

Physical Applications of Newton's 2nd law of motion (Free Body Diagrams)

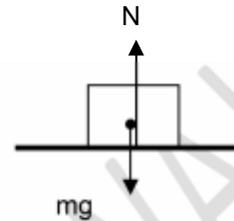
Horizontal Plane

i) Case - 1

Body kept on horizontal plane is at rest.

For vertical direction

$$N = mg \text{ (since body is at rest)}$$



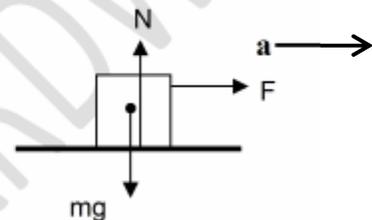
ii) Body kept on horizontal plane is accelerating horizontally under single horizontal force.

For vertical direction

$$N = mg \text{ (since body is at rest)}$$

For horizontal direction

$$F = ma$$



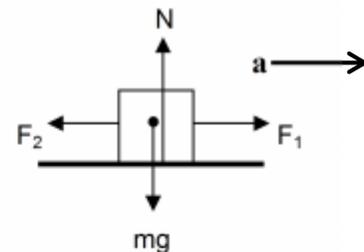
iii) Body kept on horizontal plane is accelerating horizontally towards right under two horizontal forces. ($F_1 > F_2$)

For vertical direction

$$N = mg \text{ (since body is at rest)}$$

For horizontal direction

$$F_1 - F_2 = ma$$



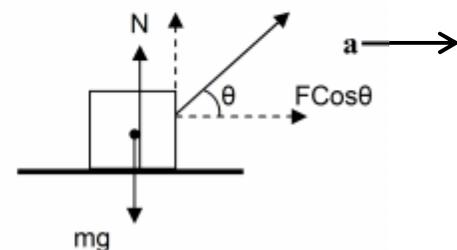
iv) Body kept on horizontal plane is accelerating horizontally under single inclined force $F \sin \theta$

For vertical direction

$$N + F \sin \theta = mg \text{ (since body is at rest)}$$

For horizontal direction

$$F \cos \theta = ma$$

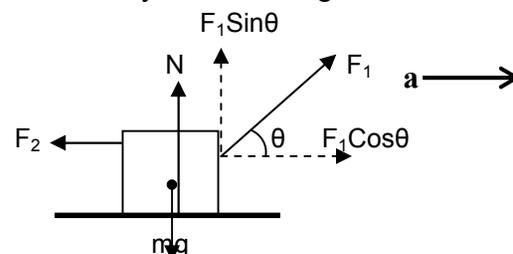


v) Body kept on horizontal plane is accelerating horizontally towards right under an inclined force and a horizontal force.

For vertical direction

$$N + F_1 \sin \theta = mg \text{ (since body is at rest)}$$

For horizontal direction $F_1 \cos \theta - F_2 = ma$



vi) Body kept on horizontal plane is accelerating horizontally towards right under two inclined forces acting on opposite sides.

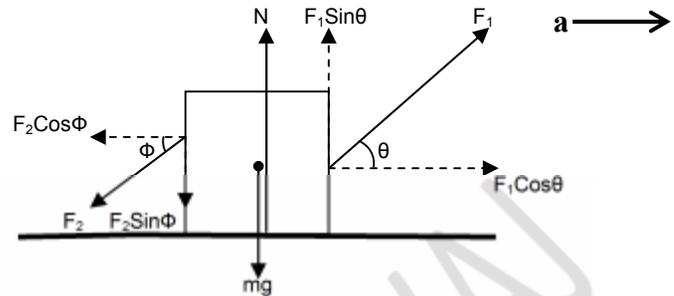
For vertical direction

$$N + F_1 \sin \theta = mg + F_2 \sin \phi$$

(since body is at rest)

For horizontal direction

$$F_1 \cos \theta - F_2 \cos \phi = ma$$



Inclined Plane

i) Case - 1

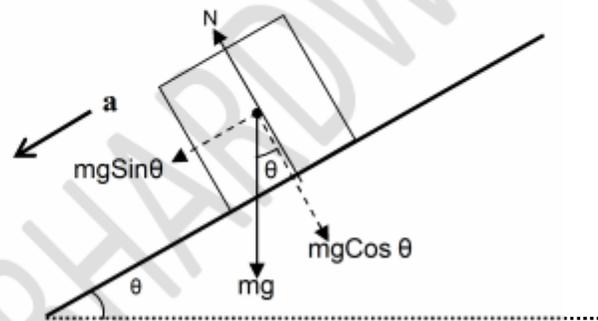
Body sliding freely on inclined plane.

Perpendicular to the plane

$$N = mg \cos \theta$$

Parallel to the plane

$$mg \sin \theta = ma$$



ii) Case - 2

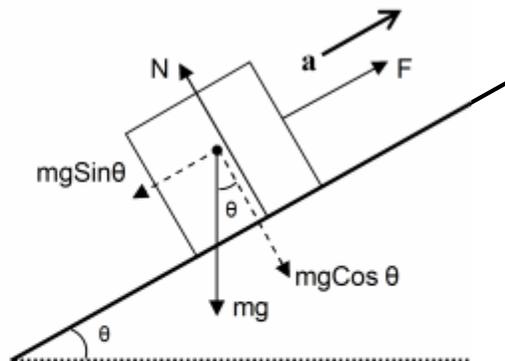
Body pulled parallel to the inclined plane.

Perpendicular to the plane

$$N = mg \cos \theta$$

Parallel to the plane

$$F - mg \sin \theta = ma$$



iii) Case - 3

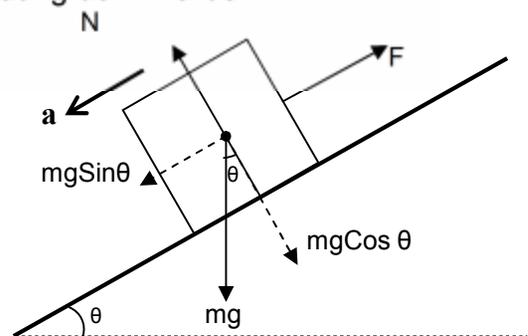
Body pulled parallel to the inclined plane but accelerating downwards.

Perpendicular to the plane

$$N = mg \cos \theta$$

Parallel to the plane

$$mg \sin \theta - F = ma$$



iv) Case - 4

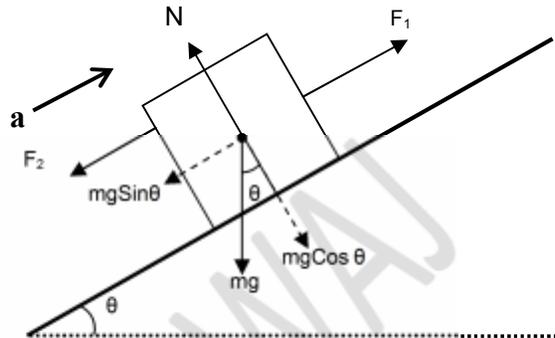
Body accelerating up the incline under the effect of two forces acting parallel to the incline.

Perpendicular to the plane

$$N = mg \cos \theta \quad (\text{since body is at rest})$$

Parallel to the plane

$$F_1 - F_2 - mg \sin \theta = ma$$



v) Case - 5

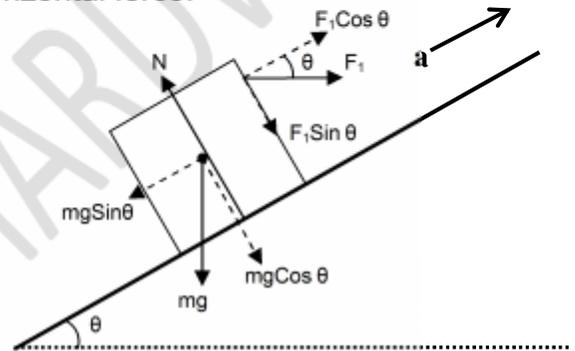
Body accelerating up the incline under the effect of horizontal force.

Perpendicular to the plane

$$N = mg \cos \theta + F_1 \sin \theta \quad (\text{since body is at rest})$$

Parallel to the plane

$$F_1 \cos \theta - mg \sin \theta = ma$$



vi) Case - 6

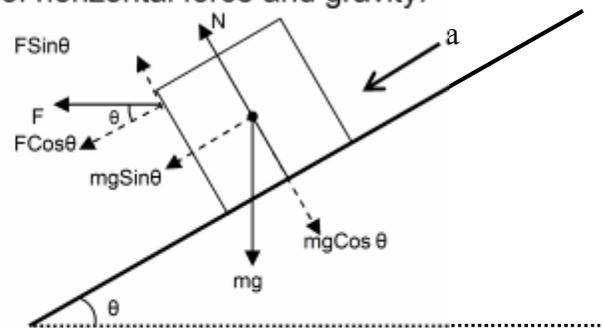
Body accelerating down the incline under the effect of horizontal force and gravity.

Perpendicular to the plane

$$N + F \sin \theta = mg \cos \theta \quad (\text{since body is at rest})$$

Parallel to the plane

$$F \cos \theta + mg \sin \theta = ma$$



vii) Case - 7

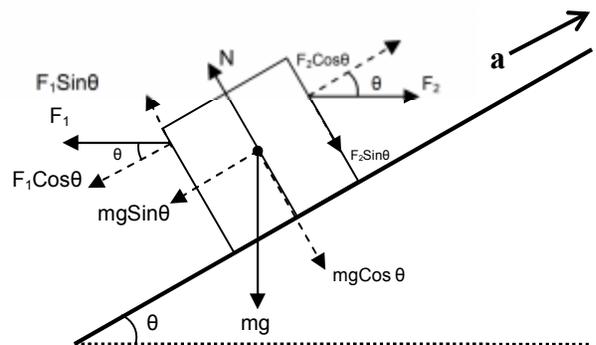
Body accelerating up the incline under the effect of two horizontal forces acting on opposite sides of a body and gravity.

Perpendicular to the plane

$$N + F_1 \sin \theta = mg \cos \theta + F_2 \sin \theta \quad (\text{since body is at rest})$$

Parallel to the plane

$$F_2 \cos \theta - F_1 \cos \theta - mg \sin \theta = ma$$



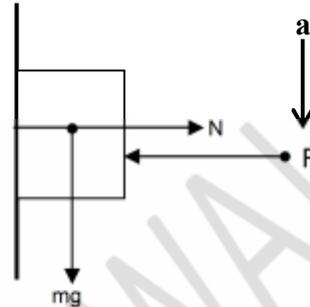
Vertical Plane

i) Case - 1

Body pushed against the vertical plane by horizontal force and moving vertically downward.

For horizontal direction
 $mg = ma$ (since body is at rest)

For vertical direction
 $F = N$

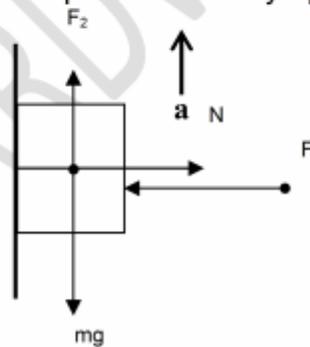


ii) Case - 2

Body pushed against the vertical plane by horizontal force and pulled vertically upward.

For vertical direction
 $F_2 - mg = ma$

For horizontal direction (since body is at rest)
 $N = F_1$

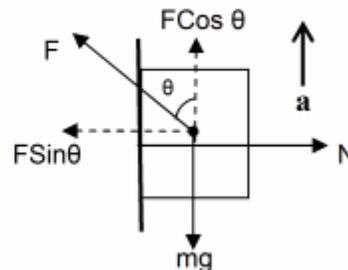


iii) Case - 3

Body pushed against the vertical plane by inclined force and accelerates vertically upward.

For horizontal direction
 $N = F \sin \theta$ (since body is at rest)

For vertical direction
 $F \cos \theta - mg = ma$

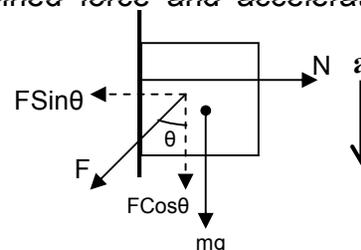


iv) Case - 3

Body pushed against the vertical plane by inclined force and accelerates vertically downward.

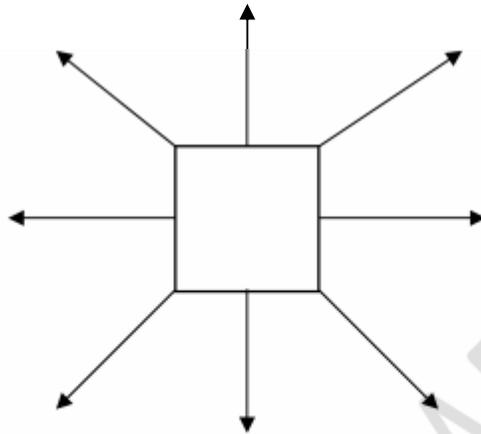
For horizontal direction
 $N = F \sin \theta$ (since body is at rest)

For vertical direction
 $F \cos \theta + mg = ma$



Tension In A Light String

Force applied by any linear object such as string, rope, chain, rod etc. is known as its tension. Since string is a highly flexible object so it can only pull the object and can never push. Hence tension of the string always acts away from the body to which it is attached irrespective of the direction.



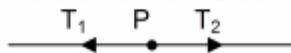
Tension of the string, being of pulling nature, always acts away from the body to which it is attached

Physical Application

- i) Flexible wire holding the lamp pulls the lamp in upward direction and pulls the point of suspension in the downward direction.
- ii) Rope holding the bucket in the well pulls the bucket in the upward direction and the pulley in the downward direction.
- iii) Rope attached between the cattle and the peg pulls the cattle towards the peg and peg towards the cattle.
- iv) When a block is pulled by the chain, the chain pulls the block in forward direction and the person holding the chain in reverse direction.

Key Point

In case of light string, rope, chain, rod etc. tension is same all along their lengths.



Consider a point P on a light (massless) string. Let tensions on either side of it be T_1 and T_2 respectively and the string be accelerating towards left under these forces. Then for point P

$$T_1 - T_2 = ma$$

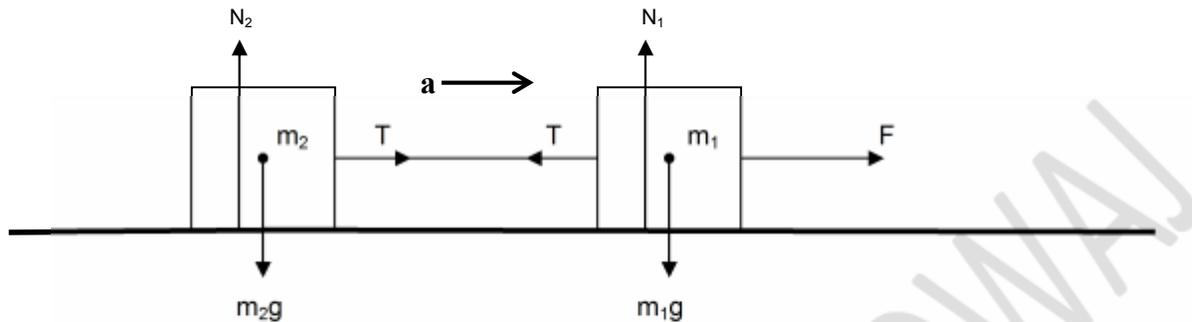
Since string is considered to be light mass m of point P is zero

or,
$$T_1 - T_2 = 0$$

or,
$$T_1 = T_2$$

i) Case - 1

Two bodies connected by a string are placed on a smooth horizontal plane and pulled by a horizontal force.



For vertical equilibrium of m_1 and m_2

$$\mathbf{N_1 = m_1g \text{ and } N_2 = m_2g}$$

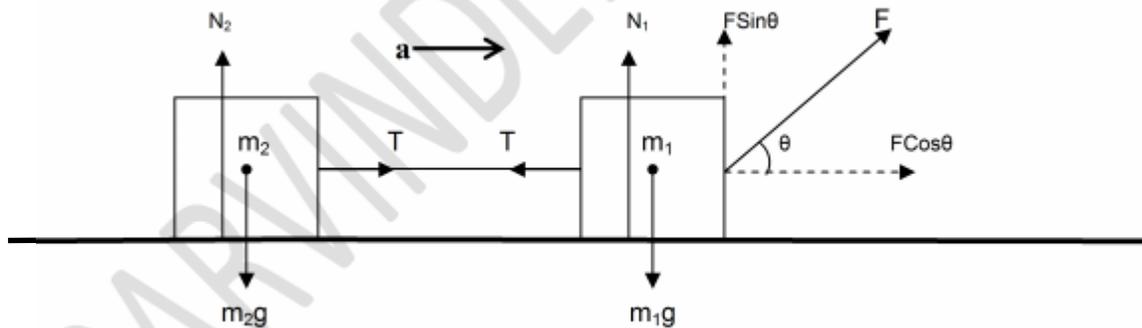
For horizontal acceleration of m_1 and m_2

$$\mathbf{F - T = m_1a \text{ and } T = m_2a}$$

(Since both the bodies are connected to the same single string they have same acceleration)

ii) Case - 2

Two bodies connected by a horizontal string are placed on a smooth horizontal plane and pulled by a inclined force.



For vertical equilibrium of m_1 and m_2

$$\mathbf{N_1 + F\sin\theta = m_1g \text{ and } N_2 = m_2g}$$

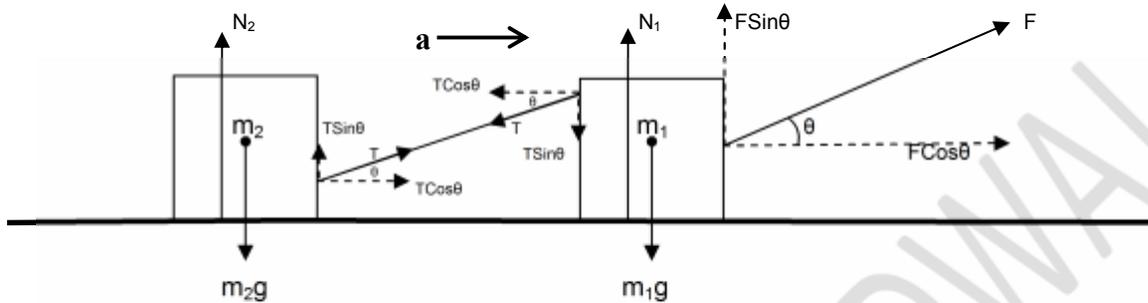
For horizontal acceleration of m_1 and m_2

$$\mathbf{F\cos\theta - T = m_1a \text{ and } T = m_2a}$$

(since both the bodies are connected to the same single string they have same accelerations)

iii) Case - 3

Two bodies connected by a inclined string are placed on a smooth horizontal plane and pulled by a inclined force.



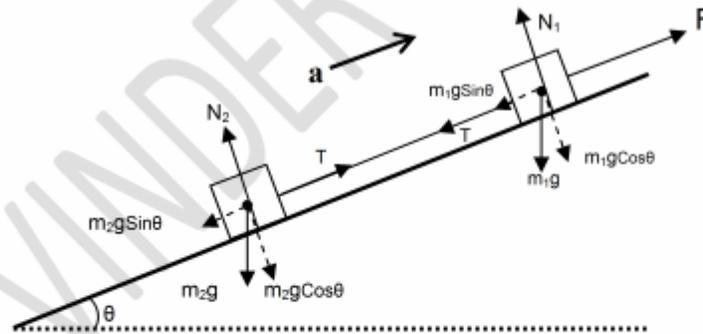
For vertical equilibrium of m_1 and m_2
 $N_1 + F\sin\theta = m_1g + T\sin\theta$ and $N_2 + T\sin\theta = m_2g$

For horizontal acceleration of m_1 and m_2
 $F\cos\theta - T\cos\theta = m_1a$ and $T\cos\theta = m_2a$

(since both the bodies are connected to the same single string they have same accelerations)

iv) Case - 4

Two bodies connected by a string made to accelerate up the incline by applying force parallel to the incline.

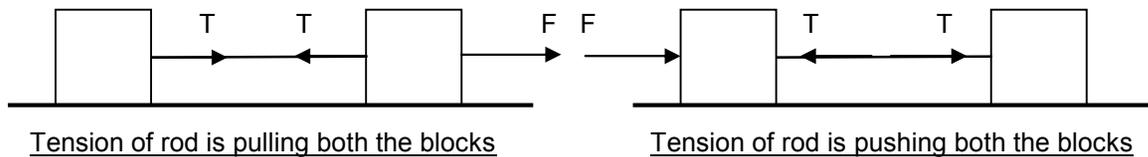


For equilibrium of m_1 and m_2 in the direction perpendicular to the plane
 $N_1 = m_1g\cos\theta$ and $N_2 = m_2g\cos\theta$

For acceleration of m_1 and m_2 up the incline
 $F - T - m_1g\sin\theta = m_1a$ and $T - m_2g\sin\theta = m_2a$

Tension of A light Rigid Rod

Force applied by rod is also known as its tension. Since rod is rigid, it cannot bend like string. Hence rod can pull as well as push. Tension of rod can be of pulling as well as pushing nature but one at a time. Tension of a rod attached to the body may be directed towards as well as away from the body.



Physical Application

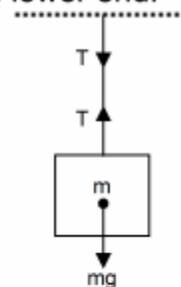
- i) Pillars supporting the house **pushes** the house in the upward direction and **pushes** the ground in the downward direction.
- ii) Wooden bars used in the chair **pushes** the ground in the downward direction and **pushes** the seating top in the upward direction.
- iii) Parallel bars attached to the ice-cream trolley **pushes** the trolley in the forward direction and **pushes** the ice-cream vendor in the backward direction. (when the trolley is being pushed by the vendor)
- iv) Rod holding the ceiling fan **pulls** the fan in the upward direction and **pulls** the hook attached to the ceiling in the downward direction.
- v) Parallel rods attached between the cart and the bull **pulls** the cart in the forward direction and **pulls** the bull in the backward direction.

Different Cases of Light Rigid Rod

i) Case - 1

Rod attached from the ceiling and supporting the block attached to its lower end.
Since the block is at rest

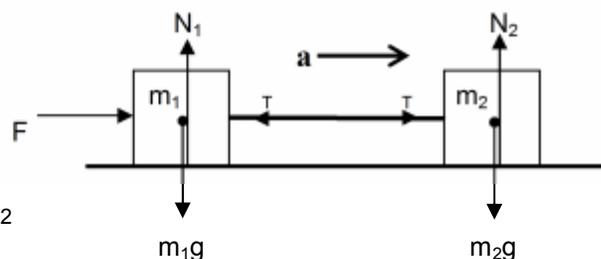
$$T = mg$$



ii) Case - 2

Rod is attached between two blocks placed on the horizontal plane and the blocks are accelerated by pushing force.

For vertical equilibrium of m_1 and m_2
 $N_1 = m_1g$ and $N_2 = m_2g$

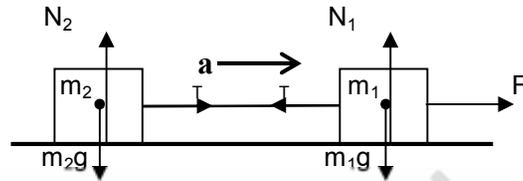


For horizontal acceleration of m_1 and m_2
 $F - T = m_1a$ and $T = m_2a$

(Since both the bodies connected to the rod will have same acceleration)

iii) Case - 3

Rod is attached between two blocks placed on the horizontal plane and the blocks are accelerated by pulling force.



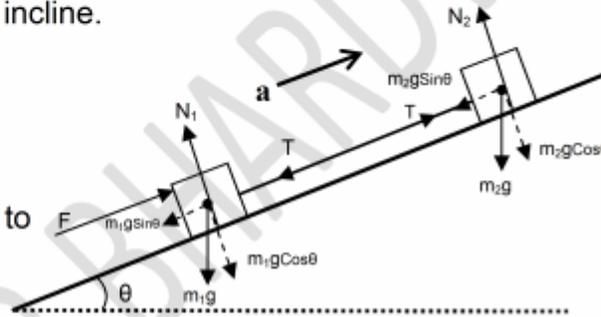
For vertical equilibrium of m_1 and m_2
 $N_1 = m_1g$ and $N_2 = m_2g$

For horizontal acceleration of m_1 and m_2
 $F - T = m_1a$ and $T = m_2a$

(Since both the bodies are connected to the same rod they have same acceleration)

iv) Case - 4

Rod is attached between two blocks placed on the incline plane and the blocks are accelerated by pushing parallel to the incline.



For vertical equilibrium of m_1 and m_2
 $N_1 = m_1g \cos \theta$ and $N_2 = m_2g \cos \theta$

For acceleration of m_1 and m_2 parallel to the incline

$$F - m_1g \sin \theta - T = m_1a,$$

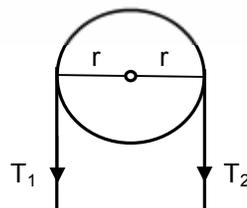
$$T - m_2g \sin \theta = m_2a$$

Fixed Pulley

It is a simple machine in the form of a circular disc or rim supported by spokes having groove at its periphery. It is free to rotate about an axis passing through its center and perpendicular to its plane.

Key Point

In case of light pulley, tension in the rope on both the sides of the pulley is same (to be proved in the rotational mechanics)



$$\text{Anticlockwise Torque} - \text{Clockwise Torque} = \text{Moment of Inertia} \times \text{Angular acceleration}$$

$$T_1 \times r - T_2 \times r = I\alpha$$

Since the pulley is light and hence considered to be massless, its moment of inertia $I = 0$

or,

$$T_1 \times r - T_2 \times r = 0$$

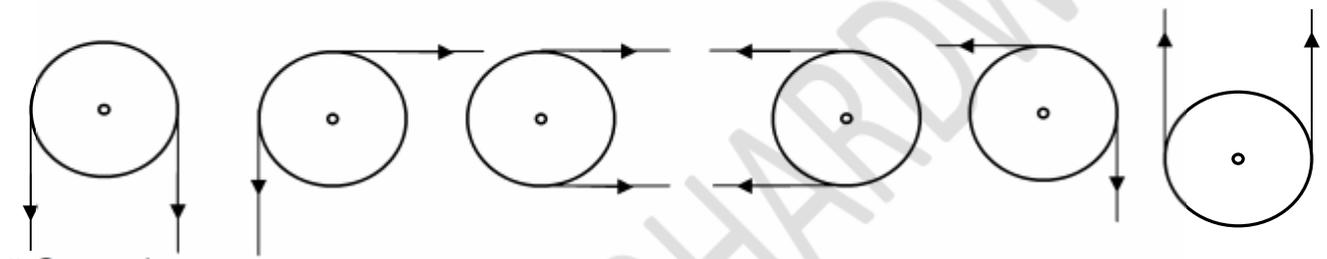
or,

$$T_1 \times r = T_2 \times r$$

or,

$$T_1 = T_2$$

Different Cases of Fixed Pulley



i) Case - 1

Two bodies of different masses ($m_1 > m_2$) are attached at two ends of a light string passing over a smooth light pulley

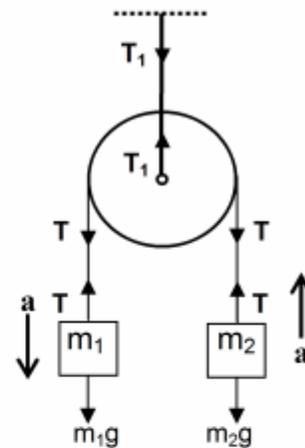
For vertical equilibrium of pulley

$$T_1 = T + T = 2T$$

For vertical acceleration of m_1 and m_2

$$m_1 g - T = m_1 a \text{ and } T - m_2 g = m_2 a$$

m_1 accelerates downwards and m_2 accelerates upwards ($m_1 > m_2$)



ii) Case - 2

Two bodies of different masses are attached at two ends of a light string passing over a light pulley. m_1 is placed on a horizontal surface and m_2 is hanging freely in air.

For vertical equilibrium m_1

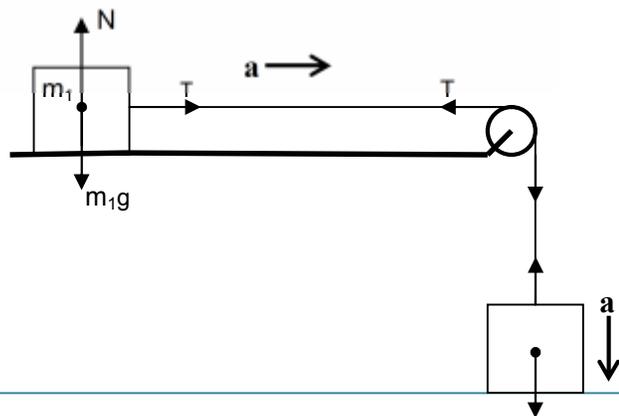
$$N = m_1 g$$

For horizontal acceleration of m_1

$$T = m_1 a$$

For vertically downward acceleration of m_2

$$m_2 g - T = m_2 a$$



iii) Case - 3

Two bodies of different masses are attached at two ends of a light string passing over a light pulley. m_1 is placed on an inclined surface and m_2 is hanging freely in air.

For equilibrium of m_1 perpendicular to incline plane

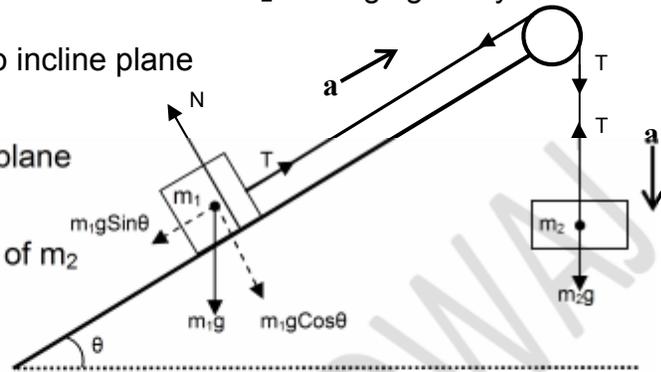
$$N = m_1 g \cos \theta$$

For acceleration of m_1 up the incline plane

$$T - m_1 g \sin \theta = m_1 a$$

For vertically downward acceleration of m_2

$$m_2 g - T = m_2 a$$



Movable Pulley

The pulley which moves in itself is known as movable pulley.

Key Point

In case of light movable pulley, acceleration of a body (pulley) goes on decreasing on increasing the number of strings attached to it. That is the body attached with two ropes moves with half the acceleration of the body attached with single rope.

Length of the string is constant

$$x + 2y + z = L \text{ (Constant)}$$

Differentiating both sides with respect to t (Time)

$$\frac{dx}{dt} + 2\frac{dy}{dt} + \frac{dz}{dt} = \frac{dL}{dt}$$

$$\text{or, } v_1 + 2v_2 + 0 = 0 \text{ (z and L are constant)}$$

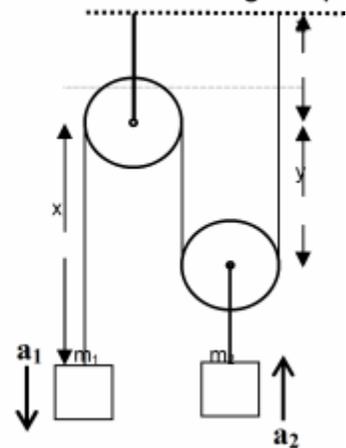
$$\text{or, } v_1 + 2v_2 = 0$$

Again differentiating both sides with respect to t

$$\frac{dv_1}{dt} + 2\frac{dv_2}{dt} = 0$$

$$\text{or, } a_1 + 2a_2 = 0$$

$$\text{or, } a_1 = -2a_2$$



That is acceleration of m_1 (body attached to a single string) is opposite and twice the acceleration of m_2 (body attached to a double string)

Different Cases of Light Movable Pulley

i) Case - 1

Mass m_1 is attached at one end of the string and the other end is fixed to a rigid support. Mass m_2 is attached to the light movable pulley.

For vertical acceleration of m_1

$$m_1g - T = m_12a \quad (m_1 \text{ is connected to a single string})$$

For vertical acceleration of m_2

$$T_2 - m_2g = m_2a$$

(m_1 accelerates downwards and m_2 accelerates upwards since $m_1 > 2m_2$)

For the clamp holding the first pulley

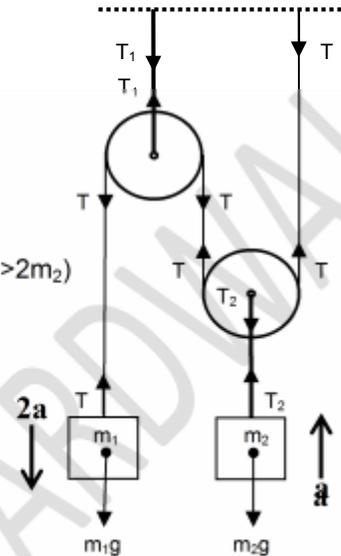
$$T_1 = 2T$$

For the clamp holding the movable pulley

$$2T - T_2 = m_{\text{pulley}}a$$

or, $2T - T_2 = 0$ (light pulley)

$$\text{or, } 2T = T_2$$



ii) Case - 2

Mass m_1 is attached at one end of the string and placed on a smooth horizontal surface and the other end is fixed to a rigid support after passing through a light movable suspended pulley. Mass m_2 is attached to the light movable pulley.

For vertical equilibrium of m_1

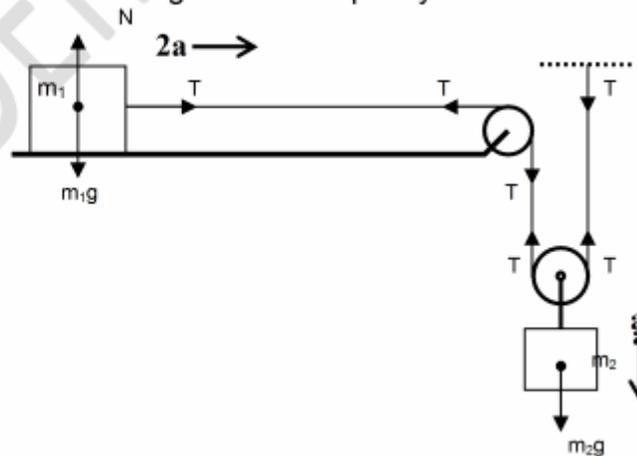
$$N = m_1g$$

For horizontal acceleration of m_1

$$T = m_12a$$

For vertical motion of m_2

$$m_2g - 2T = m_2a$$



iii) Case - 3

Mass m_1 is attached to the movable pulley and placed on a smooth horizontal surface. One end of the string is attached to the clamp holding the pulley fixed to the horizontal surface and from its other end mass m_2 is suspended.

For vertical equilibrium of m_1

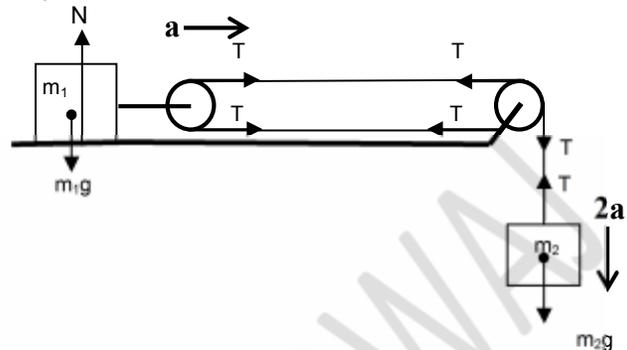
$$N = m_1g$$

For horizontal motion of m_1

$$2T = m_1a$$

For vertical motion of m_2

$$m_2g - T = m_2 \cdot 2a$$



iv) Case - 4

Mass m_1 is attached to a movable pulley and placed on a smooth inclined surface. Mass m_2 is suspended freely from a fixed light pulley.

For equilibrium of m_1 perpendicular to incline plane

$$N = m_1g \cos \theta$$

For acceleration of m_1 up the incline plane

$$2T - m_1g \sin \theta = m_1a$$

For vertically downward acceleration of m_2

$$m_2g - T = m_2 \cdot 2a$$

