them to discuss which strategy they prefer and why.

Universal Access:

Problem 2-42:

Since some students spend an inordinate amount of time drawing and labeling axes, it is a good idea to provide ready-made graphs (only label the x and y -axes) for these students. Avoid numbering the graphs; this

is an important concept students should learn.

If EL and/or special needs students have difficulty averaging intercepts and/or using the Quadratic Formula, you may consider allowing them to use the calculator OR you could pair up a stronger student with a weaker student.

If students are unfamiliar with completing the square, utilize the Further Guidance problems.

Problems 2-43, 2-44, 2-45:

These are Further Guidance problems that will provide the structure and scaffolding that EL and/or special needs students may need to review the strategy “completing the square.” Allowing struggling students to use algebra tiles will provide the tactile and visual aids some students need to make connections.

Problem 2-47:

EL and/or special needs students may need assistance understanding and justifying each step to solving the equation (particularly steps d, e, and f).

Math Notes – Methods and Meanings:

This provides a good opportunity to review two strategies to change equations to graphing form. Consider a way for students to organize and access these strategies.

Team Strategies:

When you approach a team, remember that it is important to engage all of the students in conversation with you. Specific choices you make about body language and eye contact can impact the conversation. Consider squatting down so that you are at the students’ level or, if there is an empty chair, you can actually join the team at the table.

Mathematical Background:

SOLVING EQUATIONS WITH SQUARE ROOTS

When solving $x^2 = 16$ students will often come up with the correct solution, $x = \pm 4$. But when asked why that is true, they erroneously answer that the $\sqrt{16}$ is ± 4 . However, the radical sign “ $\sqrt{}$ ” refers to the principal or positive root only. Thus, $\sqrt{16}$ equals 4 but not -4 . Therefore students often lack conceptual understanding of why the solution to $x^2 = 16$ is $x = \pm 4$.

The following approach may help develop conceptual understanding in students when solving equations that involve taking the square root of a squared variable expression, as in $x^2 = 16$, or even $25 = (x - 3)^2$.

Since the radical sign refers only to the positive root, when taking a square root we must assure that the result is positive. An expression is made positive by taking its absolute value. For example, solve:

$$25 = (x - 3)^2.$$

First, take the square root of both sides, resulting in

$$\sqrt{25} = \sqrt{(x-3)^2}.$$

Assure both sides are positive because the radical sign refers to the positive root. On the left, “25” is already positive. However, on the right “ $(x-3)$ ” could conceivably be negative. We must force it to be positive by using an absolute value, resulting in

$$5 = |x-3|.$$

Lastly, find both solutions to the absolute value equation.

$$5 = x-3 \text{ or } -5 = x-3$$

$$8 = x \quad \text{or} \quad -2 = x$$

In that same manner, $x^2 = 16$ can be rewritten as $|x| = 4$ and then $x = 4$ or $x = -4$.

With the approach of using an absolute value to force radicals that contain a variable to be positive, students are less likely to get confused about when they should use a “ \pm ” when taking a square root, and less likely to make the mistake $\sqrt{16} = \pm 4$.

Homework:

Problems 2-50 through 2-56

Note: If you use the Further Guidance and do this as a two-day lesson then assign the homework as follows:

Day 1: Problems 2-50 through 2-56

Day 2: Problems 2-57 through 2-63

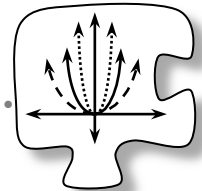
Throughout the text certain homework problems have been marked as “checkpoints” for efficient use of mathematical skills. An icon like the one at right marks each checkpoint problem.



Problem 2-53 is Checkpoint 2A for finding the distance between two points and finding the equation of a line. Students who have difficulty with this problem should get help and work on some additional practice problems. Additional practice problems for each checkpoint problem topic are available in the Core Connections, Algebra 2 Checkpoint Problems section at the back of the student text. For more information about using checkpoint problems, see the Preparing to Teach this Course tab at the front of this Teacher Edition.

Notes to Self:

2.1.4 How can I rewrite it in graphing form?



Rewriting in Graphing Form

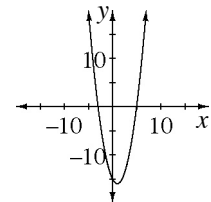
In Lesson 2.1.3, you used the method of averaging the intercepts to change the equation of a parabola from the standard form $f(x) = ax^2 + bx + c$ to the graphing form $f(x) = a(x - h)^2 + k$ by finding the x -intercepts and averaging them to find the x -value of the vertex. Next you substituted to find the y -value, and then used the coordinates of the vertex for h and k .

What can you say about a parabola that cannot be factored or that does not cross the x -axis? How can you write its equation in graphing form? In a previous course you may have learned how to complete the square for quadratics and this strategy can help you write the graphing form for a parabola.

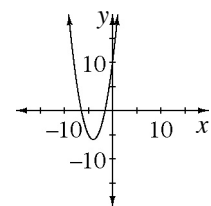
2-42. In this investigation you will compare two methods of changing a quadratic equation from standard form to graphing form.



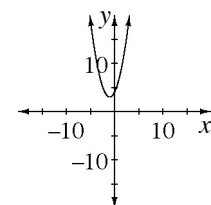
- a. Write the equation of the parabola $y = x^2 - 2x - 15$ in graphing form using two methods. First, use the method of averaging the intercepts. Then, use the method of completing the square. Find the x -intercept(s), the y -intercept(s), and the vertex of the parabola, and sketch the graph. [$y = (x - 1)^2 - 16$; Intercepts at $(-3, 0)$, $(5, 0)$, $(0, -15)$, vertex is at $(1, -16)$; graph is shown at right.]



- b. Write $y = x^2 + 8x + 10$ in graphing form. Find the intercepts and vertex, and sketch the graph. Do both strategies work for this parabola? [$y = (x + 4)^2 - 6$; Intercepts at $(-4 + \sqrt{6}, 0)$, $(-4 - \sqrt{6}, 0)$ and $(0, 10)$, vertex at $(-4, -6)$. Both strategies work because when the x -intercepts are averaged, the radicals are eliminated and the result is -4 , graph shown at right.]



- c. Can you use both methods to sketch $y = x^2 + 2x + 4$? Do both strategies still work? [$y = (x + 1)^2 + 3$; You cannot use the method of average the x -intercepts because you get the square root of a negative number when using the Quadratic Formula. This means that there is no solution for x and no x -intercepts. But you can still sketch the graph by completing the square and using the vertex $(-1, 3)$ and y -intercept $(0, 4)$; graph shown at right.]



- d. Discuss the two strategies with your team. Then respond to the following Discussion Points.

Discussion Points

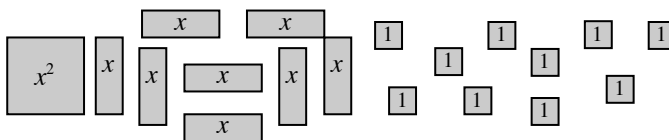
When does the method of averaging the intercepts work better?

When does the method of completing the square work better?

Which method was more efficient and why?

Further Guidance

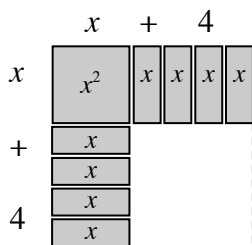
- 2-43. Jessica was at home struggling with her homework. She had missed class and could not remember how to **complete the square**. She was supposed to use the method to change $f(x) = x^2 + 8x + 10$ to graphing form. Then her precocious younger sister, Anita, who was playing with algebra tiles, said, “Hey, I bet I know what they mean.” Anita’s Algebra class had been using tiles to multiply and factor binomials. Anita explained, “ $f(x) = x^2 + 8x + 10$ would look like this,”



“Yes,” said Jessica, “I took Algebra 1 too, remember?”

Anita continued, “And you need to make it into a square!”

“OK,” said Jessica, and she arranged her tiles as shown in the picture below.



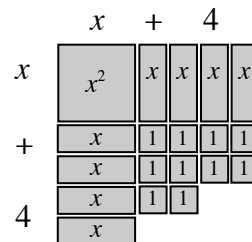
“Oh,” said Jessica. “So I just need 16 small unit tiles to fill in the corner.”

“But you only have 10,” Anita reminded her.

“Right, I only have ten,” Jessica replied. She put in the 10 small square tiles then drew the outline of the whole square and said:

“Oh, I get it! The **complete square** is $(x + 4)^2$ which is equal to $x^2 + 4x + 16$. But my original expression, $x^2 + 8x + 10$, has six fewer tiles than that, so what I have is $(x + 4)^2$, minus 6.”

“Yes,” said Anita. “You started with $x^2 + 8x + 10$, but now you can rewrite it as $x^2 + 8x + 10 = (x + 4)^2 - 6$.”

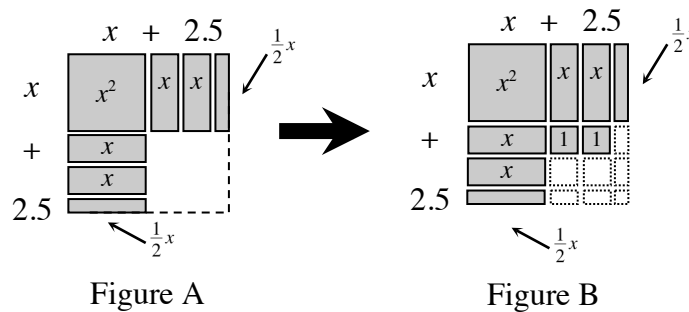


Use your graphing calculator to show that $f(x) = x^2 + 8x + 10$ and $f(x) = (x + 4)^2 - 6$ are equivalent functions. [Enter them both into the calculator and verify the tables are the same.]

- 2-44. Help Jessica with a new problem. She needs to complete the square to write $y = x^2 + 4x + 9$ in graphing form. Draw tiles to help her figure out how to make this expression into a square. Does she have too few or too many unit squares this time? Write her equation in graphing form. [$(x + 2)^2 = x^2 + 4x + 4$, so the original expression has 5 too many unit tiles; $y = (x + 2)^2 + 5$]

- 2-45. How could you complete the square to change $f(x) = x^2 + 5x + 2$ into graphing form? How would you split the five x -tiles into two equal parts?

Jessica decided to use force! She cut one tile in half, as shown below. Then she added her two unit tiles.



- How many unit tiles are in the perfect square? [$2.5 \cdot 2.5 = 6.25$]
- Does Jessica have too many or too few tiles in her original expression? How many? [**She has 4.25 tiles too few.**]
- Write the graphing form of the function. [$f(x) = (x + 2.5)^2 - 4.25$]
- Now use your work on problems 2-43 through part (b) of problem 2-45 to complete problem 2-42.

Further Guidance
section ends here.

- 2-46. Use the strategy of your choice to write each function below in graphing form.

- $f(x) = x^2 + 6x + 7$
[$f(x) = (x + 3)^2 - 2$]
- $f(x) = x^2 - 4x + 11$
[$f(x) = (x - 2)^2 + 7$]
- $f(x) = x^2 + 5x + 2$
[$f(x) = (x + 2.5)^2 - 4.25$]
- $f(x) = x^2 - 7x + 2$
[$f(x) = (x - 3.5)^2 - 10.25$]

2-47. How can you use a quadratic equation in graphing form to make a quick sketch of the parabola?

- What is the vertex and y-intercept of the graph of $y = (x - 3)^2 - 25$? Explain how you found the y-intercept. [**vertex is at (3, -25). y-intercepts (0, -16). To find the y-intercept, set $x = 0$ and evaluate y .**]
- Find the x intercepts of $y = (x - 3)^2 - 25$ algebraically. Explain how you found the x -intercepts. [**(-2, 0) and (8, 0), this can be done by setting y equal to 0 and solving for x .**]
- Obtain the Lesson 2.1.4 Resource Page and justify each step in solving the equation in part (b) for x when $y = 0$.
- Find the exact vertex, y-intercept, and x -intercepts of $y = (x + 5)^2 - 8 = 0$. Make a sketch of the parabola, then check your sketch with your graphing calculator. [**vertex (-5, -8), y-intercept (0, 17), find x -intercepts by solving $|x + 5| = \sqrt{8}$; x -intercepts at $(-5 - \sqrt{8}, 0)$ and $(-5 + \sqrt{8}, 0)$ or approximately $(-7.8, 0)$ and $(-2.2, 0)$ on the graph.**]

2-48. GENERALIZATION CHALLENGE I

Jeremy had an idea, “What if we start with the equation in standard form? I bet we can find a way to get the vertex so we don’t have to solve the equation or change it to graphing form every time. Let’s start with $y = ax^2 + bx + c$ and find the x -intercepts for its graph.”

- Solve the general quadratic equation to get the x -intercepts.
[$\left(\frac{-b + \sqrt{b^2 - 4ac}}{2a}, 0 \right), \left(\frac{-b - \sqrt{b^2 - 4ac}}{2a}, 0 \right)]$
- How can you use the x -intercepts to find the line of symmetry? What is the line of symmetry? [**Average the intercepts to get the point halfway between, thus $x = -\frac{b}{2a}$.**]
- What is the vertex of the graph of $y = ax^2 + bx + c$? [$(-\frac{b}{2a}, c - \frac{b^2}{4a})$]

2-49. GENERALIZATION CHALLENGE II

Jessica had another idea. She said, “*Couldn’t we just start with $y = ax^2 + bx + c$ and complete the square?*”

Jeremy objected, “*But what do we do about the ax^2 ?*”

- a. How can you rewrite the equation so the coefficient of x is one? Do it.
[**Divide by a ; $\frac{y}{a} = x^2 + \frac{b}{a}x + \frac{c}{a}$]**
- b. What is the square that needs to be completed? [$(x + \frac{b}{2a})^2$]
- c. What expression has to be added to complete the square? [$-\frac{b^2}{4a^2}$]
- d. Rewrite the equation in the form $\frac{y}{a} = (x + \underline{\hspace{1cm}})^2 + \underline{\hspace{1cm}}$, and multiply by a .
[$y = a(x + \frac{b}{2a})^2 + c - \frac{b^2}{4a}$]
- e. What is the vertex? Is the result the same as in problem 2-48? How do you know? [$(-\frac{b}{2a}, c - \frac{b^2}{4a})$, **either the expressions are identical or can be shown algebraically to be equivalent.**]



METHODS AND MEANINGS

Finding Graphing Form and Vertex of Parabolas

Starting with the graphing form of a quadratic equation and rewriting it to get standard form is straightforward algebra. But starting with a quadratic equation in standard form and rewriting it to get graphing form is more difficult. You have used two strategies to rewrite standard form in graphing form: **averaging the intercepts** and **completing the square**.

For example, change $y = x^2 + 3x - 10$ to graphing form.

Averaging Intercepts:

x -intercept is where $y = 0$.

$$\text{Solve } 0 = x^2 + 3x - 10$$

$$0 = (x + 5)(x - 2)$$

The x -intercepts are $(-5, 0)$ and $(2, 0)$

$$\text{Axis of symmetry: } x = \frac{-5+2}{2} = -\frac{3}{2}$$

By evaluating for x , find $y = -\frac{49}{4}$

$$\text{Vertex: } \left(-\frac{3}{2}, -\frac{49}{4}\right)$$

Completing the Square:

Make a perfect square from $x^2 + 3x$:

1.5	1.5x	2.25
x	x^2	1.5x
	x	1.5

$$(x + 1.5)^2 = x^2 + 3x + 2.25$$

The original expression, $x^2 + 3x - 10$, is 12.25 fewer than $(x + 1.5)^2$. So,

$$y = x^2 + 3x - 10 = (x + 1.5)^2 - 12.25$$

Since graphing form is $y = a(x - h)^2 + k$, vertex: $(-1.5, -12.25) = \left(-\frac{3}{2}, -\frac{49}{4}\right)$

In general, how are h and k in $y = a(x - h)^2 + k$ related to b and c in

$y = ax^2 + bx + c$? By averaging the two solutions given by the Quadratic

Formula, or by completing the square, the axis of symmetry is $y = -\frac{b}{2a}$, and

the vertex is $\left(-\frac{b}{2a}, c - \frac{b^2}{4a}\right)$, so $h = -\frac{b}{2a}$ and $k = c - \frac{b^2}{4a}$.



- 2-50. For each quadratic function below use the method of completing the square or averaging the intercepts to rewrite it in graphing form. Then, state the axis of symmetry and give the vertex of each parabola. Try to use each method at least once.

a. $f(x) = x^2 + 6x + 15$
 $[f(x) = (x + 3)^2 + 6,$
 $(-3, 6), x = -3]$

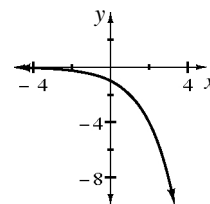
b. $y = x^2 - 4x + 9$
 $[y = (x - 2)^2 + 5,$
 $(2, 5), x = 2]$

c. $f(x) = x^2 - 8x$
 $[f(x) = (x - 4)^2 - 16,$
 $(4, -16), x = 4]$

d. $y = x^2 + 7x - 2$
 $[y = (x + 3.5)^2 - 14.25,$
 $(-3.5, -14.25), x = -13.5]$

- 2-51. Represent the number you would have to add to an expression of the form $x^2 + bx$ to make a complete square. $[\frac{b^2}{4}]$

- 2-52. How is $y = 2^x$ different from $y = -(2^x)$? Sketch the graph of $y = -(2^x)$. **[The second graph is a reflection of the first across the x -axis; see graph at right.]**



- 2-53. Throughout this book, key problems have been selected as “checkpoints.” Each checkpoint problem is marked with an icon like the one at left. These checkpoint problems are provided so that you can check to be sure you are building skills at the expected level. When you have trouble with checkpoint problems, refer to the review materials and practice problems that are available in the Checkpoint Materials section at the back of your book.



This problem is a checkpoint for finding the distance between two points and finding the equation of a line. It will be referred to as Checkpoint 2A.

For each pair of points, determine the distance between them. Then find the equation for a line through them.

a. $(-2, 4)$ and $(4, 7)$
 $[\sqrt{45} = 3\sqrt{5} \approx 6.71; y = \frac{1}{2}x + 5]$

b. $(3, 4)$ and $(3, -1)$
 $[5; x = 3]$

c. $(-7, 20)$ and $(3, -5)$
 $[\sqrt{725} \approx 26.93; y = -\frac{5}{2}x + \frac{5}{2}]$

d. $(1, -2)$ and $(5, -2)$
 $[4; y = -2]$

Problem continues on next page →

2-53. *Problem continued from previous page.*

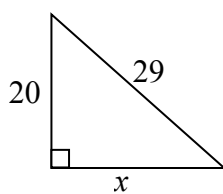
Check your answers by referring to the Checkpoint 2A materials located at the back of your book.

If you needed help solving these problems correctly, then you need more practice. Review the Checkpoint 2A materials and try the practice problems. Also, consider getting help outside of class time. From this point on, you will be expected to do problems like these quickly and easily.

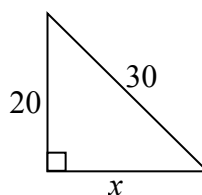
2-54. The Quadratic Formula can be used to help solve $4x^3 + 23x^2 - 2x = 0$. Show or explain how. [**After x is factored out, the other factor is a quadratic equation. After using the Quadratic Formula the solutions are $x = \frac{-23 \pm \sqrt{561}}{8}$ or 0.**]

2-55. Find the value of x . [**a: $x = 21$, b: $x = 10\sqrt{5} \approx 22.4$, c: $x = 50$**]

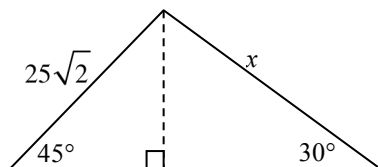
a.



b.

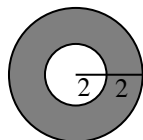


c.

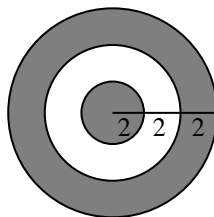


2-56. A dart hits each of these dartboards at random. What is the probability that the dart will land in the unshaded area? [**a: $\frac{1}{4}$, b: $\frac{1}{3}$**]

a.



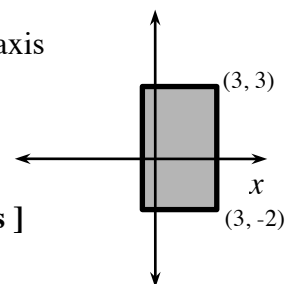
b.



2-57. If $\frac{2}{3}$ of A is $\frac{5}{12}$, and $\frac{4}{3}$ of B is $\frac{8}{9}$, which is larger, A or B? [**B**]

y

- 2-58. Examine the diagram at right. Imagine spinning the rectangle around the y-axis. Think of a rectangular flap attached to the y-axis so that the rectangle will revolve around the y-axis.



- Draw the resulting shape. [**A cylinder**]
 - Find the volume of this shape. [**$45\pi = 141.37$ cubic units**]
- 2-59. What is a line of symmetry?
- Draw a figure that has a line of symmetry. [**Answers vary.**]
 - Draw a figure that has *two* lines of symmetry. [**Answers vary.**]
 - Can you find a basic geometric shape that has an infinite number of lines of symmetry? [**A circle.**]

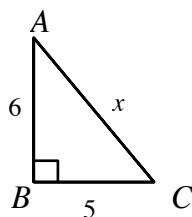
- 2-60. Find the point where $y = 3x - 1$ intersects $2y + 5x = 53$. [**(5, 14)**]

- 2-61. Lettie just got her driver's license. Her friends soon nicknamed her "Leadfoot" because she is always going 80 mph on the freeway even though the speed limit is 65 mph.

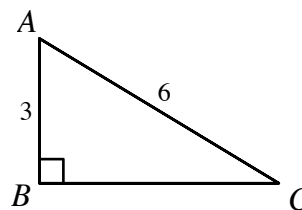
- At this speed, how long will it take her to travel 50 miles?
[**0.625 hours or about 37.5 minutes.**]
- How long would it take her if she drove the 50 miles at 65 mph?
[**0.77 hours or about 46.2 minutes.**]
- Speeding tickets carry fines of about \$200 and usually increase the cost of insurance. If Lettie gets a ticket on this trip, then what would be her cost per minute of time saved? [**About \$22.99 per minute.**]

- 2-62. Solve for the indicated value. Leave your answer in exact form.
[**a: $\sqrt{61}$, b: 30° , c: $\tan^{-1}(\frac{4}{5})$, d: $5\sqrt{3}$**]

- a. $x =$ _____



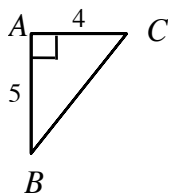
- b. $m\angle C =$ _____



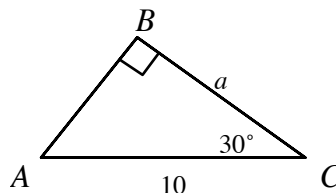
Problem continues on next page →

2-62. *Problem continued from previous page.*

c. $m\angle B = \underline{\hspace{2cm}}$



d. $a = \underline{\hspace{2cm}}$



2-63. Below are two situations that can be described using exponential functions. They represent a small sampling of the situations where quantities grow or decay by a constant percentage over equal periods of time. For each situation:

- Find an appropriate unit of time (such as days, weeks, years).
 - Find the multiplier that should be used.
 - Identify the initial value.
 - Write an exponential equation in the form $f(x) = ab^x$ that represents the growth or decay.
- a. A house purchased for \$120,000 has an annual appreciation of 6%.
[**Years; 1.06; 120,000; $120000(1.06)^x$**]
- b. The number of bacteria present in a colony is 180 at noon, and it increases at a rate of 22% per hour. [**Hours; 1.22; 180; $180(1.22)^x$**]

Lesson 2.1.5 How can I model the data?

Mathematical Modeling with Parabolas

Lesson Objective: Students will learn how to write quadratic equations for situations using the graphing form of the parabola $y = a(x - h)^2 + k$. Specifically, students will develop an algebraic strategy for finding the value of the stretch factor, a .

Mathematical Practices: model with mathematics, look for and make use of structure

Length of Activity: One day (approximately 50 minutes)

Core Problems: Problems 2-64 and 2-66 (with 2-65 if using “Further Guidance”)

Materials: None

Suggested Lesson Start teams on problem 2-64 using a Teammates Consult, which

Activity:

challenges students to write the equation of a parabola based on a situation. As you circulate, you might find that teams are stuck right away because they are unsure of where to start. This usually means that teams do not realize they can choose where to place their x - and y -axes. To help teams decide, you might ask, “*What would be a convenient location for the origin?*” or “*What if the y -axis were here instead? Would that make any difference? Why?*” Since the placement of the origin does not matter, there are at least three likely possibilities for equations. If you prefer a more guided approach, direct teams to problem 2-65.

Expect students to recognize the shape of the situation as a parabola and to conclude that it can be modeled with the equation $y = a(x - h)^2 + k$. Teams should realize that they know the vertex and thus can replace h and k in the equation. Some teams might think they are finished here. Help students realize that they need to find the stretch factor a by asking, “*How do you know your equation fits the situation? What can you change about the equation so that the vertex stays the same and the equation fits the data? How can you use what you know about the equation so far and your information about the situation to find a ?*” Students should substitute a data point into their equation to calculate the value of a . Ask, “*Have you used a similar process with other families of equations?*” to help students connect this process with substituting a point into a linear equation ($y = mx + b$) or an exponential equation ($y = ab^x$) to find the value of the unknown parameter.

Once you have teams with different equations (because they made different choices about axes placement) have them share their processes with the whole class. Students should see that the value of a is the same regardless of the choice of the axes location. Ask students why it makes sense for all of these equations to have the same value for a .

By the end of this discussion, students should understand that it is possible to model the same situation with many different equations depending on the chosen location of the axes.

Move teams on to problem 2-66, in which they will find the equation to model the path of a soccer ball. Here they will need to make their own diagram and again choose where to place their axes.

If time permits, problem 2-67 provides another opportunity for students to practice finding the equation of a parabola.

**Closure:
(10 minutes)**

Lead a brief discussion about the different forms that students know for writing the equation of a parabola and what can be learned from each form. These are standard form: $y = ax^2 + bx + c$, graphing form: $y = a(x - h)^2 + k$, and factored form: $y = a(x + b)(x + c)$. Allow students a few minutes to record their ideas in their Learning Logs in problem 2-68.

Universal Access:**Problem 2-64:**

EL and/or special needs students may not be familiar with the term “jackrabbit.” A quick Wikipedia or Google search *before* teaching the lesson will provide students with a real-life representation.

Problem 2-65:

This Further Guidance problem will provide the structure and scaffolding that EL and/or special needs students may need to come up with their equation for the Jumping Jackrabbits.

Problem 2-66:

Some students may struggle with creating an equation to model the data. Be prepared to provide prompts and cues to move learning forward.

Team Strategies:

Since there are many different problems here that are all related, it might be a good time to use a study team strategy called Red Light, Green Light. After the team completes each problem, they need to stop (Red Light) and check their answer with the teacher (or with the answers that the teacher has located somewhere in the room). If their answer is correct, they go (Green Light) to the next problem and repeat the process.

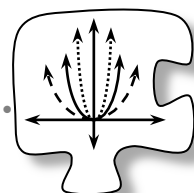
Homework:

Problems 2-69 through 2-75

Note: Problem 2-74 introduces the idea of using (h, k) as a **locator point**. This idea is revisited in Lesson 2.2.4.

2.1.5 How can I model the data?

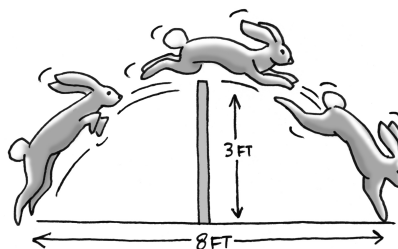
Mathematical Modeling with Parabolas



In the past few lessons, you have determined how to move graphs of parabolas around, that is, to transform them, on a set of axes. You have also learned how to write quadratic equations in graphing and in standard form. In this lesson you will put these new skills to work as you use parabolas and their equations to model situations.

2-64. JUMPING JACKRABBITS

The diagram at right shows a jackrabbit jumping over a three-foot-high fence. To just clear the fence, the rabbit must start its jump at a point four feet from the fence.



Sketch the situation and write an equation that models the path of the jackrabbit. Show or explain how you know your sketch and equation fit the situation.

[Some possibilities: $y = -\frac{3}{16}x^2$, $y = -\frac{3}{16}x^2 + 3$, $y = -\frac{3}{16}(x - 4)^2 + 3$]

Discussion Points

How can we make a graph fit this situation?

What information do we need in order to find an equation?

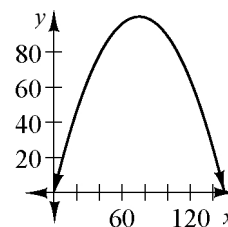
How can we be sure that our equation fits the situation?

Further Guidance

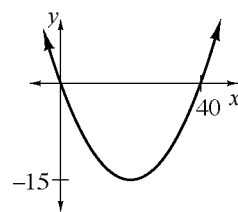
- 2-65. Sketch the path of the jackrabbit on your paper. Choose where to place the x - and y -axes in your diagram so that they make sense and make the problem easier. Label as many points as you can on your sketch.
- What is the shape of the path of the jackrabbit? What kind of equation would best model this situation? [**A parabola, a quadratic of the form $y = a(x - h)^2 + k$**]
 - What point on your graph can tell you about the values of h and k in the equation? Write the values for h and k into the general equation. Is your equation finished? [**The vertex is (h, k) . Possible equations include $y = ax^2$, $y = a(x - 4)^2 + 3$, $y = ax^2 + 3$. The equation is not finished, as a value for a is still needed.**]
 - With your team, find a strategy to find the exact value of a . Will any of the points on your diagram help? Be prepared to share your strategy with the class. [**Strategies vary. $a = -\frac{3}{16}$**]
 - What are the domain and range for your model? [**Domain and range should include only those values that correspond to the position of the rabbit from the beginning to the end of its jump.**]
 - Did any team in your class get a different equation? If so, write down their equation and show how it can also model the path of the jackrabbit. What choices did that team make differently that resulted in the different equation? [**Equations will vary depending on the choice of axes location.**]

===== *Further Guidance* =====
section ends here.

- 2-66. When Ms. Bibbi kicked a soccer ball, it traveled a horizontal distance of 150 feet and reached a height of 100 feet at its highest point. Sketch the path of the soccer ball and find an equation of the parabola that models it. [**See possible graph at right.** Possible equations include $y = -\frac{4}{225}(x - 75)^2 + 100$, $y = -\frac{4}{225}x^2 + 100$, $y = -\frac{4}{225}x^2$]



- 2-67. At the skateboard park, the hot new attraction is the *U-Dip*, a cement structure embedded into the ground. The cross-sectional view of the *U-Dip* is a parabola that dips 15 feet below the ground. The width at ground level, its widest part, is 40 feet across. Sketch the cross-sectional view of the *U-Dip*, and find an equation of the parabola that models it. [See possible graph at right. $y = \frac{3}{80}x^2 - 15$ or $y = \frac{3}{80}(x - 20)^2 - 15$ or $y = \frac{3}{80}x^2$ are three possible answers depending on where the axes are placed.]



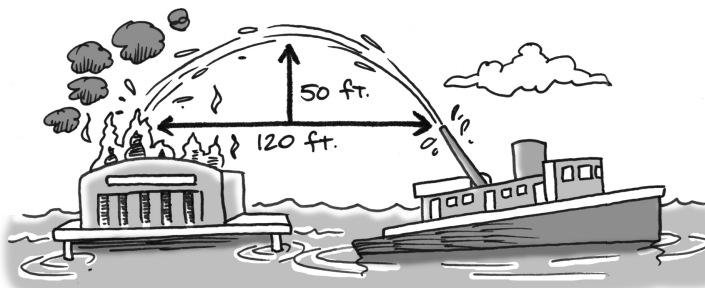
- 2-68. LEARNING LOG

With your team, discuss all of the different forms you know for the equation of a parabola. In your Learning Log, write down each form, along with a brief explanation of how that form is useful. Title this entry, “Forms of a Quadratic Function” and label it with today’s date.



- 2-69. FIRE! CALL 9-1-1!

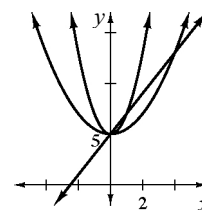
A fireboat in the harbor is helping put out a fire in a warehouse on the pier. The distance from the barrel (end) of the water cannon to the roof of the warehouse is 120 feet, and the water shoots up 50 feet above the barrel of the water cannon.



Sketch a graph and find an equation of the parabola that models the path of the water from the fireboat to the fire. Give the domain and range for which the function makes sense in relation to the fireboat. [Possible equations include $y = -\frac{1}{72}(x - 60)^2 + 50$, $y = -\frac{1}{72}x^2 + 50$, and $y = -\frac{1}{72}x^2$; domain and range should include only those values that correspond to the water passing between the boat and the warehouse.]

- 2-70. Draw accurate graphs of $y = 2x + 5$, $y = 2x^2 + 5$, and $y = \frac{1}{2}x^2 + 5$ on the same set of axes. Label the intercepts.

[See graph at right.]



- a. In the equation $y = 2x + 5$, what does the 2 tell you about the graph? [**It is the slope.**]
- b. Is the 2 in $y = 2x^2 + 5$ also the slope? Explain. [**No, because only lines have (constant) slopes. This 2 is the stretch factor.**]

- 2-71. Think about how you might sketch a parabola on a graph.

- a. Do the sides of a parabola ever curve back in like the figure at right? Explain your reasoning. [**No. Reasons vary, but may include: because there is only one height for each x or because it takes bigger x -values to get bigger y -values.**]
- b. Do the sides of the parabola approach straight vertical lines as shown in the figure at right? (In other words, do parabolas have asymptotes?) Give a reason for your answer. [**No. Reasons vary, but may include: because the domain is unlimited (any number can be squared).**]



- 2-72. Find the equation of an exponential function that passes through each pair of points.

- a. $(2, 9)$ and $(4, 324)$ [$y = 0.25 \cdot 6^x$]
- b. $(-1, 40)$ and $(0, 12)$ [$y = 12 \cdot 0.3^x$]

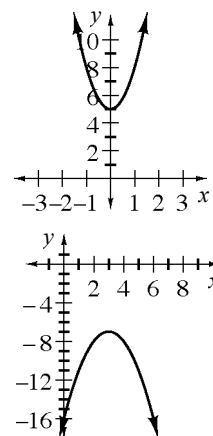
- 2-73. Find the x - and y -intercepts of the graphs of the two equations below.

- a. $y = 2x^2 + 3x - 5$ [x : $(1, 0), (-\frac{5}{2}, 0)$, y : $(0, -5)$]
- b. $y = \sqrt{2x - 4}$ [x : $(2, 0)$, y : none]

- 2-74. The vertex of a parabola, point (h, k) , locates its position on the coordinate graph. The vertex thus serves as a **locator point** for a parabola. Other families of functions that you will be investigating in this course will also have locator points. These points have different names, but the same purpose for each different type of graph. They help you place the graph on the axes.

Sketch graphs for both of the following equations. On each sketch, label the locator point. [See graphs at right.]

- a. $y = 3x^2 + 5$ [**stretched parabola, vertex $(0, 5)$**]
- b. $f(x) = -(x - 3)^2 - 7$ [**inverted parabola, vertex $(3, -7)$**]



2-75. If $g(x) = x^2 - 5$, find the value(s) of x so that:

a. $g(x) = 20$

[$x = \pm 5$]

b. $g(x) = 6$

[$x = \pm\sqrt{11}$]

Lesson 2.2.1 How can I transform any graph?

Transforming Other Parent Graphs

Lesson Objective:

Students will transform the graphs of $y = b^x$, $y = \frac{1}{x}$, $y = \sqrt{x}$, $y = |x|$, and $y = x^3$.

Mathematical Practices:

reason abstractly and quantitatively, look for and make use of structure, look for and express regularity in repeated reasoning

Length of Activity:

Three days (approximately 150 minutes)

Core Problems:

Problems 2-76 and 2-80 (with 2-77 through 2-79 if using “Further Guidance”)

Materials:

Poster-making materials

Suggested Lesson Activity:

Lesson Overview: This lesson consists of a three-part investigation followed by presentations in the form of stand-alone posters. It is designed to span three days. The investigation involves investigating parent graphs, transforming parent graphs, and writing general equations.

If you are concerned about time, it is possible to assign Part 1 of this investigation (the parent graphs) as homework the night before the lesson. This will allow you to start by having students share ideas about their parent graph with their team and then begin investigating transformations. It is recommended, however, that you use this option only if you are confident that all of your students will do the assignment.

Day 1: Start class by having a volunteer read the lesson introduction and then having students do a Think-Pair-Share about everything they know about transforming $y = x^2$. End with a whole-class discussion. Ask student volunteers to describe how they can move the graph of $y = x^2$. Now is a good time to introduce the graph of $y = x^2$ as the parent graph of a parabola. Ask, “Why do you think we refer to $y = x^2$ as the parent graph?” Students should be developing the idea that the parent graph $y = x^2$ is the simplest graph of a parabola, and every other parabola that opens up or down can be seen as the result of changes to the equation $y = x^2$. You might finish the discussion by highlighting for students that they have spent the first section of this chapter making shifts to the parent graph $y = x^2$, looking at the relationship between the equation of a parabola and its graphs, and modeling situations using parabolas. Let

them know that now they are going to do all these same things with some other functions and at the end of this section they will be able to transform any function.

Problem 2-76 asks each student in the team to choose one of the new parent graphs to investigate. You can have each team member do one of the first four equations and then assign a different value for b to each team member and have all four of them do the investigation of $y = b^x$. Students should use only positive integer values for b for this investigation. To complete this part of the investigation, students should make a graph, identify its domain and range, and label any important points or asymptotes. You can choose to do this as an open-ended investigation or to use the Further Guidance in problems 2-77 through 2-79.

Students will use what they know about transforming the graph of a parabola to transform their new functions. Students should use graphing calculators to check their conjectures. As you circulate, you might notice that the students shifting $y = \frac{1}{x}$ and $y = b^x$ are grappling with shifting horizontally for two different reasons:

- If students do not use parentheses correctly for $y = \frac{1}{x-h}$, they will get the graph of $y = \frac{1}{x} - h$, which is a vertical shift down.
- Students might struggle with the exponential graph because the horizontal shift is less obvious.

Note: The exponential graph is also complicated by the stretch. Expect that some teams may decide to work with transformations of the form $y = b^{x-h} + k$ or $y = a \cdot b^x + k$. Accept whichever forms are making sense for the teams. If multiple forms come up, note this as a possibility for future discussions.

After drawing some conclusions about transformations, teams should write general equations for each of the new parent graphs:

Parent	Family	General Equation
$y = x^3$	Cubic	$y = a(x - h)^3 + k$
$y = \frac{1}{x}$	Hyperbola	$y = a\left(\frac{1}{x-h}\right) + k$
$y = \sqrt{x}$	Square Root	$y = a\sqrt{x-h} + k$
$y = x $	Absolute Value	$y = a x-h + k$
$y = b^x$	Exponential	$y = ab^{(x-h)} + k$

If students can see how the new shifts are related to shifts of a parabola, then writing the general equation for the new parent graphs should follow. To help students see how the parabola is connected to the shifts of the new functions, you might ask questions like: “What is an example of an equation of a parabola shifted to the left?” “What in the general

equation of the parabola shifts the graph to the right?” “How is your example equation of a shift to the right for this new section the same as an equation of a parabola shifted to the right?” “How is the equation of this graph shifted to the left different from the equation of a parabola shifted to the left?” or “What about a quadratic equation makes it the equation of a parabola?”

Days 2 and 3: In problem 2-80, students are asked to create a poster that shows what they have found for each new function, how it moves, how it is connected to its sketch, and its general equation. Students should use color-coding, arrows, etc., to show how each example equation and sketch are connected and how each new general equation is connected to the parent graph equation. Also, students are asked to include problems for other teams to solve.

Closure:
(20 minutes)

Day 1 or 2: If you did the Team Roles exercise (see the Team Strategies section below), consider reviewing the list that you created, and encourage students to add additional phrases that their team members used in each role that helped the group move forward.

Day 3: Hang the posters around the room for a Gallery or Museum Walk and a discussion. Allow teams a few minutes to view posters and solve some of the problems posed by their classmates. Then have students agree as a class what the appropriate general equation is for each new parent.

Questions that might come up are:

- *“Why does the exponential general equation look different from the others?”* At this time you can ask students to pick an example graph of an exponential function from one of the posters and ask how they might write an equation for that same graph using $y = ab^{x-h} + k$ to show how the ideas can be combined. Please note that it is not intended for students to master all of the transformations of exponentials. Students should know how to shift and flip an exponential.
- *“Do these transformations work for all graphs?”* Ask students what their predictions are about this and let them know that they will investigate other kinds of graphs later in the chapter and as the course continues.

Also, consider using problem 2-95 from the “Review and Preview” section as a team quiz, as a lesson close, or as an introduction problem for the next lesson. This can be done as a Carousel, Hot Potato, or a Pairs Check.

Universal Access:

Problem 2-76:

EL and/or special needs students may not understand the term “parent graph.” You could explain that the parent graph is the original or base graph. A graph that has been changed in shape and/or position is a transformation of that parent/original graph.

Depending on your class, you may opt to show the process of transforming one of the parent graphs; either the hyperbola parent graph or the exponential parent graph (chosen due to their complexity). This demonstration will help clarify the progression of a complete investigation.

Another option could be to assign each team one new parent graph to investigate as opposed to each student on the team choosing one the new parent graphs to investigate.

If students are still struggling with identifying the domain and range, don't forget the "framing" strategy identified in Chapter 1.

Problems 2-77, 2-78, and 2-79:

These are Further Guidance problems that will provide the structure and scaffolding that EL and/or special needs students may need to investigate their parent graph. Consider having the dynamic graphing tool (introduced in lesson 2.1.3) available for struggling students to utilize. This will not only provide a visual representation, it will continue to reinforce connections between changing parameters and graph orientation.

Problem 2-80:

To ensure all team members assist in organizing their work into a poster, you could assign each team member a specific task.

Team Strategies:

You can do a Participation Quiz during Day 1. Remind students of their Team Roles before beginning this investigation. As an alternate to using the Lesson 1.1.2A Resource Page ("Team Roles"), you can work with students to quickly create their own list of norms before beginning the activity. On the board or overhead display, write one of the four roles (often, it is easy to start with the Facilitator). Ask the class, "*What kind of things should I hear Facilitators saying today?*" Jot down two or three quotes and phrases that students share on the overhead. Then, move on to another role and ask the same question. If students have trouble coming up with ideas for any particular role, you can ask, "*What should a _____ be doing?*" or "*What is _____'s responsibility? What does that look or sound like?*" Keep the list and project it for students to look over before beginning work each day of this lesson. Consider including a review of the list in closure on the first or second day, and encourage students to add additional phrases that their teammates used in each role that helped the group move forward.

A Participation Quiz is suggested in Lesson 2.2.2. This "Team Roles" exercise makes a good lead-in to the Participation Quiz.

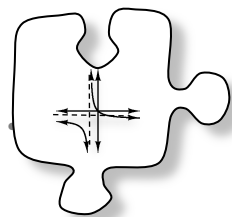
Homework:

Day 1: Problems 2-81 through 2-87

Day 2: Problems 2-88 through 2-94

Day 3: Problems 2-95 through 2-101

2.2.1 How can I transform any graph?



Transforming Other Parent Graphs

You have been learning how to move a parabola around a set of axes, write equations, sketch graphs, and model situations. The graph of $y = x^2$ is called the **parent graph** for the family of parabolas because every other parabola can be seen as a transformation of that one graph.

2-76. In this investigation you will use what you have learned about transforming the graph of $y = x^2$ to transform four other parent graphs. In fact, your team will figure out how to use what you have learned to transform the graph of *any* function!



Your Task: As a team, determine how you can make the graph of any function move left, right, up, and down and how you can stretch it vertically, compress it vertically, and flip it. Each team member should investigate one of the following parent functions: $y = x^3$, $y = \frac{1}{x}$, $y = \sqrt{x}$, $y = |x|$, and $y = b^x$. (If you are investigating $y = b^x$, your teacher will give you a value to use for b .)

- Remember that to investigate completely, you should sketch graphs, identify the domain and range, and label any important points or asymptotes.
- Then graph and write an equation to demonstrate each transformation you find.
- Finally, find a general equation for your family of graphs.

Discussion Points

How can we move a parabola?

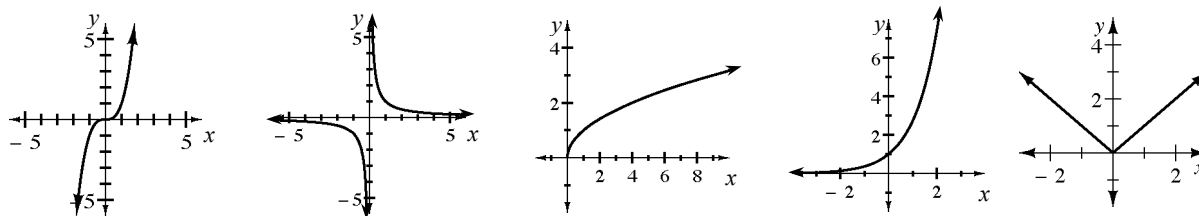
How can we use our ideas about moving parabolas to move other functions?

What changes can we make to the equation?

Further Guidance

2-77. First, investigate your parent graph.

- a. Graph your equation on a full sheet of graph paper. [See graphs below.]
- b. As a team, place your parent graphs into the middle of your workspace. For each graph, identify the domain and range and label any important points or asymptotes.



2-78. For your parent graph:

- a. Find and graph an equation that will shift your parent graph left or right.
- b. Find and graph an equation that will shift your parent graph up or down.
- c. Find and graph an equation that will stretch or compress your parent graph vertically.
- d. Find and graph an equation that will flip your parent graph upside-down.

2-79. One way of writing an equation for a parabola is to use graphing form:
 $y = a(x - h)^2 + k$. This equation tells you how to shift or stretch the parent graph,
 $y = x^2$, to get any other parabola.

- a. Explain what each parameter (a , h , and k) represents for the graph of a parabola.
[a is the stretch/compression factor, h is the x -coordinate of the vertex, and k is the y -coordinate of the vertex.]
- b. As a team, write general equations for each given parent equation. Be ready to explain how your general equations work; that is, tell what effect each part has on the orientation (right-side-up or upside-down), relative size (stretched or compressed), horizontal location (left or right shift), and vertical location (up or down shift).

===== *Further Guidance* =====
section ends here.

2-80. As a team, organize your work into a large poster that shows clearly:

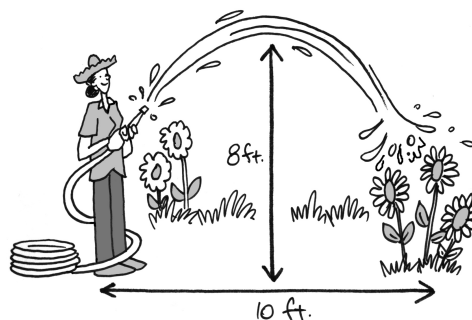
- Each parent graph you worked with,
- Examples of each transformation you found, and
- Each general equation.

Use tools such as colors, arrows, and shading to show all of the connections you can find. Then add the following problems for other teams to solve:

- Show the graph of a function in your family for which other teams need to find the equation.
- Give an equation of a function in your family that other teams will graph.



2-81. While watering her outdoor plants, Maura noticed that the water coming out of her garden hose followed a parabolic path. Thinking that she might be able to model the path of the water with an equation, she quickly took some measurements. The highest point the water reached was 8 feet, and it landed on the plants 10 feet from where she was standing. Both the nozzle of the hose and the top of the flowers were 4 feet above the ground. Help Maura



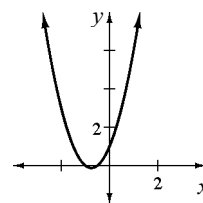
write an equation that describes the path of the water from the hose to the top of her plants. What domain and range make sense for the model? [**Possible equation:** $y = -\frac{4}{25}(x - 5)^2 + 8$, standing at $(0, 0)$; domain: $0 \leq x \leq 10$; range: $4 \leq y \leq 8$]

2-82. Draw the graph of $y = 2x^2 + 3x + 1$. [See graph at right.]

a. Find the x - and y -intercepts. [$x : (-\frac{1}{2}, 0), (-1, 0)$; $y : (0, 1)$]

b. Where is the line of symmetry of this parabola? Write its equation. [$x = -\frac{3}{4}$]

c. Find the coordinates of the vertex. [$(-\frac{3}{4}, -\frac{1}{8})$ or $(-0.75, -0.125)$]



2-83. Change the equation in problem 2-82 so that the parabola has only one x -intercept. [**Move it up 0.125 units:** $y = 2x^2 + 3x + 1.125$]

2-84. Simplify each expression. Remember you can simplify radicals by removing perfect square factors (e.g. $\sqrt{12} = \sqrt{4 \cdot 3} = 2\sqrt{3}$).

a. $\frac{\sqrt{24}}{[2\sqrt{6}]}$

b. $\frac{\sqrt{18}}{[3\sqrt{2}]}$

c. $\frac{\sqrt{3} + \sqrt{3}}{[2\sqrt{3}]}$

d. $\frac{\sqrt{27} + \sqrt{12}}{[5\sqrt{3}]}$

2-85. Below are two more situations that can be described using exponential functions. They represent a small sampling of the situations where quantities grow or decay by a constant percentage over equal periods of time. For each situation:

- Find an appropriate unit of time (such as days, weeks, years).
- Find the multiplier that should be used.
- Identify the initial value.
- Write an exponential equation in the form $f(x) = ab^x$ that represents the growth or decay.

a. The value of a car with an initial purchase price of \$12,250 depreciates by 11% per year. [**Years; 0.89; 12250; 12250(0.89)^x**]

b. An investment of \$1000 earns 6% annual interest, compounded monthly. [**Months; 1.005; 1000; 1000(1.005)^x**]

2-86. Rewrite each of the following expressions so that your answer has no negative or fractional exponents.

a. $16^{5/4}$
[**32**]

b. $(x^5y^4)^{1/2}$
[**$x^2y^2\sqrt{x}$**]

c. $(x^2y^{-1})(x^{-3}y)^0$
[**$\frac{x^2}{y}$**]

2-87. Harvey's Espresso Express, a drive-through coffee stop, is famous for its great house coffee, a blend of Colombian and Mocha Java beans. Their archrival, Jojo's Java, sent a spy to steal their ratio for blending beans. The spy returned with a torn part of an old receipt that showed only the total number of pounds and the total cost, 18 pounds for \$92.07. At first Jojo was angry, but then he realized that he knew the price per pound of each kind of coffee (\$4.89 for Colombian and \$5.43 for Mocha Java). Show how he could use equations to figure out how many pounds of each type of beans Harvey's used.

[**$c + m = 18$ and $\$4.89c + \$5.43m = \$92.07$; 10.5 lbs. of Colombian and 7.5 lbs. of Mocha Java.]**

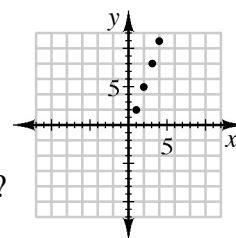
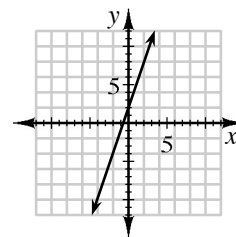


2-88. Lilia wants to have a circular pool put in her backyard. She wants the rest of the yard to be paved with concrete.

- If her yard is a 50 ft. by 30 ft. rectangle, what is the radius of the largest pool that will fit in her yard? [**15 ft**]
- If the concrete is to be 8 inches thick, and costs \$2.39 per cubic foot, what is the cost of putting in the concrete? No concrete will be used in the pool. (Reminder: Volume = (Base Area) · Depth). [**Surface area of concrete: 793.14 sq. ft.; 528.76 cu. ft.; \$1,263.74**]

2-89. Consider a line with a slope of 3 and a y-intercept at (0, 2).

- Sketch the graph of this line. [**See graph at right.**]
- Write the equation of the line. [**$y = 3x + 2$**]
- Find the initial term and the next three terms of the sequence $t(n) = 3n - 1$. Plot the terms on a new set of axes next to your graph from part (a) above. [**2, 5, 8, 11**]
- Explain the similarities and differences between the graphs and equations in parts (a) through (c). Are both continuous? [**One is continuous and one is discrete. They have the same slope so the “lines” are parallel, but they have different intercepts.**]



2-90. The Gross National Product (GNP) of the United States in 1960 was $1.665 \cdot 10^{12}$ dollars. Until 1989 it increased at a rate of 3.17% per year. Use this information to answer each of the questions below.

- What was the GNP in 1989? [**$4.116 \cdot 10^{12}$**]
- Write an equation to represent the GNP t years after 1960, assuming that the rate of growth remained constant. [**$y = 1.665(10^{12})(1.0317)^t$**]
- Do you think the rate of growth really remains constant? Explain. [**Explanations vary.**]

2-91. Write each expression in simpler radical form.

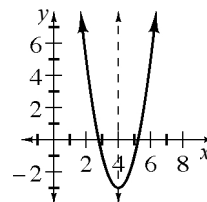
- $\sqrt{x} + \sqrt{y} + 5\sqrt{x} + 2\sqrt{y}$
[**$6\sqrt{x} + 3\sqrt{y}$**]
- $(2\sqrt{8})^2$
[**32**]
- $\frac{\sqrt{50}}{\sqrt{2}}$ [**5**]
- $\sqrt{\frac{3}{4}}$ [**$\frac{\sqrt{3}}{2}$**]

2-92. Multiply each of the following expressions.

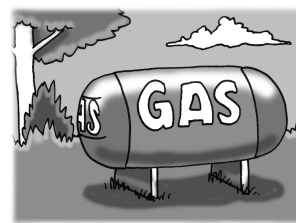
a. $2x^2(3x + 4x^2y)$
 $[6x^3 + 8x^4y]$

b. $(x^3y^2)^4(x^2y)$
 $[x^{14}y^9]$

2-93. Sketch a graph and draw the line of symmetry for the equation $y = 2(x - 4)^2 - 3$. What is the equation of the line of symmetry? [See graph at right. line of symmetry $x = 4$]



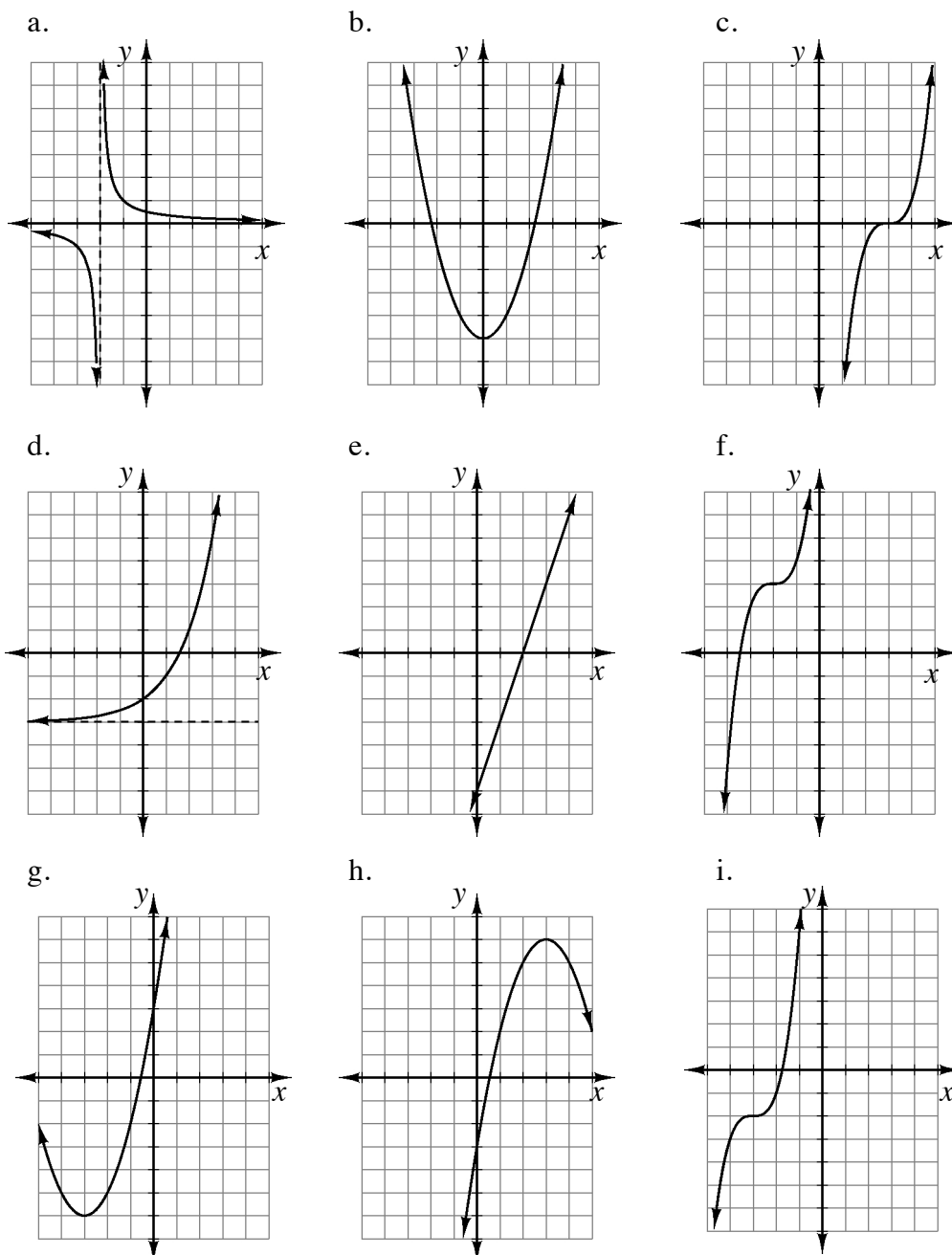
2-94. People who live in isolated or rural areas often have their own tanks that hold propane gas to run appliances like stoves, washers, and water heaters. Some of these tanks are made in the shape of a cylinder with two hemispheres on the ends, as shown in the picture at right. (Recall that a hemisphere is half of a sphere, and the volume of a sphere is found by using $V = \frac{4}{3}\pi r^3$.)



The Inland Propane Gas Tank Company wants to make tanks with this shape, and to offer models in different sizes. The cylindrical portion of each of the different tanks will be 4 meters long. However, the radius, r , will vary among the different models.

- One of the tank models has a radius of 1 meter. What is its volume?
 $[4\pi + \frac{4}{3}\pi \approx 16.755 \text{ m}^3]$
- If the radius is doubled, will the volume double? Explain. Then calculate the volume of the larger tank with $r = 2$ m.
 $[\text{No; } r, r^2, r^3 \text{ relationship; } V = \frac{80\pi}{3} \approx 83.776 \text{ m}^3]$
- Write an equation that will let the Inland Propane Gas Tank Company determine the volume of a tank with any size radius. $[V = \frac{4}{3}\pi r^3 + 4\pi r^2]$

- 2-95. Write a possible equation for each of these graphs. Assume that one mark on each axis is one unit. When you are in class, check your equations on a graphing calculator and compare your results with your teammates. [**a:** $y = \frac{1}{x+2}$; **b:** $y = x^2 - 5$; **c:** $y = (x - 3)^3$; **d:** $y = 2^x - 3$; **e:** $y = 3x - 6$; **f:** $y = (x + 2)^3 + 3$; **g:** $y = (x + 3)^2 - 6$; **h:** $y = -(x - 3)^2 + 6$; **i:** $y = (x + 3)^3 - 2$]



- 2-96. By mistake, Jim graphed $y = x^3 - 4x$ instead of $y = x^3 - 4x + 6$. What should he do to his graph to get the correct one? [**He should move it up 6 units or redraw the axes 6 units lower.**]

2-97. Simplify each radical expression.

- a. $(3\sqrt{2})^2$
[**18**]
- b. $\sqrt{\frac{9}{4}}$
[$\frac{3}{2}$]
- c. $\sqrt{\frac{1}{3}}$
[$\frac{1}{\sqrt{3}}$ or $\frac{\sqrt{3}}{3}$]
- d. $(3 + \sqrt{2})^2$
[**$11 + 6\sqrt{2}$**]

2-98. Factor each of the following expressions. Look for the difference of squares and common factors.

- a. $4x^2 - 9y^2$
[**$(2x - 3y)(2x + 3y)$**]
- b. $8x^3 - 2x^7$
[**$2x^3(2 + x^2)(2 - x^2)$**]
- c. $x^4 - 81y^4$
[**$(x^2 + 9y^2)(x - 3y)(x + 3y)$**]
- d. $8x^3 + 2x^7$
[**$2x^3(4 + x^4)$**]
- e. Did you use a shortcut to factor the expressions in parts (a) through (c)? If so, describe it. If not, what pattern do you see in these expressions? How can you use that pattern to factor quickly? [**They all contain a factor that is a difference of squares.**]

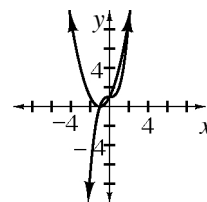
2-99. Solve for x : $ax + by^3 = c + 7$. [$x = \frac{-by^3 + c + 7}{a}$]

2-100. Write an equation for each of the following sequences.

- a. 20, 14, 8, ...
[**$t(n) = -6n + 26$**]
- b. -6, -24, -96, ...
[**$t(n) = -1.5(4)^n$ or $-6(4)^{n-1}$**]

2-101. Given $f(x) = x^3 + 1$ and $g(x) = (x + 1)^2$:

- a. Sketch the graphs of the two functions. [**See graph at right.**]
- b. Solve $f(x) = 9$.
[**2**]
- c. Solve $g(x) = 0$.
[**-1**]
- d. Solve $f(x) = -12$.
[**$\sqrt[3]{-13}$**]
- e. Solve $g(x) = -12$.
[**no solution**]
- f. For how many values of x does $f(x)$ equal $g(x)$? Explain.
[**Three because the graphs cross three times.**]
- g. Find and simplify an expression for $f(x) - g(x)$. [**$x^3 - x^2 - 2x$**]



Lesson 2.2.2 What is the significance of (h, k) ?

Describing (h, k) for Each Family of Functions

Lesson Objective: Students will identify the point (h, k) for graphs of parabolic, hyperbolic, cubic, absolute value, exponential, and square root functions and relate it to the point-slope form of a line. They will consolidate their understanding of parent graphs and general equations in the Parent Graph Toolkit.

Mathematical Practices: reason abstractly and quantitatively, look for and make use of structure

Length of Activity: Two days (approximately 100 minutes)

Core Problems: Problems 2-102 through 2-105

Materials: Lesson 2.2.2 Resource Page (“Parent Graph Toolkit”), four copies per student

Suggested Lesson Activity: This lesson is designed as an opportunity for students to stop and consolidate their ideas about shifting parent graphs. A Participation Quiz is suggested for Day 1.

Day 1: Start with a brief introduction of the objectives of this lesson. Then start teams on problem 2-102, in which students will review their knowledge about transforming parabolas. In part (c), teams are asked to justify *why* replacing x with $(x - h)$ shifts a graph horizontally. Expect this to challenge your students. As you circulate, encourage discussion by asking questions such as, “*How can you find the x -intercepts of a parabola? Why does the factor $(x - 3)$ give an x -intercept of positive 3?*” Teams may think of horizontal shifts as relocating the original point where $x = 0$ on the parent graph to a new point where $x = h$ on the transformed graph. In the case of the parabola, when $x = h$, then $(x - h)^2$ is equal to 0^2 . Debrief this question with the whole class, allowing teams to share their ideas. Validate however students are seeing it at this point.

Then move teams on to problems 2-103 using a Think-Ink-Pair-Share in which students should be able to name the seven functions in the bulleted list below.

Move students on to problem 2-104 and distribute four copies of the Lesson 2.2.2 Resource Page to each student. Students will complete the Parent Graph Toolkit for all of the parent functions that they know. They are also asked to think about the importance of the point (h, k) for each of the parent graphs.

Note about (h, k) : For parabolas, the point (h, k) from the general equation $y = a(x - h)^2 + k$ is the vertex, which is a clear point that students can use to help identify how a parabola has shifted from the

origin. Other parent graphs have points (h, k) as well that play different roles as listed below.

- Parabola: (h, k) is the vertex.
- Line: (h, k) is any point on the line.
- Hyperbola: (h, k) is the point where the asymptotes intersect.
- Cubic (simple): (h, k) is the ‘center’ point, or the point where the curve changes from being concave up to concave down or vice versa (the point of inflection).
- Square Root: (h, k) is the point where the graph begins or ends.
- Absolute Value: (h, k) is the vertex.
- Exponential: There is some ambiguity for this function. Encourage students to see it in whatever way makes the most sense to them.
 - i. Some students choose to locate the graph by y-intercept. This point does not have the coordinates (h, k) in the general equation $y = ab^{(x-h)} + k$, and they will have to adjust for the value of a .
 - ii. Some students choose to move the asymptote and adjust the graph based on the position of the new asymptote.

Teams will probably not finish problem 2-104 on Day 1. Have them save their materials in a safe place and return to the problem on Day 2 in this case.

Day 2: If needed, direct teams to complete problem 2-104 and then move on to problem 2-105, which connects the general equation of a line to point-slope form. Be sure to leave time for closure and the Learning Log in problem 2-106. To share ideas from their Parent Graph Toolkit, you could use a Swapmeet or Traveling Salesman.

Closure:
(10-15 minutes each day)

Day 1: If you used a Participation Quiz, reserve a few minutes at the end of class for students to read the written feedback and to reflect about their team’s process during the activity.

Day 2: Ask teams to share their conclusions about (h, k) for each of the parent graphs. You could do this with a Walk and Talk (or proximity partners) first followed by a whole-class discussion. This allows individuals the opportunity to talk about (h, k) before the class discussion. Allow discussion about the unusual nature of the exponential function. Solicit ideas about what it is about that function that causes its transformations to appear different. Then direct teams to complete the Learning Log entry in problem 2-106.

Universal Access: **In general:** Some students spend an inordinate amount of time drawing and labeling axes. It is a good idea to provide ready-made graphs (with the x and y -axis labeled only) for these students. Avoid numbering the graphs; this is an important concept students should learn.

Problem 2-102:

EL and/or special needs students may need a visual representation to complete this problem. Students could sketch the graphs, use a calculator, and/or have access to the dynamic graphing tool.

Problem 2-103:

A Whip Around would ensure that all teams have listed the seven families of function.

Problem 2-104:

While the Resource Page 2.2.2 is a beneficial graphic organizer/toolkit for all students, it may be a bit busy and/or confusing for EL and/or special needs students. Options are: cut the resource page in half (give each student eight half-pages); take one of the half-pages and enlarge it to a full-size page; and/or enlarge the coordinate planes so students have more room to sketch their graphs.

Problem 2-105:

The Math Notes – Methods and Meanings section could be helpful for students struggling with this problem.

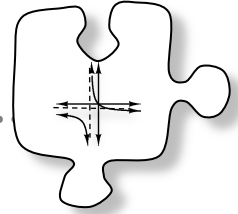
Math Notes – Methods and Meanings:

An important concept is explained in this section. Consider a way for students to organize and access this information.

Team Strategies: Consider supporting your students' teamwork by conducting a Participation Quiz on Day 1. In a Participation Quiz, the *quality of the teamwork* is documented and assessed directly by the teacher, rather than the mathematical content. Because these big problems require students to collaborate to bring together all the ideas they have learned so far in this chapter, it is well suited for a Participation Quiz. If you use this structure, reserve a few minutes at the end of class for students to read the written feedback and to reflect on their team's process during the activity.

Homework: Day 1: Problems 2-107 through 2-113
Day 2: Problems 2-114 through 2-120

2.2.2 What is the significance of (h, k) ?



Describing (h, k) for Each Family of Functions

In Lesson 2.2.1, you learned that you could apply your knowledge of transforming parabolas to transform several other parent functions. In this lesson, you will consolidate your knowledge of each of the parent functions that you know and you will identify the importance of the point (h, k) for each parent function and its family.

- 2-102. Think about the parent graph for parabolas, $y = x^2$.
- Write the equation of a parabola that will be the same as the parent graph, but shifted four units to the right. [$y = (x - 4)^2$]
 - Does the strategy you used to move parabolas horizontally also work for other parent graphs? Justify your answer. [**Yes; justifications vary.**]
 - You have learned that the general equation for a parabola is $y = a(x - h)^2 + k$. To move the graph of $y = x^2$ h units to the *right*, you replaced x^2 with $(x - h)^2$. Work with your team to justify why replacing x with $(x - h)$ moves a graph to the right. Think about multiple representations as you discuss this and be prepared to share your ideas with the class. [**Answers vary. Students could look at the value that will make the quantity in the parentheses equal to zero.**]

- 2-103. With your team, brainstorm a list of all of the families of functions that you have learned about so far in your study of algebra.
[**Possibilities include: lines, parabolas, hyperbolas, cubics, square root functions, exponentials, and absolute value functions**]




- 2-104. Obtain copies of the Parent Graph Toolkit (Lesson 2.2.2 Resource Page) from your teacher. Work with your team to complete a Toolkit entry for each of the parent graphs you have studied so far in this course.

- 2-105. What is the equation of the parent graph of a line? Use what you have learned about transforming parent graphs to write the general equation of a transformed line.
 $[y = x ; y = a(x - h) + k]$
- Use this general equation of a line to write the equation of a line with slope $\frac{4}{5}$ that passes through the point $(3, 9)$. $[y = \frac{4}{5}(x - 3) + 9]$
 - A line passes through the points $(-1, 5)$ and $(8, -2)$. Substitute each of these into the general equation to create a system of equations. Now solve this system to find the slope. Is this how you have found slope in the past?
 $[y = m(x + 1) + 5 ; -2 = m(8 + 1) + 5 , m = -\frac{7}{9}]$

2-106. LEARNING LOG

What can the point (h, k) tell you about the how to graph a function from its equation? How can it help you write the equation for a function given its graph? Discuss these questions with your team and then answer them in a Learning Log entry. Be sure to include examples to help you illustrate your ideas. Title this entry “How to use (h, k) ” and label it with today’s date.





METHODS AND MEANINGS

Point-Slope Equations for Lines

MATH NOTES

If you think of $y = x$ as a parent equation, then the general equation for the family of lines can be written as

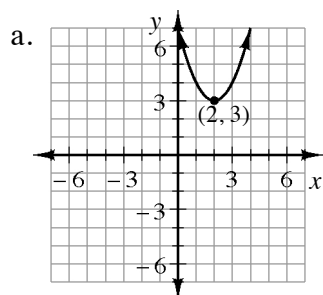
$$y = a(x - h) + k .$$

When this equation is rewritten as $y - k = a(x - h)$ it is often called the **point-slope** form of the equation for a line that contains the point (h, k) and has slope a .

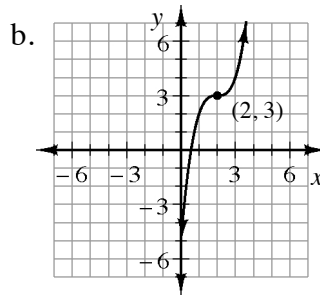
For example, if you know a line contains the point $(7, -8)$ and has slope -4 then the equation can be written $y - (-8) = -4(x - 7)$ or $y + 8 = -4(x - 7)$.



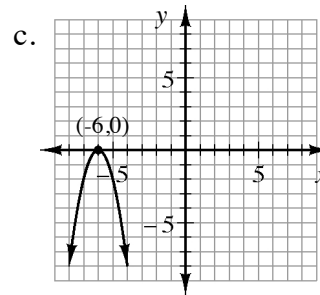
- 2-107. Use the point (h, k) to help you write a possible equation for each graph shown below.



[$y = (x - 2)^2 + 3$]



[$y = (x - 2)^3 + 3$]



[$y = -2(x + 6)^2$]

- 2-108. Find the domain and range for each of the graphs in the previous problem. [a: **D: all real numbers, R: $y \geq 3$** ; b: **D and R: all real numbers**; c: **D: all real numbers, R: $y \leq 0$**]

- 2-109. For each of the following equations, describe how d transforms the parent graph.

a. $y = dx^3$
[**compresses or stretches**]

b. $y = x^2 - d$
[**shifts up or down**]

c. $y = (x - d)^2 + 7$
[**shifts left or right**]

d. $y = \frac{1}{x} + d$
[**shifts up or down**]

- 2-110. Find the equation of an exponential function that passes through each pair of points.

a. (3, 0.05) and (5, 0.0125)
[$y = 0.4 \cdot 0.5^x$]

b. (1, 16) and (4, 128)
[$y = 8 \cdot 2^x$]

- 2-111. Rewrite each of the following expressions so that your answers have no negative or fractional exponents.

a. $5^{-2} \cdot 4^{1/2}$
[$\frac{2}{25}$]

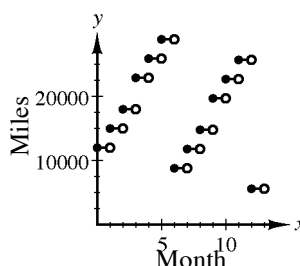
b. $\frac{3xy^2z^{-2}}{(xy)^{-1}z^2}$
[$\frac{3x^2y^3}{z^4}$]

c. $(3m^2)^3(2mn)^{-1}(8n^3)^{2/3}$
[$54m^5n$]

d. $(5x^2y^3z)^{1/3}$
[$y\sqrt[3]{5x^2z}$]

- 2-112. Tino is a businessman who flies to sales conferences regularly. He flies from Seattle to Kansas City once each month and from Seattle to Los Angeles once every 3 months (March, June, September, and December). The flight to Kansas City adds 1500 miles each way to his frequent flier account, while flying to Los Angeles adds 950 miles each way to his account. In January last year, he started with 12,000 miles in his account. In June and December he withdrew 25000 miles from his account for a ticket to Florida for vacation.

- a. Make a table and a graph that shows the balance in Tino's frequent flier account at the end of each month last year.
[See table and graph.]



Month	Miles
1	15,000
2	18,000
3	22,900
4	25,900
5	28,900
6	8,800
7	11,800
8	14,800
9	19,700
10	22,700
11	25,700
12	5,600

- b. What was the highest number of miles that Tino had in his account during the year? In which month did this occur?
[He had 28,900 miles in May.]
- c. How many miles did Tino have in his account at the beginning of this year?
[5600 miles]
- d. If Tino continues this same pattern of flying will he have enough miles to go on both of his usual vacations this year? Why or why not? [No, he will not be able to go in December, he will only have 24,200 miles.]

- 2-113. Solve each equation for x (that is, put it in $x = \underline{\hspace{1cm}}$ form).

a. $y = 2(x - 17)^2$
[$x = \pm\sqrt{\frac{y}{2}} + 17$]

b. $y + 7 = \sqrt[3]{x + 5}$
[$x = (y + 7)^3 - 5$]

- 2-114. Where do the following pairs of lines intersect?

a. $y = 5x - 2$ [(10, 48)]
 $y = 3x + 18$

b. $y = x - 4$ [($\frac{29}{5}$, $\frac{9}{5}$)]
 $2x + 3y = 17$

- 2-115. Write each expression below in simplest radical form.

a. $\sqrt{75} + \sqrt{27}$
[$8\sqrt{3}$]

b. $\sqrt{x} + 2\sqrt{x}$
[$3\sqrt{x}$]

c. $(\sqrt{12})^2$
[12]

d. $(3\sqrt{12})^2$
[108]

2-116. If $g(x) = x^2 - 5$, find:

a. $g\left(\frac{1}{2}\right)$

[$g\left(\frac{1}{2}\right) = -4.75$]

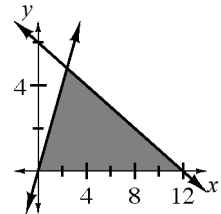
b. $g(h+1)$

[$g(h+1) = h^2 + 2h - 4$]

2-117. Graph these two lines on the same set of axes: $y = 2x$ and $y = -\frac{1}{2}x + 6$. [See graph at right.]

a. Find the x - and y -intercepts for each equation.

[$y = 2x$: $(0, 0)$, $y = -\frac{1}{2}x + 6$: $(0, 6)$, $(12, 0)$]



b. Shade the region bounded by the two lines and the x -axis.

[It should be a triangle with vertices $(0, 0)$, $(12, 0)$, and $(2.4, 4.8)$.]

c. What are the domain and range of the region?

How did you find these values?

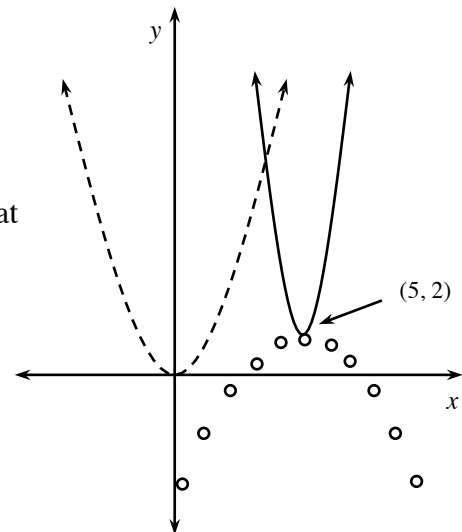
[Domain $0 \leq x \leq 12$, Range $0 \leq y \leq 4.8$]

d. Find the area of this region.

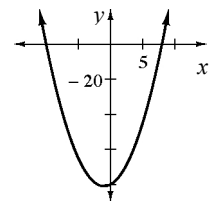
Round your answer to the nearest tenth.

[$A = \frac{1}{2}(12)(4.8) = 28.8$ square units]

2-118. The graph of $y = x^2$ is shown as a dashed curve at right. Estimate the equations of the two other parabolas. [$y \approx 2(x - 5)^2 + 2$ and $y \approx -\frac{1}{2}(x - 5)^2 + 2$]



2-119. Find the x - and y -intercepts and the vertex of $y = x^2 + 2x - 80$. Then sketch the graph and write the equation in graphing form. [See graph at right; $y = (x + 1)^2 - 81$; x -intercepts: $(-10, 0)$, $(8, 0)$, y -intercept: $(0, -80)$; vertex: $(-1, -81)$]



2-120. Is -578 a term in the sequence defined by $t(n) = -5n + 7$? Justify your answer.

[Yes, when $n = 117$.]

Lesson 2.2.3 How can I move a function?

Transformations of Functions

Lesson Objective: Students investigate one more transformation, $f(-x)$, the reflection of $f(x)$ across the y -axis. Then they will compare $f(-x)$ and $-f(x)$ for a variety of parent graphs and develop the definitions for even and odd functions.

Mathematical Practices: reason abstractly and quantitatively, look for and make use of structure

Length of Activity: One day (approximately 50 minutes)

Core Problems: Problems 2-121 through 2-123

Materials: None

Suggested Lesson Activity: Use Reciprocal Teaching to lead a brief discussion of what students remember about transformations from Geometry, where they investigated translations, reflections, rotations, and dilations.

Have teams begin problem 2-121. When most teams have finished, debrief by asking how the parameters of the transformations of $f(x)$ in $a \cdot f(x-h) + k$ are related to the geometric transformations. They should come up with (h, k) as creating translations, $|a|$ accounting for dilations (stretching and shrinking), and $-f(x)$, where $a = -1$, as a reflection across the x -axis (or maybe, more generally $a < 0$ as a reflection across a horizontal line). Use this brief discussion to lead into problem 2-122 in which they will explore $f(-x)$, a reflection across the y -axis.

Next, put teams to work on problems 2-122 and 2-123 where they will compare $f(x)$, $f(-x)$, and $-f(x)$ to develop the idea of even and odd functions.

In part (a) of problem 2-122 students may find, especially if they are using graphing calculators, that the graph of $f(x) = \sqrt{x}$ can be reflected across the y -axis, and may even write $f(x) = \sqrt{-x}$ for the equation of the reflected graph. If they do this, ask them to write the domain for the reflected graph, which is $x \leq 0$.

In problem 2-123 students should come to realize that they cannot create the graph of $\sqrt{-x}$ by substituting $-x$ for x in $f(x) = \sqrt{x}$. The function $f(x) = \sqrt{x}$ has a domain of $x \geq 0$. For that domain, $f(-x) = \sqrt{-x}$ is not defined, except for when $x = 0$. So $f(-x)$ is therefore not equal to $f(x) = \sqrt{x}$, nor is it equal to $-f(x) = -\sqrt{x}$. Thus, students should conclude that $f(x) = \sqrt{x}$ is neither even nor odd.

If students are curious about how the other transformations they learned in Geometry are represented algebraically, tell them they will learn more

about reflections across the line $y = x$ in Chapter 5 when they study inverse functions, more about ‘sideways dilations’ or horizontal stretches and shrinks in Chapter 7 when they study sine waves, and more about rotations in pre-calculus.

Move on to the closure activity.

Closure:
(8 minutes)

Discuss the results of problems 2-122 and 2-123 using a Swapmeet. Possibly put two more functions such as $g(x) = -x^2 + 5$ or $h(x) = (x - 3)^2$ or $p(x) = x^3 + x$ on the board and ask if they can tell quickly whether they are even or odd and how they know. Possible responses include that the transformation $f(-x)$ is a reflection across the y -axis. An even function reflects across the y -axis onto itself, and you can tell from the equation that a function is even if $f(-x)$ algebraically simplifies to $f(x)$.

Have students write their entries in their Learning Logs, which is problem 2-124. An expected response would include that $f(-x)$ is a reflection across the y -axis of $f(x)$. An even function has a line of symmetry on the y -axis, thus, reflecting across the y -axis does not change the function. In the equation, substituting an x and a $-x$ results in the same output, for example, when squaring or taking the absolute value of the x part of the function.

Universal Access:

Problem 2-121:

EL and/or special needs students may need further review in regards to transformations from Geometry. Depending on your class, you may want to provide examples of translations (slide), reflections (flip), rotations (turn around a fixed point), and dilations (reduce or enlarge) and/or prepare a few problems to use as formative assessment.

If students have difficulty visualizing the movements, you could provide a piece of patty paper (tracing paper) and/or a piece of a transparency. Students can trace the original shape, and then move it accordingly.

Giving EL and/or special needs students the ability to use hand gestures (e.g., slide their hand across their desk/table, flip their hand from palm up to palm down, etc) while they speak allows for non-verbal communication to take place, as opposed to speaking only. As you assist them in developing their mathematical vocabulary through modeling, repetition, and class discussions, the students’ ability to verbally describe their ideas will be broadened. It is important that EL and special needs students speak and use mathematical vocabulary everyday.

Problem -122:

Some students spend an inordinate amount of time drawing and labeling axes. It is a good idea to provide ready-made graphs (with the x - and y -axis labeled only) for these students. Avoid numbering the graphs; this

is an important concept students should learn.

Problem 2-123:

EL and/or special needs students may need further explanation in regards to categorizing functions as even or odd. This is explained in the Math Notes – Methods and Meanings section of Lesson 2.2.5.

Math Notes – Methods and Meanings:

This section displays the general equations for the families of functions. Consider a way for students to organize this information (e.g. graphic organizer, chart/poster in the classroom, flash cards for review).

Team Strategies:

It is sometimes necessary (and desirable) to leave a team with some of its questions unanswered. You may need to attend to another team or issue or you may decide that the team needs a chance to try out some new ideas. In this case, let the team know that you will return and that they have made useful progress. Give them an interesting question or a goal and be clear that you will expect an answer when you return. It can be useful to ask teams, “*Do you have enough information to keep working?*” before you leave them so that it is clear that they will be able to move forward after you leave. Make sure that you check back with them in a timely manner.

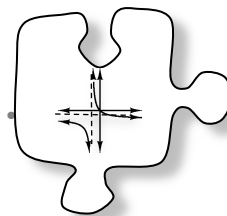
Homework:

Problems 2-125 through 2-131

Notes to Self:

2.2.3 How can I move a function?

Transformations of Functions



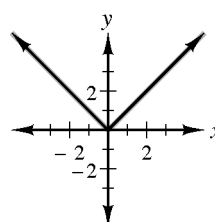
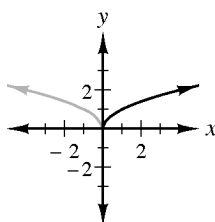
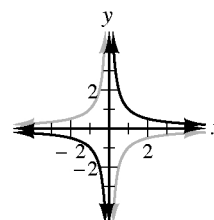
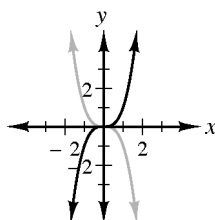
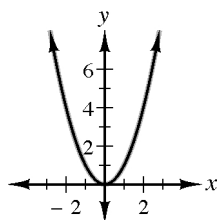
In your Geometry course you transformed figures just like you transformed parent graphs in this course. Today you will look more at geometric transformations and you will explore what happens when you take the opposite of x before applying the operations of the function. That is, you will investigate $f(-x)$.



- 2-121. In Geometry, you called the transformation of figures “translations,” “reflections,” “rotations,” and “dilations.” Refer to your Parent Graph Toolkit from problem 2-104 and/or your Learning Log entry in problem 2-106 as you complete parts (a) through (d) below.
- What kind of a geometric transformation have you made when you replace $f(x)$ with $f(x) + k$? Be as specific as you can. [**A vertical translation of a distance k .**]
 - What kind of geometric transformation occurs when you replace $f(x)$ with $-f(x)$? Be as specific as you can. [**A reflection across the x -axis.**]
 - What kind of transformation is $f(x - h)$? [**A horizontal translation of a distance h .**]
 - What kind of transformation is $a \cdot f(x)$? Be specific. [**A vertical dilation with a stretch (or shrink) factor of a . If a is negative, there is also a reflection across the x -axis.**]

2-122. Investigate the transformation $y = f(-x)$ as directed below.

- For each of the parent graphs you have investigated so far, investigate what happens to the graph when you replace x with $-x$. For each parent function, draw the original and the new graph on the same set of axes in different colors. [See sample graphs below.]
- For each parent equation, substitute $-x$ for x and algebraically simplify the result. [$f(-x) = (-x)^2 = x^2$, $f(-x) = (-x)^3 = -x^3$, $f(-x) = \frac{1}{-x} = -\frac{1}{x}$, $f(-x) = \sqrt{-x}$ for $x \leq 0$ cannot be simplified, $f(-x) = |-x| = |x|$, $f(-x) = b^{-x} = \frac{1}{b^x}$]
- Describe the geometric transformation that occurs when you replace $f(x)$ with $f(-x)$. [A reflection across the y-axis.]



2-123. Functions can be categorized as **even** or **odd functions**. With your team sort the functions you investigated in problem 2-122 into the following three groups:

EVEN FUNCTIONS: All functions where $f(-x) = f(x)$. [x^2 and $|x|$]

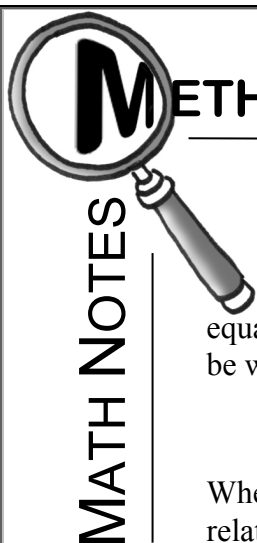
ODD FUNCTIONS: All functions where $f(-x) = -f(x)$. [x^3 , $\frac{1}{x}$]

FUNCTIONS THAT ARE NEITHER EVEN nor ODD.

[b^x , \sqrt{x} . \sqrt{x} is neither because $f(x) = \sqrt{x}$ has the domain $x \geq 0$, and for that domain $f(-x)$ is defined only for $x = 0$. Thus $f(-x)$ neither equals $f(x)$ nor $-f(x)$.]



How will the graph of a function change when $-x$ replaces x in the function $f(x)$? How can you tell from its graph whether a function will be even? How can you tell from its equation whether a function will be even? Discuss these questions with your team and then answer them in a Learning Log entry. Be sure to include examples. Title this entry “Reflections and Even Functions” and label it with today’s date.



METHODS AND MEANINGS

General Equations for Families

If $y = f(x)$ is an equation for a parent graph, then the general equation for the family of functions with similar characteristics as $f(x)$ can be written as:

$$y = a \cdot f(x - h) + k$$

Where (h, k) is the point corresponding to $(0, 0)$ in the parent graph and, relative to the parent graph, the function has been:

- Vertically stretched if the absolute value of a is greater than 1.
- Vertically compressed if the absolute value of a is less than 1.
- Reflected across the x -axis if a is less than 0.

So far in this chapter you have worked with the following families of functions:

Parent	Family	General Equation
$y = x$	Line	$y = a(x - h) + k$
$y = x $	Absolute Value	$y = a x - h + k$
$y = x^2$	Parabola	$y = a(x - h)^2 + k$
$y = x^3$	Cubic	$y = a(x - h)^3 + k$
$y = \frac{1}{x}$	Hyperbola	$y = a(\frac{1}{x-h}) + k$
$y = \sqrt{x}$	Square Root	$y = a\sqrt{x - h} + k$
$y = b^x$	Exponential	$y = ab^{(x-h)} + k$



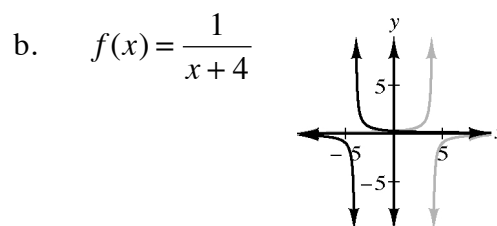
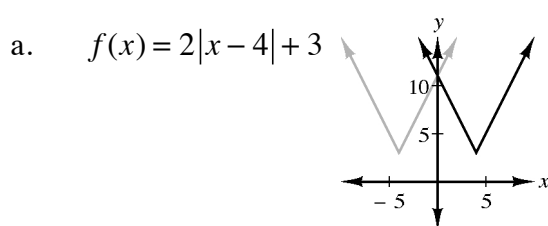
2-125. Decide whether each of the following functions is even, odd, or neither. Show or explain your reasoning.

a. $y = \frac{2}{3}x + 1$
[Neither]

b. $y = (x + 2)^2$
[Neither]

c. $y = |x| - x^2$
[Even]

2-126. For each of the following functions sketch the graph of the original and of $y = f(-x)$.



c. Is either of these functions odd or even? Justify your answer.
[Neither function is odd nor even.]

2-127. A parabola has vertex $(2, 3)$ and contains the point $(0, 0)$. Find an equation that represents this parabola. [$y = -\frac{3}{4}(x - 2)^2 + 3$]

2-128. For each equation below, find the x - and y -intercepts and the locator point (h, k) , then write the equations in graphing form.

a. $y = 7 + 2x^2 + 4x - 5$
[x : $(-1, 0)$, y : $(0, 2)$,
V: $(-1, 0)$, $y = 2(x + 1)^2$]

b. $x^2 = 2x + x(2x - 4) + y$
[x : $(0, 0)$, $(2, 0)$, y : $(0, 0)$,
V: $(1, 1)$, $y = -(x - 1)^2 + 1$]

2-129. Consider the system of equations at right:

$$3y - 4x = -1$$

a. What is the parent of each equation? [$y = x$]

$$9y + 2x = 4$$

b. Solve this system algebraically. [$(\frac{1}{2}, \frac{1}{3})$]

c. Find where the two graphs intersect. [$(\frac{1}{2}, \frac{1}{3})$]

d. Explain the relationship between parts (b) and (c) above. [The solution to the system is the point at which the lines intersect.]

2-130. Write an equation for each of the following sequences.

a. $10, 2.5, 0.625, \dots$

[$t(n) = 20(\frac{1}{4})^n$ or $10(\frac{1}{4})^{n-1}$]

b. $-2, -8, -14, \dots$

[$t(n) = -6n + 4$]

2-131. Find the intercepts, the locator point (h, k) , the domain, and the range for each of the following functions.

a. $y = |x - 4| - 2$

[$x: (2, 0), (6, 0)$

$y: (0, 2)$

vertex $(4, -2)$

D: all real numbers

R: $y \geq -2$]

b. $y = -|x + 1| + 3$

[$x: (-4, 0), (2, 0)$

$y: (0, 2)$

vertex $(-1, 3)$

D: all real numbers

R: $y \leq 3$]

Lesson 2.2.4 How can I transform circles?

Transforming Non-Functions

Lesson Objective: Students will use what they know about transforming parabolas to make conjectures about transforming non-functions, specifically a circle.

Mathematical Practices: make sense of problems and persevere in solving them, reason abstractly and quantitatively, look for and make use of structure, look for and express regularity in repeated reasoning

Length of Activity: Two days (approximately 100 minutes)

Core Problems: Problems 2-132 through 2-136

Materials: None

Suggested Lesson Activity: In this two-day lesson, students will investigate parabolas of the form $x = y^2$ (we will call them “sleeping parabolas”) as well as circles, and they will learn how to distinguish between a function and a non-function. They will also learn how to shift the circle using what they know about transforming functions.

Day 1: Begin the first day with a short class discussion about why $y = (x + 4)^2$ will cause the parabola to move left instead of right. Consider doing this lesson using the Red Light, Green Light strategy.

Then direct teams to begin problem 2-132. Tell students that they will be investigating and describing two new parent equations. Looking at how to transform the graphs of these parent equations will come up later in the lesson. Students should create graphs for these equations without using graphing technology. Do not intervene and let students know what shape the graphs *should* have, as they will come to these

conclusions as they work through the problems. There are opportunities built into the problems for them to recognize omissions in earlier parts of the problems and to revisit and correct that work.

After teams complete problem 2-132, bring the class together to have students share their observations about the graph and the relationships. This is another opportunity to discuss what it means for a relationship to be a function. At this time you can expect students to make the following observations about non-functions:

- Some or all x -values have two y -values in the table and on the graph.
- They are not necessarily in $y =$ form.
- The y could be squared.
- You cannot easily graph it on the graphing calculator.
- It may be easier to substitute values for y rather than x to generate ordered pairs to graph it.

Students might have variations on the above list, but it is essential that they have something comparable to the first bulleted item above.

To graph the equation of these non-functions with a calculator in problem 2-133, students will need to rewrite the equations in $y=$ form. Most calculators can only graph functions. As students solve the equation for y , verify that they are using an absolute value and thus coming up with two functions to graph each of these non-functions:

$x = y^2$	$x^2 + y^2 = 25$
$\sqrt{x} = \sqrt{y^2}$	$y^2 = 25 - x^2$
$\sqrt{x} = y $	$\sqrt{y^2} = \sqrt{25 - x^2}$
$y = +\sqrt{x}$	$ y = \sqrt{25 - x^2}$
or $y = -\sqrt{x}$	$y = +\sqrt{25 - x^2}$
	or $y = -\sqrt{25 - x^2}$

Give teams time to enter the equations into their graphing calculators. To see the full graph in the calculator, students will have to enter both equations into their calculators. Note that most graphing calculators will display graphs of semicircles that do not look perfectly semicircular because different scales are used for the axes. A graph with the same scale on both axes can be created on a TI83+/84+ calculator by pressing **ZOOM** "5:ZSquare".

This is a good place to end Day 1. Have students put their materials in a safe place for the next day.

Day 2: To open the second day, give students two minutes to remind each other of what they are working on. You can do this with a

Reciprocal Teaching. Then get them started on problem 2-134.

Problem 2-134 turns the focus of the lesson to transforming the graphs of circles. This investigation asks students to use what they know about transforming graphs of parent functions to see which ideas translate to the circles with which they are now working. Note that students are expected to find just the transformations of a circle and not an ellipse. While they may experiment with adding a coefficient a before the radical to vertically stretch or compress the graph, it is not necessary for them to include this transformation. You can ask questions such as, “Does this maintain the shape of the graph?” Students will manipulate the radius of the circle later in the lesson.

Many teams may struggle with the algebraic manipulations necessary for problem 2-135. In this case, consider stopping to ask a team to lead a discussion in which the class can work together to rewrite one or both of the general equations. Another possibility would be to do an I Spy or Swapmeet and send members from each team to other teams to get ideas. When teams have completed problem 2-135, ask them to share their ideas about the significance of the locator point (h, k) .

Then move teams on to problem 2-136, which focuses on the impact of the radius as a parameter for the graph.

If there is time, problem 2-137 is provided as additional practice.

Problem 2-138 considers the transformation of the “sleeping parabola” $x = y^2$ from problem 2-133. Teams are likely to get $x = \frac{(y-k)^2}{a^2} + h$. Help them to see that, since a represents an unknown constant and $\frac{1}{a^2}$ is just another unknown constant, the equation can be rewritten as $x = b(y-k)^2 + h$, which is similar in form to the vertically oriented parabola and is more useful. The parameter b gives the horizontal stretch, k gives the vertical shift, and h gives the horizontal shift.

Closure:
(10 minutes)

Ask students to compare and contrast the significance of the locator point (h, k) in the various families of functions. Compare and contrast the position of h and k in the general equations for each of the families. This is also a good place to consider the impact of a coefficient a in the general equation for the circle $y = \pm a\sqrt{25 - (x-h)^2} + k$. You could also ask what happens to the circle when a is negative. (The circle simply flips onto itself.) Challenge students to observe the connections as well as identify *why* the equations and graphs behave as they do.

Universal Access: Problems 2-132, 2-133 and 2-134:

Depending on your class, some students may struggle with rewriting the equations in graphing form. You might consider calling a Huddle, Sending a Spy, and/or having a class discussion. You could have a student or team lead the discussion. However, if no one is able to rewrite the equations, you may have to provide prompts and cues to guide the students through the steps of rewriting the equations in

graphing form. You could also refer the students back to the Math Notes section from Lesson 2.2.3 or possibly a chart on the wall (if you have one in the classroom).

Problems 2-135 and 2-136:

EL and/or special needs students may need a quick review about the characteristics of a circle, and the equation of a circle. This could be accomplished with a Whip Around or Reciprocal Teaching.

If students are struggling with these questions, be sure to have specific prompts/cues and questions ready to move students forward.

Team Strategies:

If you are using Team Roles, remind students that actively performing their roles will help their teams work most effectively. Teams will need to discuss and share ideas openly as they try to apply tools for transforming graphs to the two non-functions in this lesson. Based on the particular needs of your class, you may want to highlight a particular norm or behavior that you think needs attention to improve teamwork.

Homework:

Day 1: Problems 2-139 through 2-145

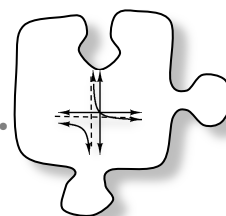
Day 2: Problems 2-146 through 2-152

Note: Problem 2-152 is Checkpoint 2B for solving linear systems in two variables.

Notes to Self:

2.2.4 How can I transform circles?

Transforming Non-Functions



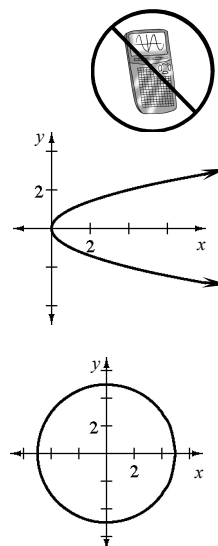
In this lesson, you consider two new parent equations that are different from the ones you have seen in the past because they are not functions. You will investigate them and apply the knowledge you have gained in this chapter to transform them. You will identify ways in which these new equations are different from the functions with which you have been working.

2-132. Begin by fully investigating $x = y^2$ and $x^2 + y^2 = 25$ as follows.

- a. Without using your graphing calculator, make a table and a graph for each equation. [See tables and graphs at right.]
- b. Marabel and Lissa were working on this problem. Marabel was making a table for $x = y^2$. For an x -value of 4, she found a y -value of 2. Lissa was watching and said, “Wait! When x is 4, there is also another possible value for y .” What did Lissa mean? Look back at your tables and decide if there are more points you could add.
[-2]

x	y
0	0
1	1
4	2
9	3
16	4

x	y
-5	0
-2	$\sqrt{21}$
0	5, -5
2	$\sqrt{21}$
5	0



- c. Now describe $x = y^2$ and $x^2 + y^2 = 25$ completely. This includes finding the domain and range of each equation, finding the important points such as intercepts, and describing what happens to y as x increases.
- d. How are these relationships different from others you have been working with?
[They are not functions. Students may say that some x -values have more than one y -value, or that the y value is squared.]

2-133. Rewrite $x = y^2$ and $x^2 + y^2 = 25$ so that you can graph them with your graphing calculator. When you have rewritten both equations, try graphing them using your calculator. Do they look like the graphs you made in problem 2-132? [Graph two separate functions for each relationship since they must be in the form “ $y =$ ” and most calculators do not have a \pm button; $y_1 = \sqrt{x}$, $y_2 = -\sqrt{x}$ and $y_1 = \sqrt{25 - x^2}$, $y_2 = -\sqrt{25 - x^2}$; within the limitations of the window settings they should resemble the graphs in problem 2-132.]

2-134. TRANSFORMATIONS OF NON-FUNCTIONS

In order to graph the equation of the circle on your graphing calculator, you had to express the non-function as two functions. Now apply your knowledge of transforming functions to learn about transforming circles.

Your Task: As a team, transform the graphs of $y = \pm\sqrt{25 - x^2}$ horizontally and vertically. Then find a general equation for this family of circles using h , and k . Be prepared to share your findings and your strategies with the class. [Students are most likely to find general equations of the form $y = \pm\sqrt{25 - (x - h)^2} + k$, at this point.]

Discussion Points

How did we change the equation in other families so that the graph moves vertically?
So that it moves horizontally?

How can we rewrite the two functions for a circle the same way?

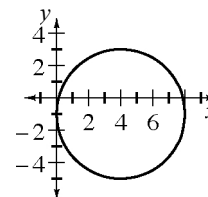
- 2-135. Write your general equations for a circle in standard form by rewriting the equation $y = \pm\sqrt{-(x-h)^2 + 25} + k$ to isolate 25 on one side of the equation. What information does the locator point (h, k) give about the graph of the circle?
[**$25 = (x-h)^2 + (y-k)^2$; the center of the circle is (h, k) .**]

- 2-136. A circle has a special characteristic, its radius, which defines its size.

- a. Refer back to the graph of $x^2 + y^2 = 25$. What is the radius? How is the radius of the circle related to the equation?
[**The radius is 5; it is the square root of 25.**]
- b. What would be the equation of a circle that has its center at $(5, -7)$ with radius 10? With radius 12? [**$(x-5)^2 + (y+7)^2 = 100$, $(x-5)^2 + (y+7)^2 = 144$.**]
- c. Now generalize the connection between the radius and the equation of a circle. Write a general equation for a circle with any center (h, k) and radius r .
[**$(x-h)^2 + (y-k)^2 = r^2$**]
- d. Given the equation $(x-3)^2 + (y+7)^2 = 169$, how can you find the radius of the circle? [**Take the square root of 169 to get a radius of 13.**]

- 2-137. Consider the equation $(x-4)^2 + (y+1)^2 = 16$.

- a. What is the shape of the graph? How can you tell? [**The graph is a circle with radius of 4. It is in the form $(x-h)^2 + (y-k)^2 = r^2$.**]
- b. What information can you learn about the graph just by looking at the equation?
[**The circle's center is at $(4, -1)$ and the radius is 4.**]
- c. Sketch a graph of $(x-4)^2 + (y+1)^2 = 16$.
[**See graph at right.**]



- 2-138. Look at your work from problem 2-133. The non-function $x = y^2$ had a graph that is called a “sleeping parabola.”

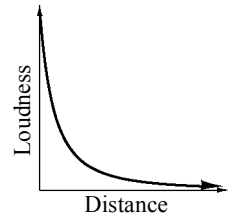
- How could you transform the equation $y = \pm\sqrt{x}$ to move the graph horizontally and vertically? How could you transform the equation to stretch or compress the graph, or to “flip” it vertically? [**Answers will vary as students experiment with a , h , and k .**]
- Write a general equation for transforming the sleeping parabola family $y = \pm\sqrt{x}$ by using a , h , and k . [$y = \pm a\sqrt{x-h} + k$]
- Write the equation for a sleeping parabola in standard form by isolating x on one side of the equation. [**See the “Suggested Lesson Activity”.**
 $x = b(y-k)^2 + h$]



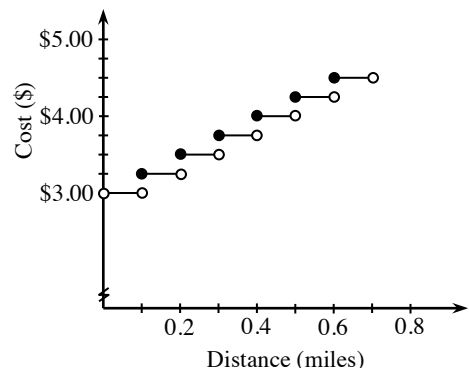
2-139. Write the equation $y = x^2 + 7x - 8$ in graphing form. [$y = (x + 3.5)^2 - 20.25$]

2-140. You are standing outside the school, waiting to cross the street, when you hear booming music coming from an approaching car.

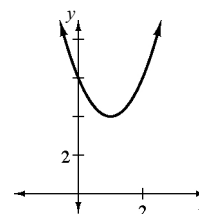
- Sketch a graph that shows the relationship between how far away from you the car is and the loudness of the music.
[**See graph at right. Note that this graph is not very intuitive—a distance versus loudness graph starts high and decreases. Many students will graph a bell shape. If so, this is a good time to do some whole-class graphical interpretation. The bell shape could be argued as appropriate if the student sees his or her position as the origin and the negative side of the x -axis as representing direction. Students should talk about what they are visualizing. Seeing themselves with the distance first decreasing then increasing is different from the way distance is usually graphed on the x -axis, small to large.**]
- Which is the dependent variable and which is the independent variable?
[**Loudness depends on distance.**]



2-141. The Green Streak Taxi Company charges a \$3.00 base fee plus \$2.50 per mile. The cab driver sets his meter at \$3.00 and the meter adds \$0.25 each one-tenth of a mile. Draw a graph to represent this fare structure. Describe the domain and range of your graph. [**See graph at right. The domain is all positive numbers (or $d > 0$). The range is all real numbers greater than 3 and that are multiples of 0.25.**]

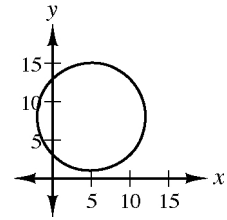


- 2-142. Write an equation for a function that is odd, and explain how you can tell it is odd from its graph, its table and its equation. [**Answers will vary.**]
- 2-143. Explain the difference between the graphs of $y = \frac{1}{x}$ and $y = 4(\frac{1}{x+5}) + 7$. [**The second graph shifts the first 5 units left and 7 units up and stretches it by a factor of 4.**]
- 2-144. Multiply the expressions in parts (a) through (c) to remove the parentheses.
- a. $(x-1)(x+1)$ b. $2x(x+1)(x+1)$ c. $(x-1)(x+1)(x-2)$
 [$x^2 - 1$] [$2x^3 + 4x^2 + 2x$] [$x^3 - 2x^2 - x + 2$]
- d. Find the x - and y -intercepts of $y = x^3 - 2x^2 - x + 2$. The factors in part (c) should be useful. [y : (0, 2), x : (1, 0), (-1, 0), (2, 0)]
- 2-145. Solve the following systems of equations. In other words, find values of a and b that make each system true. Be sure to show your work or explain your thinking clearly.
- a. $2 = a \cdot b^0$ b. $\frac{1}{2} = a \cdot b^0$
 $\frac{1}{2} = a \cdot b^2$ $2 = a \cdot b^2$
 [$(a, b) = (2, \pm \frac{1}{2})$] [$(a, b) = (\frac{1}{2}, \pm 2)$]
- 2-146. A parabola has vertex (3, 5) and contains the point (0, 0).
- a. If this parabola is a function, find its equation. [$y = -\frac{5}{9}(x-3)^2 + 5$]
- b. Suppose this parabola is not a function, but is a “sleeping” parabola. Find its equation. [$x = -\frac{3}{25}(y-5)^2 + 3$]
- 2-147. Sketch the graph of $y = 2(x-1)^2 + 4$. [**See graph at right.**]
- a. Now rewrite the equation $y = 2(x-1)^2 + 4$ without parentheses. Remember the Order of Operations!
 [$y = 2x^2 - 4x + 6$]
- b. What would the difference be between the graphs of the two equations above? This is sort of a trick question, but explain your reasoning. [**There is no difference, but the explanations vary.**]
- c. What is the parent function of $y = 2(x-1)^2 + 4$? [$y = x^2$]
- d. What is the parent function of $y = 2x^2 - 4x + 6$? [$y = x^2$]



2-148. Consider the equation $(x-5)^2 + (y-8)^2 = 49$.

- a. What can you tell about the graph just by looking at the equation? [**The graph will be a circle with a center at (5, 8) and a radius of 7.**]
- b. Sketch a graph of $(x-5)^2 + (y-8)^2 = 49$.
[**See graph at right.**]



2-149. A line passes through the points (0, 2) and (1, 0).

- a. Find the slope of the line. [**-2**]
- b. Find the slope of a line parallel to the given line. [**-2**]
- c. Find the slope of a line perpendicular to the given line. [$\frac{1}{2}$]
- d. Find the product of the slopes you found in parts (b) and (c). [**-1**]
- e. Make a conjecture about the product of the slopes of any two perpendicular lines. Test your conjecture by creating more examples. [**The product of the slopes of any two perpendicular lines is -1.**]

2-150. Give the equations of two functions, $f(x)$ and $g(x)$, so that $f(x)$ and $g(x)$ intersect at exactly: [**Answers vary.**]

- a. One point.
- b. Two points.
- c. No points.

2-151. Find the x - and y -intercepts for the following parabolas.

- a. $y = (x+12)^2 - 144$
[**(0, -144), (0, 0) and (24, 0)**]
- b. $y = (x-8)^2 - 4$
[**(0, -144), (0, 0) and (24, 0)**]

2-152. This problem is a checkpoint for solving linear systems in two variables. It will be referred to as Checkpoint 2B.



Solve the system of linear equations at right. [**(3, 2)**]

$$5x - 4y = 7$$

$$2y + 6x = 22$$

Check your answers by referring to the Checkpoint 2B materials located at the back of your book.

If you needed help solving these problems correctly, then you need more practice. Review the Checkpoint 2B materials and try the practice problems. Also, consider getting help outside of class time. From this point on, you will be expected to do problems like these quickly and easily.

Lesson 2.2.5 Can I combine functions?

Transforming Piecewise-Defined Functions

Lesson Objective:	Students will use what they know about transformations to relocate and reorient a piecewise-defined function.
Mathematical Practices:	make sense of problems and persevere in solving them, reason abstractly and quantitatively, attend to precision, look for and make use of structure
Length of Activity:	One day (approximately 50 minutes)
Core Problems:	Day 1: Problems 2-153 through part (c) of problem 2-156 Day 2: Part (d) of problem 2-156 through 2-157
Materials:	Lesson 2.2.5A Resource Page, one per student Lesson 2.2.5B Resource Page, one for teacher Poster-making materials, optional
Suggested Lesson Activity:	<p>Day 1: Begin class by having students read the introduction in their teams. Have a brief discussion of other situations where a step function would be convenient. Then have students begin problems 2-153 and 2-154. Problem 2-153 introduces piecewise-defined functions in the context of more familiar step functions. Guide students to start writing the domain and range formally, as in the solution to part (a) of problem 2-153. This is also prompted in part (b). This would be a good day to use Red Light, Green Light, so the students will need to check their solutions to each problem before moving on.</p> <p>When teams have completed problem 2-154, where they create their own piecewise-defined functions, verify that they have created an actual function: each x input must have only one y output, although there can be x values that are not defined.</p> <p>When most teams have completed problem 2-154, move them on to problem 2-155 where they will practice writing a series of equations to define the different pieces of a piecewise-defined function.</p> <p>When teams begin work on problem 2-156, pass out the Lesson 2.2.5A Resource Page. (The Lesson 2.2.5B Resource Page contains solutions for your convenience.) In this problem, students will investigate and transform the piecewise-defined function, $F(x)$, that they defined in problem 2-155. Their work will probably be based on shifting either the graph or the table, point-by-point, rather than trying to deal with the four equations and restricted domain that are required to write the “equations” for each new function. They should write the equations, but the tables and graphs will probably be easier to work with for most students. The investigation should include a table, a graph, the domain and range, and important points for each graph.</p>

Students will probably finish part (c) of problem 2-156 on Day 1.

Day 2: Teams should continue where they left off on Day 1.

The most difficult transformation will be part (d) of problem 2-156 which is $F(x - 4)$, the horizontal shift. Check in with teams to see how they are thinking about this problem. Based on their experience with the translations of other graphs, they might visualize the whole graph as a rigid piece and slide it to the right 4 units. In this case they need to recognize that the new domain is $0 \leq x \leq 11$. Some students will use the table. In this case they will have to grapple with the order of operations. For example, to figure out that the transformed point for $x = 3$ in the function $F(x - 4)$ they will find $F(3 - 4) = F(-1) = -3$. Therefore the point $(3, -3)$ is in the table and on the graph for $F(x - 4)$. Also, the transformed point for $x = -4$ does not exist because $-4 - 4 = -8$, and -8 is not part of the domain of the function, but the transformation of the point for $x = 11$ does.

Once teams have a graph and table for each transformation, they can use them to adjust the equations and their domains. Questions such as *“How do you write an equation of a parabola that is shifted to the right?”* and *“How would you shift other parent graphs to the right?”* or *“How could you use the table to get the y-value for an x-value of 5 or 10 or -3?”* will help students focus on the graph or the table as a way to start.

Students explore the graphs of odd and even functions in problem 2-157. When teams are about halfway through part (b) of problem 2-157, you may want to stop for a quick whole-class discussion. Teams can share how they are using the calculator to efficiently find whether a function is odd, even, or neither. Then move teams on to making conjectures in part (c) and testing them in part (d) of problem 2-157.

Depending on how much time you have left, some teams could complete problems 2-158 and 2-159, while other teams could complete problem 2-160.

Any of these problems could be used to create a stand-alone poster. If time is short, however, teams should move directly to the Learning Log in problem 2-161.

Closure:
(8 minutes)

If students had time to complete problems 2-158, 2-159, or 2-160, have them post their work and discuss any questions. This could also be done using a Traveling Salesman.

Refer students to problem 2-161, which asks them to write in their Learning Log about how to transform graphs no matter what the function. Remind them that at the end of this chapter they should be able to transform any function without a calculator as long as they know the shape of the graph. Today’s Learning Log prompt will help them think about what they do when transforming graphs. This might be a

good opportunity to do a Peer Edit. See the following “Team Strategies” section.

Universal Access: Problems 2-153 and 2-154:

Some students may not recognize the terms “piecewise-defined functions” and “step functions.” For both terms, you could connect each of them to their meaning. Piecewise-defined functions have more than one “piece”; this means that they have more than one formula/expression/equation. Step functions are functions whose graphs look like “steps”; they are a series of horizontal line segments that look like small steps.

Some students may not understand why some points on the graph are open and some are shaded in. Explain that they are like red and green traffic lights, show where the line segments start and stop, and indicate whether the endpoint is included (shaded dot) or not (open dot). Be prepared to provide further explanation, prompts/cues and questions to move students forward.

Also, if students are still struggling with identifying the domain and range, do not forget the “framing” strategy identified in Chapter 1.

Problem 2-155:

Depending on your class, some students may struggle with writing equations based on the graph and table in part (b). In addition, identifying the domain and range for each part of the piecewise-defined function could be problematic. You might consider Sending a Spy, a Traveling Salesman, and/or having a class discussion. You could have a student or team lead the discussion. Be prepared to provide prompts/cues and ask thought-provoking questions to guide the students through the steps of writing the equations and identifying the domain and range.

Problem 2-156:

If students have difficulty visualizing the movements, you could provide a piece of patty paper (tracing paper) and/or a piece of transparency. Students can trace the original shape and then move it accordingly.

Some students may struggle because the equations are not in graphing form. Remind students of the general equation for the family of functions and how each variable affects the graph.

Problem 2-157:

If students are having difficulty distinguishing between even and odd functions, refer them to the Math Notes – Methods and Meanings for this lesson.

Math Notes – Methods and Meanings:

This provides a good opportunity to review even and odd functions.

Consider a way for students to organize and access this information.

Team Strategies:

Peer Edit allows peers to give each other feedback either orally or in writing. It can be used especially when taking notes on Math Notes or writing Learning Log entries. After the students consolidated their thoughts in writing, they find a partner and do a Peer Edit using one of the following formats.

One way to do Peer Edit is to have students work in pairs. Peer A reads their paragraph to peer B. Peer B then asks questions for clarification and gives suggestions or compliments. Then they switch roles. Reading out loud can offer a different perspective than writing or reading silently does.

The other way to do a Peer Edit is to have the two peers switch papers and then read and edit the work. Then they return the paper. The peers should read the comments, and rewrite, if needed.

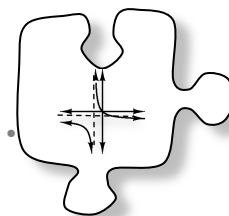
Homework:

Problems 2-162 through 2-169

Notes to Self:

2.2.5 Can I combine functions?

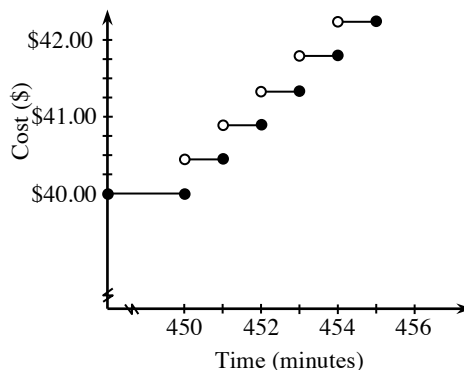
Transforming Piecewise-Defined Functions



Often the equation for a single familiar function describes a part of a situation, but then is not a good description for the rest of the situation. A step graph is one kind of **piecewise-defined function**. The graphs of these situations are functions, but a single equation is not sufficient to describe them. Describing them requires two (or more) different equations for different inputs. Today you will build new functions by using pieces of familiar functions. Phone plans and water rates are situations that can be modeled using step functions.

- 2-153. The Horizon Phone Company offers a basic monthly voice phone plan where you pay \$40.00 for the first 450 minutes and then \$0.45 per minute after that. The graph at right shows how the plan works.

- a. This is a piecewise-defined function with many pieces. Describe each piece and the domain and range for the function overall. Then describe the domain and range for the first few pieces.



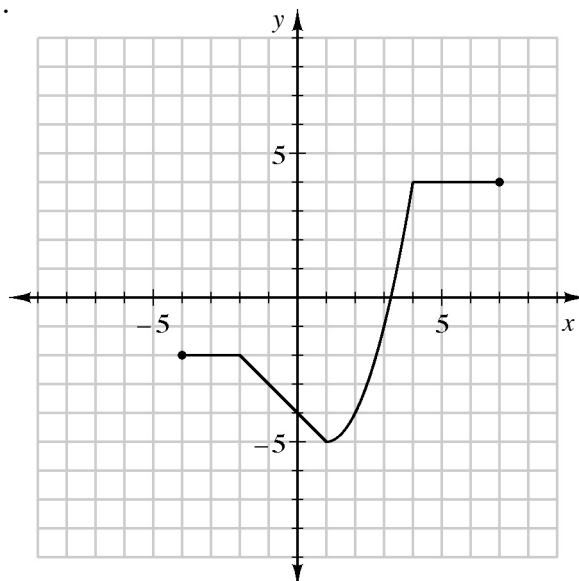
Problem continues on next page →

- 2-153. *Problem continued from previous page.*
 [Overall $x \geq 0$, $y = 40.00, 40.45, 40.90, 41.35, \dots$.
First piece: $0 \leq x \leq 450$, $y = 40$
Second piece: $450 < x \leq 451$, $y = 40.45$
Third piece: $451 < x \leq 452$, $y = 40.90$]

b. Write an equation for each part of the domain. [See solution to part (a).]

- 2-154. With your team, create a piecewise-defined function with at least three “pieces.” The function does not need to be a step-function with horizontal line segments, but it needs to meet the definition of a function. Make a table and a graph for your function, and write an equation for each part. Be sure to state the domain for each part, as well as the domain for the whole function. [**Answers vary.**]

- 2-155. Here is another piecewise-defined function, $F(x)$, defined in the domain $-4 \leq x \leq 7$ by the graph below.



- a. Because there is no single equation that represents the whole graph, it is often useful to make a more complete table than you might usually make. On your paper, fill in a table for the function like the one below. [**Solutions listed in bold in the table below.**]

x	-4	-3	-2	-1	0	1	2	3	4	5	6	7
$F(x)$	-2	-2	-2	-3	-4	-5	-4	-1	4	4	4	4

- b. Use the graph and the table that you made to write equations for each part of the piecewise-defined function. Be sure to state the domain for each part.
 [$-5 \leq x < -2$, $y = -2$; $-2 \leq x < 1$, $y = -4 - x$; $-1 \leq x \leq 7$, $y = (x - 1)^2 - 5$; $4 \leq x \leq 7$, $y = 4$.]

2-156. TEAM TRANSFORMATION CHALLENGE

Obtain the Lesson 2.2.5A Resource Page. Use the resource page to show the graph, the table, and the equations for each of the following transformations of the piecewise-defined function in problem 2-155. [**The Lesson 2.2.5B Resource Page contains solutions.**]

a. $y = -F(x)$

b. $y = \frac{1}{2} F(x)$

c. $y = F(x) + 4$

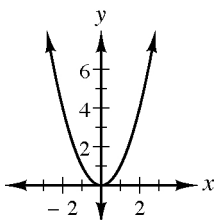
d. $y = F(x - 4)$

2-157. GRAPHS OF ODD AND EVEN FUNCTIONS

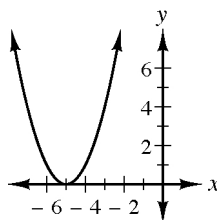
Your goal in this investigation is to determine whether a function is odd or even by looking at its graph.

- a. Use your graphing calculator to graph the following functions, and make a quick sketch of each graph on your paper. Be sure to label each graph.

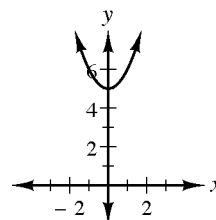
$$f(x) = x^2$$



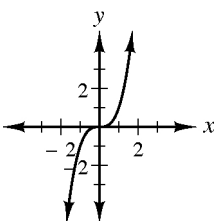
$$f(x) = (x + 5)^2$$



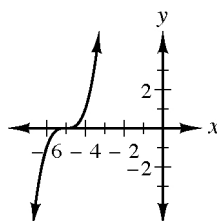
$$f(x) = x^2 + 5$$



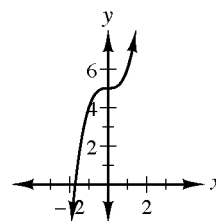
$$f(x) = x^3$$



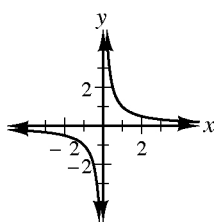
$$f(x) = (x + 5)^3$$



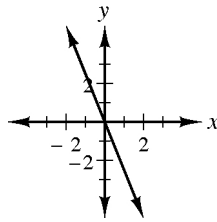
$$f(x) = x^3 + 5$$



$$f(x) = \frac{1}{x}$$



$$f(x) = -2.5x$$



Problem continues on next page →

2-157. *Problem continued from previous page.*

- b. Determine which of the functions above are odd, even, or neither. Can you find an efficient way to do this with your graphing calculator?

[**One efficient method is to examine the tables for $f(-x)$ and $-f(x)$ for each function, and determine which are equivalent to the table for $f(x)$.**

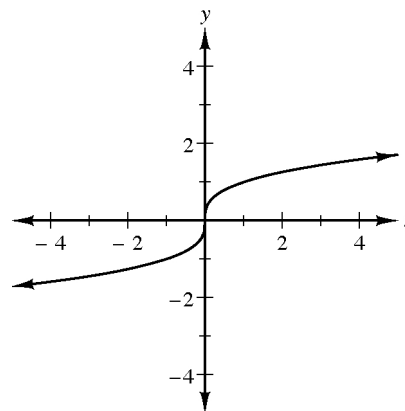
Even: $f(x) = x^2$, $f(x) = (x + 5)^2$; **Odd:** $f(x) = x^3$, $f(x) = \frac{1}{x}$, $f(x) = -2.5x$;

Neither: $f(x) = x^2 + 5$, $f(x) = (x + 5)^3$, $f(x) = x^3 + 5$.]

- c. How can you tell by looking at the graph whether a function is odd, even, or neither?

[**Even functions have reflective symmetry across the y-axis, and odd functions have 180-degree rotational symmetry about the origin.**]

- d. Classify the function at right as odd, even, or neither. Explain. [**Odd, because it has 180-degree rotational symmetry about the origin.**]



2-158. Write an equation for an even function of your own. Now write another function of your own that is odd. Show that your functions meet the even/odd criteria. The functions you use do not have to be parent functions.

2-159. Write an equation for a function that is neither even nor odd. Show that it is neither.

2-160. AN ADDITIONAL CHALLENGE

In general, a transformation for the parent function $F(x)$ in problem 2-155 can be represented by $y = aF(x - h) + k$. With your team, choose your own values for a , h , and k and show the table, graph, and equations for your transformed $F(x)$. [**Answers vary. a will stretch or compress the graph vertically, and if a is negative the graph will be flipped vertically. h will move the graph horizontally. k will move the graph vertically. If x is negated, the graph will reflect across the y-axis.**]

2-161. LEARNING LOG

In the last few lessons, you have developed the ability to create a family of functions by transforming *any* parent function.

Does the function you start with (the parent) affect how you will transform it? If so, how? If not, why not? Are there any parent graphs that are hard for you to transform? Why or why not? Write a Learning Log entry answering these questions. Title it “Transform Any Function” and label it with today’s date.





METHODS AND MEANINGS

Even and Odd Functions

When a function $f(-x) = f(x)$, the function f is called an **even function**. For example, for the function $f(x) = x^2$:

$$f(-x) = (-x)^2 = x^2 = f(x)$$

Thus, $f(x) = x^2$ is an even function.

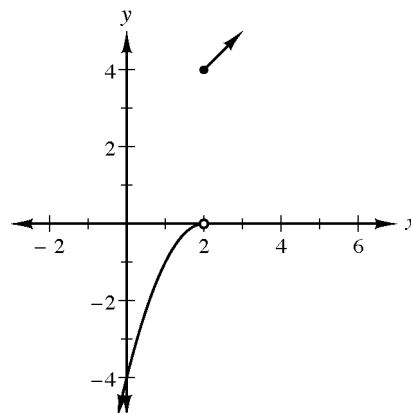
When $f(-x) = -f(x)$, the function f is called an **odd function**. For example for the function $f(x) = x^3$:

$$f(-x) = (-x)^3 = -x^3 = -f(x)$$

Therefore, $f(x) = x^3$ is an odd function.

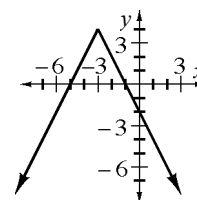


- 2-162. Write a set of equations for the piecewise-defined function shown on the graph at right. Be sure to include the domain for each part of the function.
 $[x < 2, y = -(x - 2)^2; 2 \leq x, y = |x + 2|]$



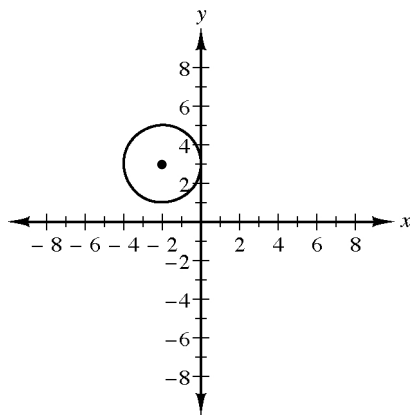
- 2-163. Write an equation for a function that is even. Then explain how you can tell it is even from its graph, its table, and its equation.

- 2-164. Use your knowledge of absolute value functions to find the equation of the graph at right. $[y = -2|x + 3| + 4]$

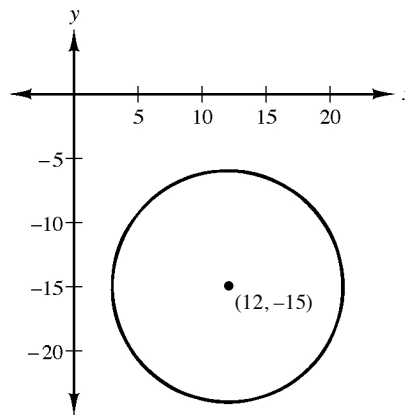


2-165. Write an equation for each of the circles shown in the graphs below.

a. [$(x+2)^2 + (y-3)^2 = 4$]



b. [$(x-12)^2 + (y+15)^2 = 81$]

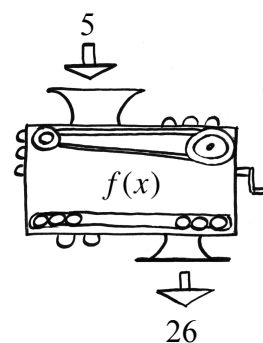
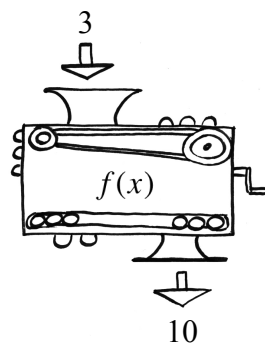
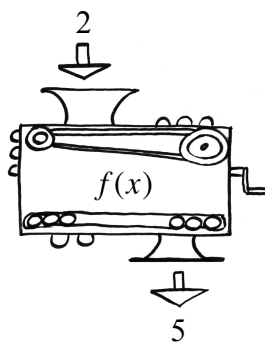
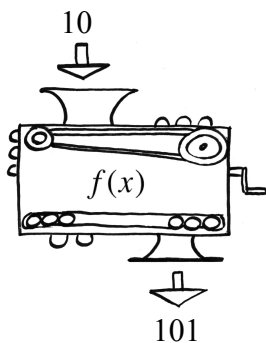


2-166. Use the technique of completing the square to express $y = x^2 - 5x + 7$ in graphing form and state the vertex. [$y = (x - 2.5)^2 + 0.75$, vertex $(2.5, 0.75)$]

2-167. Shortcut Shuneel claims he has a shortcut for finding the vertex of a parabola. While using his shortcut on $y = 2x^2 + 3x + 1$, he ended up with $y = 2(x + \frac{3}{4})^2 - \frac{7}{2}$. Is Shuneel's new equation correct? Why or why not? [**He is incorrect. Justifications vary.**]



2-168. Remember function machines? Each of the following pictures shows how the same machine changes the given x -value into a corresponding $f(x)$ value. Find the equation for this machine. [$f(x) = x^2 + 1$]



2-169. If $x^2 + kx + 18$ is factorable, what are the possible values of k ? [$\pm 11, \pm 9, \pm 19$]

Chapter 2 Closure What have I learned?

Reflection and Synthesis

Read more about the different types of activities in the Closure section in the front matter of this Teacher Edition.

Closure Objective: The objective of the chapter closure depends on which closure activity you choose to use. However, some of the main objectives of each activity are for the students to reflect about what they have learned and for the teacher to be able to assess informally where the students are in their understanding.

Length of Activity: Varies

Materials: **For teachers using *Portfolio*:**

Chapter 2 Closure Resource Page: Transformations GO

**① Team
Brainstorm**

In this chapter, *Team Brainstorm* includes an additional portion for students to consider the Standards for Mathematical Practice: “What Mathematical Practices did you use in this chapter? When did you use them? Give specific examples.”

See the Closure section of this Teacher Edition for more information about *Team Brainstorm*.

**② Making
Connections**

See the Closure section of this Teacher Edition for more information about *Making Connections*.

**③ Portfolio with
Growth Over Time**

In this section, students are given the chance to summarize their understanding of one or more central topics of the chapter. For general *Portfolio* ideas, see *both guidebooks* of the Closure and Assessment tab in the front matter of this Teacher Edition.

Growth Over Time:

Explain everything you know about $y = x^2 - 4$ and $y = \sqrt{x + 4}$

This problem will appear again in Chapter 6.

Graphic Organizer (primarily for English Language Learners): For students who need extra English language support, the “Transformations GO” (graphic organizer) is available as a resource page. This could allow students to show their understanding of the meaning of transformations in a more structured way.

**④ What Have I
Learned?**

This section gives students the opportunity to see if they can work with the current topics at the expected level.

Universal Access:

The Chapter 2 Concept Map Cards can be used in a variety of ways. Simple flash cards can be made to further support EL and/or special needs students with definitions on one side and a diagram/example on

the back.

Portfolio Problems are critical to helping students solidify their understanding of important concepts, can be used as formative assessment, and can provide evidence of mathematical proficiency.

If you haven't already, this is a good time to get your students familiar with the Common Core State Standards for Mathematical Practices. You and your students should identify the practices used in this chapter. Be specific: identify problems/lessons that those practices were used.

Chapter 2 Closure What have I learned?

Reflection and Synthesis

The activities below offer you a chance to reflect about what you have learned during this chapter. As you work, look for concepts that you feel very comfortable with, ideas that you would like to learn more about, and topics you need more help with. Look for connections between ideas as well as connections with material you learned previously.



① TEAM BRAINSTORM

What have you studied in this chapter? What ideas were important in what you learned? With your team, brainstorm a list. Be as detailed as you can. To help get you started, a list of Learning Log entries and Math Notes boxes are below.

What topics, ideas, and words that you learned *before* this chapter are connected to the new ideas in this chapter? Again, be as detailed as you can.

Next consider the Standards for Mathematical Practice that follow Activity ③: Portfolio. What Mathematical Practices did you use in this chapter? When did you use them? Give specific examples. How long can you make your list? Challenge yourselves. Be prepared to share your team's ideas with the class.



Learning Log Entries

- Lesson 2.1.5 – Forms of a Quadratic Function
- Lesson 2.2.2 – How to use (h, k)
- Lesson 2.2.3 – Reflections and Even Functions
- Lesson 2.2.5 – Transform Any Function

Math Notes

- Lesson 2.1.1 – Exponential Functions
- Lesson 2.1.3 – Forms of Quadratics
- Lesson 2.1.4 – Finding Graphing Form and Vertex of Parabolas
- Lesson 2.2.2 – Point-Slope Equations for Lines
- Lesson 2.2.3 – General Equations for Families
- Lesson 2.2.5 – Even and Odd Functions

②

MAKING CONNECTIONS

Below is a list of the vocabulary used in this chapter. Make sure that you are familiar with all of these words and know what they mean. Refer to the glossary or index for any words that you do not yet understand.

compress	dilate	domain
even function	function	general equation
graphing form	(h, k)	horizontal shift
odd function	parameter	parent graph
piecewise-defined function	range	reflection
shrink	standard form	step function
stretch	transformation	translation
variable	vertex	vertex form
vertical shift		

Make a concept map showing all of the connections you can find among the key words and ideas listed above. To show a connection between two words, draw a line between them and explain the connection. A word can be connected to any other word as long as you can justify the connection.

While you are making your map, your team may think of related words or ideas that are not listed here. Be sure to include these ideas on your concept map.

③

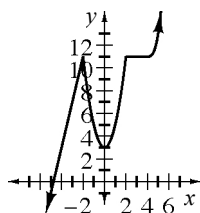
PORTFOLIO: EVIDENCE OF MATHEMATICAL PROFICIENCY

This section gives you an opportunity to show growth in your understanding of key mathematical ideas over time as you complete this course. Several options are presented below. Follow your teacher's directions for which options you should complete.

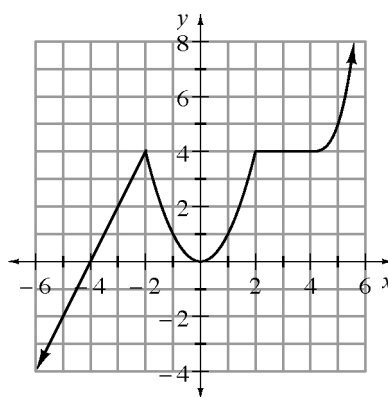
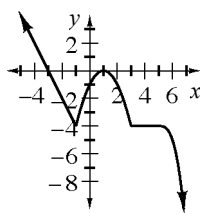


- Explain everything that you know about $y = x^2 - 4$ and $y = \sqrt{x + 4}$.
- Your teacher may give you the Chapter 2 Closure Resource Page: Transformations Graphic Organizer to complete. Use it to organize and show everything that you know about transforming the parent graph(s) your teacher gives you.
- The function $g(x)$ is graphed at right. Graph each of the following functions.

$$y = 2g(x) + 3$$



$$y = -g(x - 1)$$



- In Section 2.1, you generalized about how to write equations in graphing form by completing the square. For example, you learned how to complete the square by analyzing actual squares that represented the algebraic quantities you were working with, such as in the expression represented by the tiles at right. Then you generalized the process to complete the square for expressions involving negative and fractional terms, which are not easy to represent with tiles. Consider this as you answer the questions below.

	x	+	4	
x	x^2	x	x	x
+	x	1	1	1
	x	1	1	1
	x	1	1	
4	x			

- In general, what can you do to complete the square for a quadratic function, no matter what the x term is?
 - Use the general strategy you described in part (i) above to complete the square for the quadratic equation $f(x) = x^2 - 4.5x + 17$
 $[f(x) = (x - 2.25)^2 + 11.9375]$
- Consider the Standards for Mathematical Practice that follow. What Mathematical Practices did you use in this chapter? When did you use them? Give specific examples.

BECOMING MATHEMATICALLY PROFICIENT

The Common Core State Standards For Mathematical Practice

This book focuses on helping you use some very specific Mathematical Practices. The Mathematical Practices describe ways in which mathematically proficient students engage with mathematics everyday.



Make sense of problems and persevere in solving them:

Making sense of problems and persevering in solving them means that you can solve problems that are full of different kinds of mathematics. These types of problems are not routine, simple, or typical. Instead, they combine lots of math ideas and everyday situations. You have to stick with challenging problems, try different strategies, use multiple representations, and use a different method to check your results.

Reason abstractly and quantitatively:

Throughout this course, everyday situations are used to introduce you to new math ideas. Seeing mathematical ideas within a context helps you make sense of the ideas. Once you learn about a math idea in a practical way, you can “**reason abstractly**” by thinking about the concept more generally, representing it with symbols, and manipulating the symbols. **Reasoning quantitatively** is using numbers and symbols to represent an everyday situation, taking into account the units involved, and considering the meaning of the quantities as you compute them.

Construct viable arguments and critique the reasoning of others:

To **construct a viable argument** is to present your solution steps in a logical sequence and to justify your steps with conclusions, relying on number sense, facts and definitions, and previously established results. You communicate clearly, consider the real-life context, and provide clarification when others ask. In this course, you regularly share information, opinions, and expertise with your study team. You **critique the reasoning of others** when you analyze the approach of others, build on each other’s ideas, compare the effectiveness of two strategies, and decide what makes sense and under what conditions.

Model with mathematics:

When you **model with mathematics**, you take a complex situation and use mathematics to represent it, often by making assumptions and approximations to simplify the situation. Modeling allows you to analyze and describe the situation and to make predictions. For example, you model when you use multiple representations, including equations, tables, graphs, or diagrams to describe a situation. In situations involving the variability of data, you model when you describe the data with equations. Although a model may not be perfect, it can still be very useful for describing data and making predictions. When you interpret the results, you may need to go back and improve your model by revising your assumptions and approximations.

Use appropriate tools strategically:

To **use appropriate tools strategically** means that you analyze the task and decide which tools may help you model the situation or find a solution. Some of the tools available to you include diagrams, graph paper, calculators, computer software, databases, and websites. You understand the limitations of various tools. A result can be checked or estimated by strategically choosing a different tool.

Attend to precision:

To **attend to precision** means that when solving problems, you need to pay close attention to the details. For example, you need to be aware of the units, or how many digits your answer requires, or how to choose a scale and label your graph. You may need to convert the units to be consistent. At times, you need to go back and check whether a numerical solution makes sense in the context of the problem.

You need to **attend to precision** when you communicate your ideas to others. Using the appropriate vocabulary and mathematical language can help make your ideas and reasoning more understandable to others.

Look for and make use of structure:

To **looking for and making use of structure** is a guiding principal of this course. When you are involved in analyzing the structure and in the actual development of mathematical concepts, you gain a deeper, more conceptual understanding than when you are simply told what the structure is and how to do problems. You often use this practice to bring closure to an investigation.

There are many concepts that you learn by looking at the underlying structure of a mathematical idea and thinking about how it connects to other ideas you have already learned. For example, you understand the underlying structure of an equation such as $y = a(x - h)^2 + b$ which allows you to graph it without a table.

Look for and express regularity in repeated reasoning:

To **look for and express regularity in repeated reasoning** means that when you are investigating a new mathematical concept, you notice if calculations are repeated in a pattern. Then you look for a way to generalize the method for use in other situations, or you look for shortcuts. For example, the pattern of growth you notice in a geometric sequence results in being able to write a general exponential equation that highlights the growth and starting point.

④

WHAT HAVE I LEARNED?

Most of the problems in this section represent typical problems found in this chapter. They serve as a gauge for you. You can use them to determine which types of problems you can do well and which types of problems require further study and practice. Even if your teacher does not assign this section, it is a good idea to try these problems and find out for yourself what you know and what you still need to work on.



Solve each problem as completely as you can. The table at the end of the closure section has answers to these problems. It also tells you where you can find additional help and practice with problems like these.

- CL 2-170. Chucky and Angelica were reviewing equations of parabolas for their upcoming math test. They disagreed on what the equation would look like for a parabola whose vertex was at $(-4, 3)$.
- Help them write an equation for a parabola that opens upward from its vertex at $(-4, 3)$. What is the equation of its line of symmetry?
 - Chucky wants the same parabola to open down and Angelica wants it to be compressed. Show them how to change your original equation to meet both of their desires. Does the line of symmetry change?
 - Move your parabola from part (b) 7 units to the right and 8 units down and stretch it vertically so that it is thinner than the original parabola. What is the equation of the parabola? What is the equation of its line of symmetry?
- CL 2-171. For each equation, give the locator point (h, k) and the equation of any asymptotes, and then draw the graph.
- | | |
|----------------------------|--------------------------|
| a. $f(x) = - x + 2 - 1$ | b. $y = \frac{1}{x} + 2$ |
| c. $y = \frac{1}{x+5} - 2$ | d. $y = -x^3 + 5$ |
- CL 2-172. For each of the functions in problem 2-171 sketch the graph of $y = f(-x)$.
- CL 2-173. Gloria the grasshopper is working on her hops. She is trying to jump as high and as far as she can. Her best jump so far was 28 cm long, and she reached a height of 20 cm. Sketch a graph and write an equation of the parabola that describes the path of her jump.

- CL 2-174. Use what you know about transforming parent graphs to write an equation for each of the graphs described below.
- A parabola stretched by a factor of 0.25, opening downward and shifted 12 units down and 3 units left.
 - A cubic with a stretch factor of 2 and a locator point at $(-6, 1)$.
 - A hyperbola, $y = \frac{1}{x}$, but with asymptotes at $y = -6$ and $x = 2$.
- CL 2-175. Find the equation of the exponential functions with a horizontal asymptote at $y = 0$ through the following pairs of points.
- $(2, 99)$ and $(6, 8019)$
 - $(-1, 50)$ and $(2, 25.6)$
- CL 2-176. Write an equation for each of the following sequences.
- 10, 7, 4, ...
 - 2, -8, -32, ...
- CL 2-177. For each of the equations below, complete the following:
- Find the x - and y -intercepts.
 - Find the vertex.
 - Sketch a graph of each parabola on its own set of axes.
 - Write the equation in graphing form.
- $y = x^2 + 8x + 12$
 - $y = (x - 4)(x + 2)$
 - $y = x^2 - 6x - 9$
 - $y = x^2 + 5x + 1$
- CL 2-178. Factor each of the following expressions.
- $2x^2 + 7x - 4$
 - $8x^2 + 24x + 10$
- CL 2-179. Dinner at David's costs \$8.95 today and has been increasing an average of 7% per year.
- What will it cost in 10 years?
 - What did it cost 10 years ago?
- CL 2-180. If $g(x) = (x + 1)^2$, complete each part below.
- $g(5)$
 - $g(2m + 4)$
 - x if $g(x) = 9$

CL 2-181. Solve each equation for y .

a. $4 - 2(x + y) = 9$

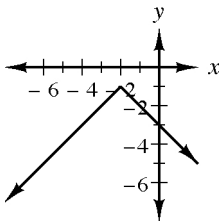
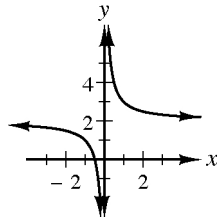
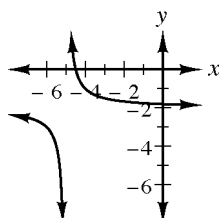
b. $x = 2(y - 1)^2 + 2$

CL 2-182. Check your answers using the table below. Which problems do you feel confident about? Which problems were hard? Have you worked on problems like these in math classes you have taken before? Use the table to make a list of topics you need help on and a list of topics you need to practice more.

Answers and Support for Closure Activity #4

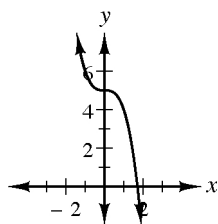
What Have I Learned?

Note: MN = Math Note, LL = Learning Log

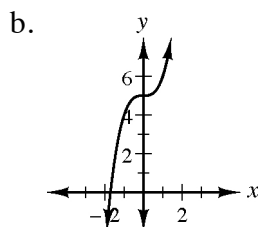
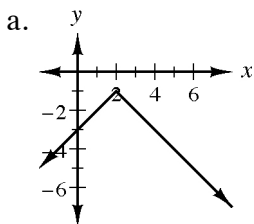
Problem	Solution	Need Help?	More Practice
CL 2-170.	<p>a. Answers may vary but should be in the form of $y = a(x + 4)^2 + 3$, where a is any positive number. $x = -4$</p> <p>b. $y = a(x + 4)^2 + 3$, where a is between 0 and -1; Line of symmetry does not change.</p> <p>c. $y = a(x - 3)^2 - 5$, where a is less than -1. $x = 3$</p>	<p>Lessons 2.1.2, and 2.1.3</p> <p>MN: 2.1.3 and 2.1.4</p>	<p>Problems 2-31, 2-32, 2-74, and 2-124</p>
CL 2-171.	<p>a. $(-2, -1)$</p>  <p>b. $(0, 2)$ $x = 0$; $y = 2$</p>  <p>c. $(-5, -2)$ $x = -5$; $y = -2$</p> 	<p>Lessons 2.2.1 and 2.2.2</p> <p>MN: 2.2.3</p> <p>LL: 2.2.2 and 2.2.5</p>	<p>Problems 2-74, 2-107, 2-109, 2-118, 2-131, and 2-164</p>

Problem	Solution	Need Help?	More Practice
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d. (0,5)



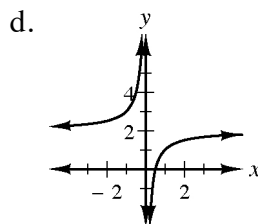
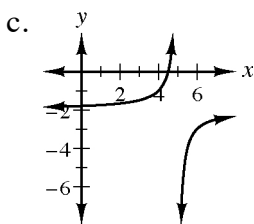
CL 2-172.



Lesson 2.2.3

LL: 2.2.3

Problems 2-122,
2-126, and 2-156

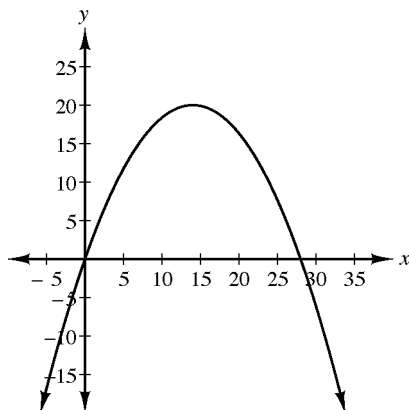


CL 2-173. $y = -\frac{5}{49}x(x-28) = -\frac{5}{49}x^2 + \frac{20}{7}x$

Lesson 2.1.5

LL: 2.1.5

Problems 2-64,
2-66, 2-67, 2-69,
and 2-81



CL 2-174. a. $y = -0.25(x+3)^2 - 12$

b. $y = 2(x+6)^3 + 1$

c. $y = \frac{1}{x-2} - 6$

Lesson 2.2.1
and 2.2.2

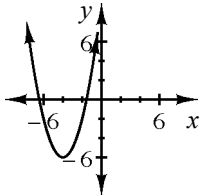
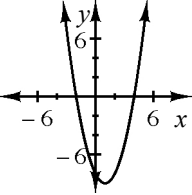
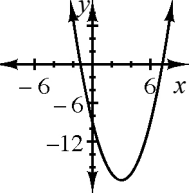
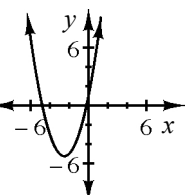
Problems 2-25,
2-52, 2-95, 2-96,
2-109, 2-118,
and 2-143.

CL 2-175. a. $y = 11 \cdot 3^x$

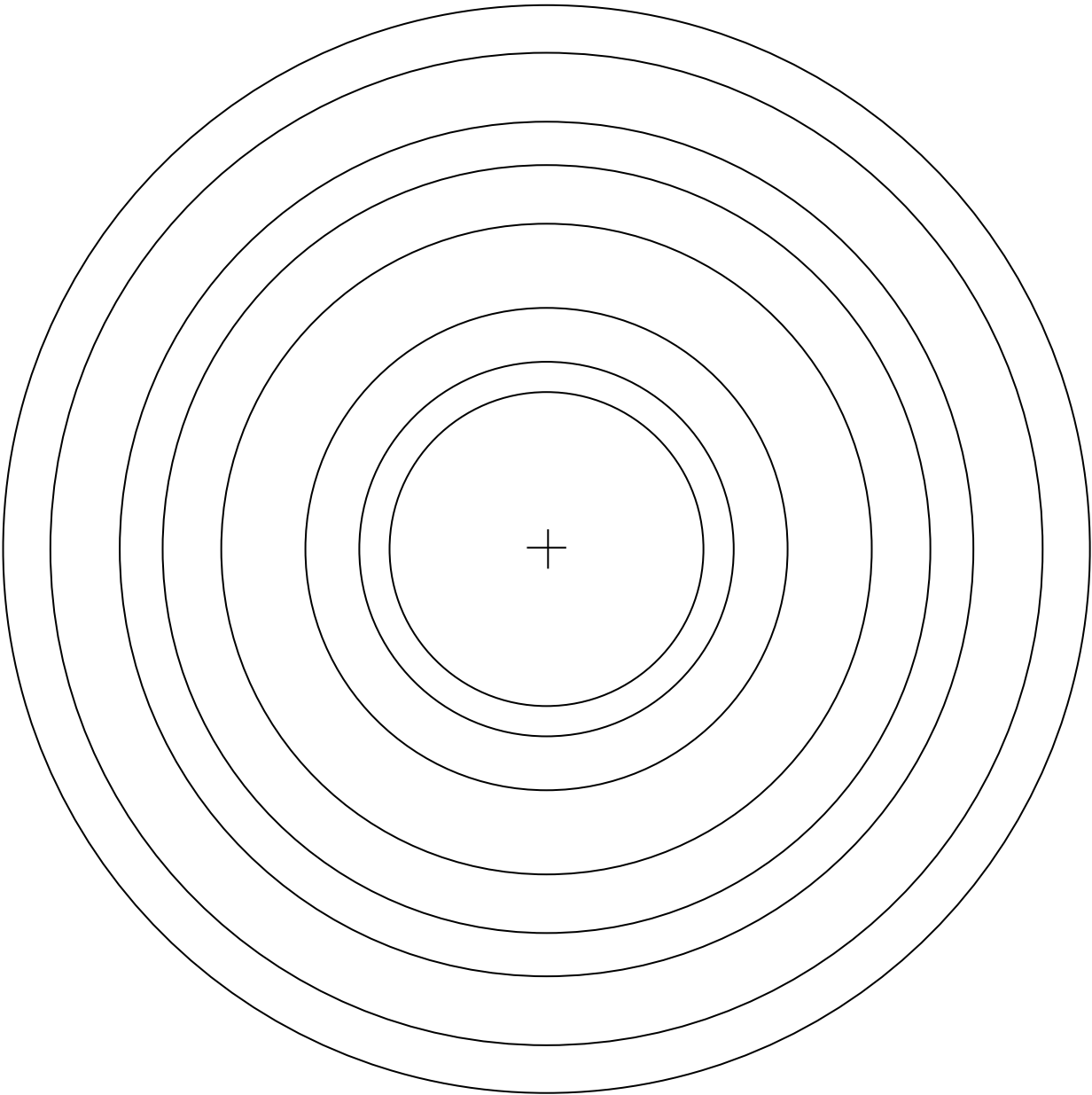
b. $y = 40(0.8)^x$

Appendix B
Lesson B.2.2

Problems 2-72
and 2-110

Problem	Solution	Need Help?	More Practice
CL 2-176.	<p>a. $t(n) = -3n + 13$</p> <p>b. $t(n) = -\frac{1}{2}(4)^n$ or $-2(4)^{n-1}$</p> <p>Both using $t(1)$ as the first term.</p>	<p>Appendix A Lessons A.2.2 and A.3.2</p> <p>MN: A.3.2 and B.2.3</p>	Problems 2-8, 2-100, 2-120, and 2-130
CL 2-177.	<p>a. x-int: $(-6, 0), (-2, 0)$; y-int: $(0, 12)$; vertex: $(-4, -4)$ $y = (x + 4)^2 - 4$</p> <p>b. x-int: $(-2, 0), (4, 0)$; y-int: $(0, -8)$; vertex: $(1, -9)$; $y = (x - 1)^2 - 9$</p> <p>c. x-int: $(3 \pm \sqrt{18}, 0)$; y-int: $(0, -9)$; vertex: $(3, -18)$; $y = (x - 3)^2 - 18$</p> <p>d. x-int: $(\frac{-5 \pm \sqrt{21}}{2}, 0)$; y-int: $(0, 1)$; vertex: $(-2.5, -5.25)$; $y = (x + 2.5)^2 - 5.25$</p>	    <p>Lessons 2.1.2, 2.1.3, and 2.1.4</p> <p>MN: 2.1.3 and 2.1.4</p> <p>LL: 2.1.5</p>	Problems 2-17, 2-34, 2-50, 2-73, 2-82, 2-119, and 2-166
CL 2-178.	<p>a. $(2x - 1)(x + 4)$</p> <p>b. $2(2x + 5)(2x + 1)$</p>	<p>Explanations and practice of topics from previous courses are available in the <i>Core Connections Algebra Parent Guide with Extra Practice</i>, available free at www.cpm.org.</p>	Problems 2-35, 2-98, and 2-169

Problem	Solution	Need Help?	More Practice
CL 2-179. a. \$17.61 b. \$4.55		Appendix B Lesson B.1.3	Problems 2-29, 2-93, 2-63, and 2-85
CL 2-180. a. 36 b. $4m^2 + 20m + 25$ c. 2, -4		Section 1.1	Problems 2-75, 2-101, 2-116, and 2-168
CL 2-181. a. $y = -x - \frac{5}{2}$ b. $y = \pm\sqrt{\frac{x-2}{2}} + 1$		Topic from previous course.	Problems 1-37, 1-72, 2-99, and 2-113

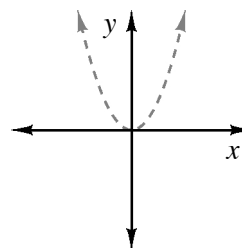


Transforming Parabolas

To make the graph of $y = x^2$ _____, you can _____.

The new equation would look like:

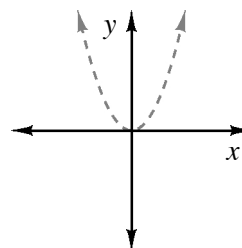
I know this because:



To make the graph of $y = x^2$ _____, you can _____.

The new equation would look like:

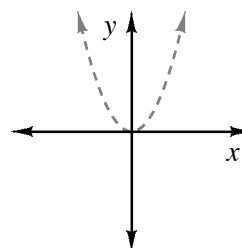
I know this because:



To make the graph of $y = x^2$ _____, you can _____.

The new equation would look like:

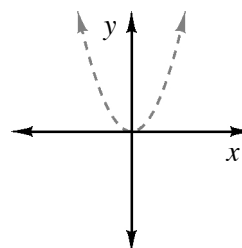
I know this because:



To make the graph of $y = x^2$ _____, you can _____.

The new equation would look like:

I know this because:



Instructions for the application: *Transfrm*, Transformation Graphing v1.03

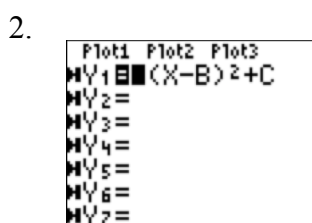
The TI-83 Plus and all the TI-84 calculators have a free Application called **Transfrm**, Transformation Graphing v1.03, which can be used on any function entered in the $\boxed{Y=}$ screen. If you press the $\boxed{\text{APPS}}$ button and scroll or press $\boxed{\text{ALPHA}}\boxed{4}$, T, you can check to see if **Transfrm** is loaded on the calculator. Transfer **Transfrm** from calculator to calculator with a link cord or download from www.education.ti.com using a computer link cord. Note: This application uses names A , B , and C for the parameters. For parabolas $A = a$, $B = h$ and $C = k$.

If you choose to use **Transfrm**, here are a few basic steps.

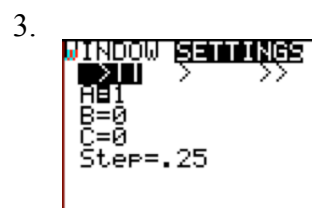
1. Press $\boxed{\text{APPS}}$. Scroll or press $\boxed{\text{ALPHA}}\boxed{4}$, T, to find **Transfrm**, press $\boxed{\text{ENTER}}$. (Screen 1.)



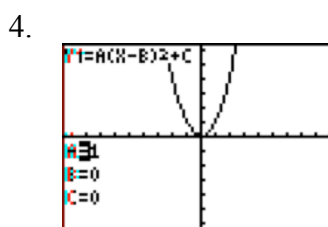
2. Go to $\boxed{Y=}$. The icon to the left of the $\boxed{Y=}$ should now look different. (Screen 2.)
 Type $A(x - B)^2 + C$ into Y_1 .



3. Press $\boxed{\text{WINDOW}}$ then $\boxed{\text{F1}}$ to select SETTINGS. You can select the type of transformation:
 $\boxed{\text{F1}}$ (manual), $\boxed{\text{F2}}$ (animation), $\boxed{\text{F3}}$ (quick animation). You can adjust the value of A, B, C, Step, and Max. The Step is the amount of change that is added or subtracted to the parameter selected and the Max is used with animation. (Screen 3.)



4. Press $\boxed{\text{GRAPH}}$. Use $\boxed{\text{F1}}$ to change the A value and $\boxed{\text{F2}}$ to change parameters. (Screen 4.)



5. To close **Transfrm**, Press $\boxed{\text{APPS}}$ to find: Transfrm, select 1: Uninstall, and press $\boxed{\text{ENTER}}$. This will not remove **Transfrm** from your calculator. (Screen 5.)



Lesson 2.1.4 Resource Page
Problem 2-47

Find the x -intercepts of $y = (x - 3)^2 - 25$

a. $y = (x - 3)^2 - 25$

justification: the x -intercepts are where $y = 0$

b. $0 = (x - 3)^2 - 25$

justification:

c. $25 = (x - 3)^2$

justification:

d. $\sqrt{25} = \sqrt{(x - 3)^2}$

justification:

e. $5 = |x - 3|$

justification:

f. $5 = x - 3$ $-5 = x - 3$

justification:

g. $\boxed{8 = x}$ $\boxed{-2 = x}$

h. The x -intercepts are $(8, 0)$ and $(-2, 0)$.

Parent Graph Name:

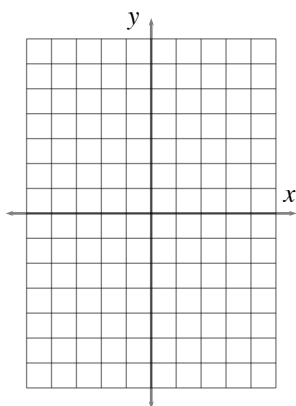
Parent Graph Table

Parent Graph

Parent Equation:

Equation of Transformed Function (Graphing Form):

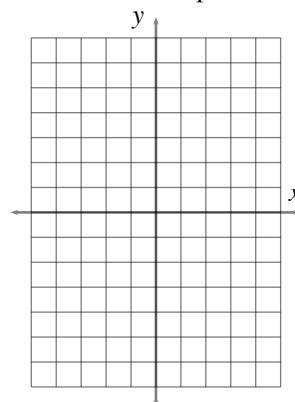
Sketch of Transformed Graph



Domain and Range of Transformed Function:

Interesting Properties:
(asymptotes, symmetry, endpoints, maxima, etc.)Describe where (on the transformed graph) the locator point (h, k) is:

x	y
-5	
-4	
-3	
-2	
-1	
0	
1	
2	
3	
4	
5	



Parent Graph Name:

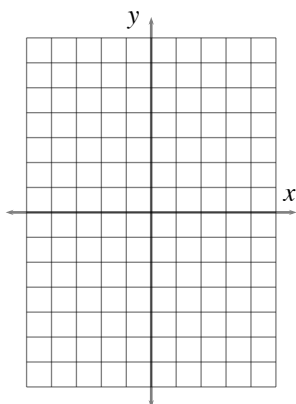
Parent Graph Table

Parent Graph

Parent Equation:

Equation of Transformed Function (Graphing Form):

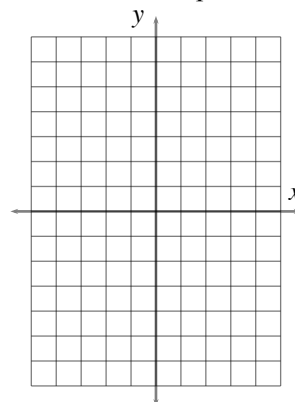
Sketch of Transformed Graph



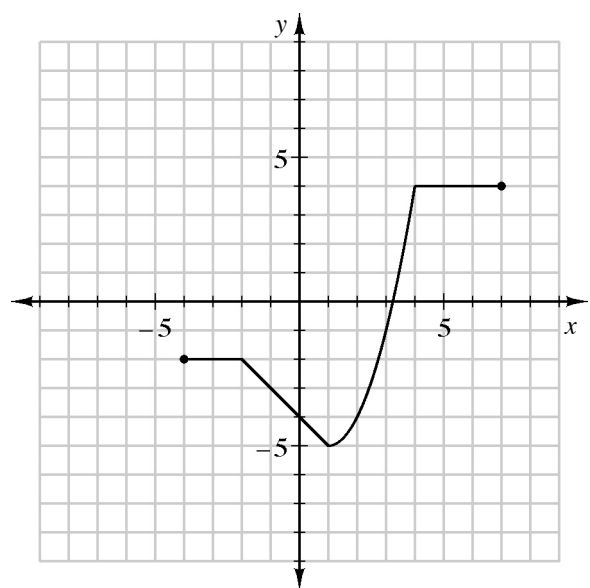
Domain and Range of Transformed Function:

Interesting Properties:
(asymptotes, symmetry, endpoints, maxima, etc.)Describe where (on the transformed graph) the locator point (h, k) is:

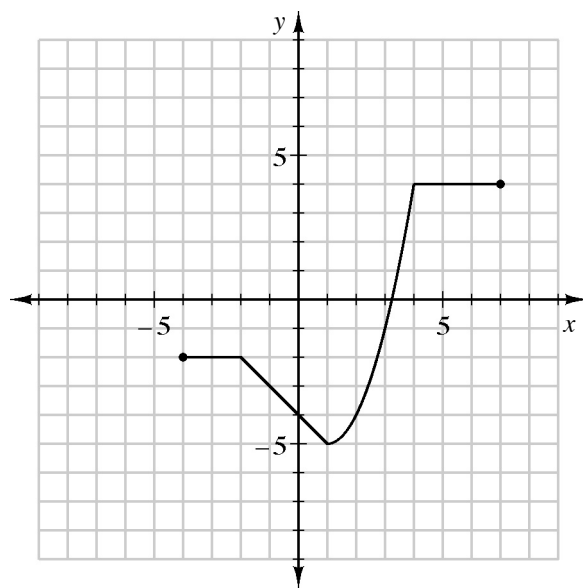
x	y
-5	
-4	
-3	
-2	
-1	
0	
1	
2	
3	
4	
5	



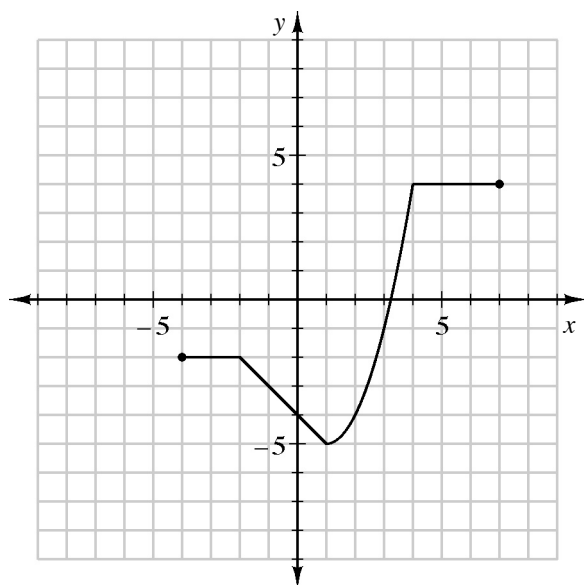
Lesson 2.2.5A Resource Page



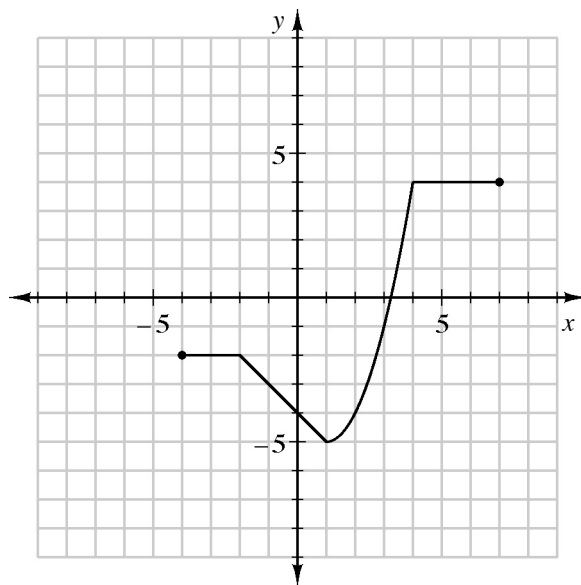
x	-4	-3	-2	-1	0	1	2	3	4	5	6	7
$F(x)$												
$-F(x)$												



x	-4	-3	-2	-1	0	1	2	3	4	5	6	7
$F(x)$												
$\frac{1}{2}F(x)$												

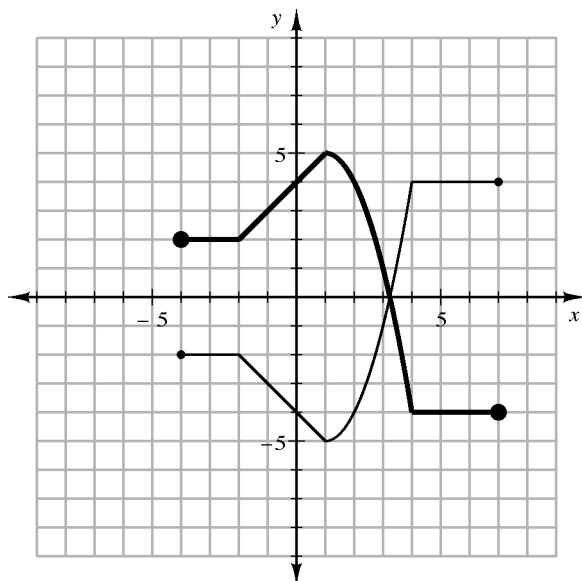


x	-4	-3	-2	-1	0	1	2	3	4	5	6	7
$F(x)$												
$F(x)+4$												

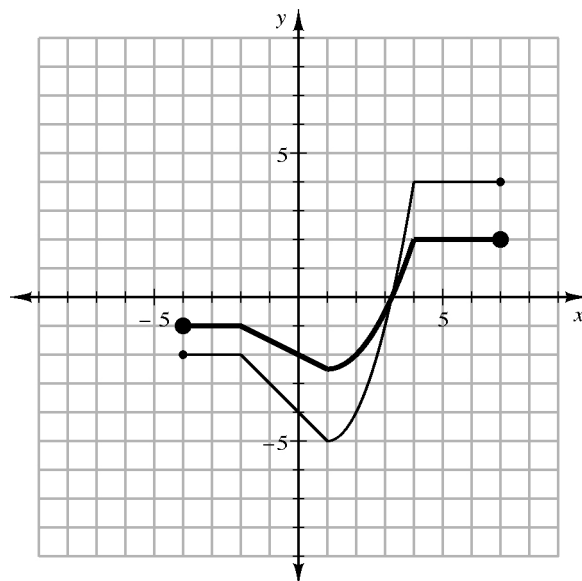


x	-4	-3	-2	-1	0	1	2	3	4	5	6	7
$F(x)$												
$F(x-4)$												

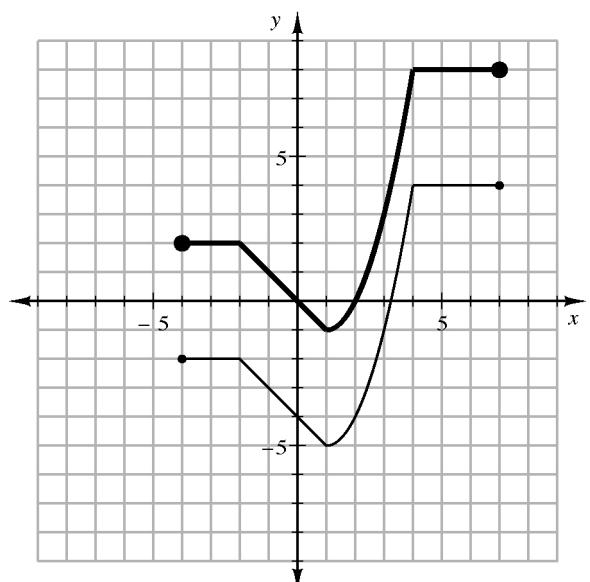
SOLUTIONS



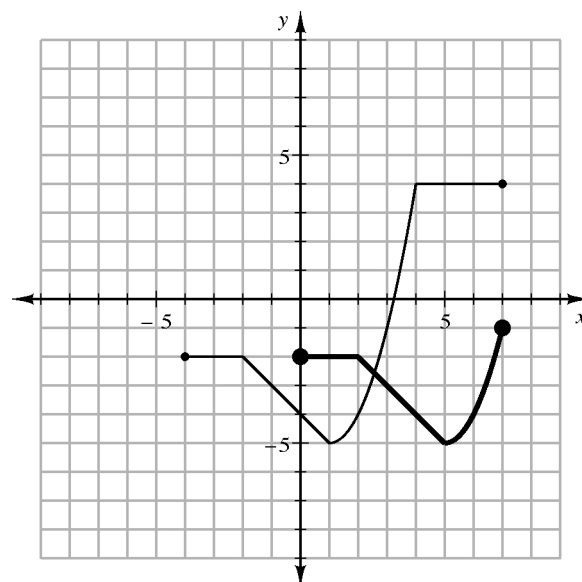
x	-4	-3	-2	-1	0	1	2	3	4	5	6	7
$F(x)$	-2	-2	-2	-3	-4	-5	-4	-1	4	4	4	4
$-F(x)$	2	2	2	3	4	5	4	1	-4	-4	-4	-4



x	-4	-3	-2	-1	0	1	2	3	4	5	6	7
$F(x)$	-2	-2	-2	-3	-4	-5	-4	-1	4	4	4	4
$\frac{1}{2}F(x)$	-1	-1	-1	-1.5	-2	-2.5	-2	-0.5	2	2	2	2



x	-4	-3	-2	-1	0	1	2	3	4	5	6	7
$F(x)$	-2	-2	-2	-3	-4	-5	-4	-1	4	4	4	4
$F(x)+4$	2	2	2	1	0	-1	0	3	8	8	8	8



x	-4	-3	-2	-1	0	1	2	3	4	5	6	7
$F(x)$	-2	-2	-2	-3	-4	-5	-4	-1	4	4	4	4
$F(x-4)$					-2	-2	-2	-3	-4	-5	-4	-1

compress	dilate
domain	even function
function	general equation
graphing form	(h, k)
horizontal shift	odd function
parameter	parent graph

piecewise- defined function	range
reflection	shrink
standard form	step function
stretch	transformation
translation	variable

vertex	vertex form
vertical shift	

