



# Exploring Space Through ALGEBRA


**EDUCATOR EDITION**
**Algebra I**

## Space Shuttle Ascent

### Instructional Objectives

Students will

- create scatterplots from a data table;
- determine correlation and interpret its meaning;
- find linear and quadratic regression equations;
- select an appropriate regression to fit the data;
- find the slope and y-intercept from an equation; and
- communicate the meanings of slope and y-intercept as they relate to a real-world problem.

### Prerequisites

Students should have a good knowledge of scatterplots. They should understand the different types of correlations and what they mean in a problem. They should also have a good understanding of linear equations and their graphs. They should understand the meaning of slope and y-intercept and know how to find them. They should also understand the basic details of quadratic equations and their graphs. Prior to this activity, students should have experience using a graphing calculator or spreadsheet application to create scatterplots and find regression equations.

### Background

*This problem is part of a series of problems that apply Algebra principles to U.S. Space Exploration policy.*

Exploration provides the foundation of our knowledge, technology, resources, and inspiration. It seeks answers to fundamental questions about our existence, responds to recent discoveries and puts in place revolutionary techniques and capabilities to inspire our nation, the world, and the next generation. Through NASA, we touch the unknown, we learn and we understand. As we take our first steps toward sustaining a human presence in the solar system, we can look forward to far-off visions of the past becoming realities of the future.

The vision for space exploration includes returning the space shuttle safely to flight, completing the International Space Station, developing a new exploration vehicle and all the systems needed for embarking on extended missions to the Moon, Mars, and beyond.

Since its first flight in 1981, the space shuttle has been used to extend research, repair satellites, and help with building the International Space

**Grade Level**  
8-12

**Subject Area**  
Mathematics: Algebra I

**Key Concept**  
Modeling data with linear and quadratic regression equations

**Teacher Prep Time**  
5 minutes

**Problem Duration**  
50-60 minutes

**Technology**  
TI-83 Plus family;  
TI-84 Plus family;  
graphing calculator; or  
spreadsheet application

**Materials**  
- Student Edition

**Degree of Difficulty**  
Basic to Moderate

**Skill**  
Use a graphing calculator to create scatterplots, find regression equations; interpret graphs and equations of linear and quadratic data

**NCTM Principles and Standards**  
- Number and Operations  
- Algebra  
- Data Analysis and Probability  
- Problem Solving  
- Communication  
- Connections



Station, or ISS. However, by 2010 NASA plans to retire the space shuttle in favor of a new Crew Exploration Vehicle, or CEV. Until then, space exploration depends on the continued success of space shuttle missions. Critical to any space shuttle mission is the ascent into space.

The ascent phase begins at liftoff and ends at insertion into a circular or elliptical orbit around the Earth. To reach the minimum altitude required to orbit the Earth, the space shuttle must accelerate from zero to 8000 meters per second (almost 18,000 miles per hour) in eight and a half minutes. It takes a very unique vehicle to accomplish this.



*Figure 1: Space Shuttle Discovery at Liftoff*

There are three main components of the space shuttle that enable the launch into orbit. The orbiter is the main component. It not only serves as the crew's home in space and is equipped to dock with the International Space Station, but it also contains maneuvering engines for finalizing orbit. The external tank, the largest component of the space shuttle, supplies the propellant to the orbiter's three main maneuvering engines. The two Solid Rocket Boosters, the third component, provide the main thrust at launch and are attached to the sides of the external tank (Figure 1). The components of the space shuttle experience changes in position, velocity and acceleration during the ascent into space. These changes can be seen when one takes a closer look at the entire ascent process (Figure 2).

The ascent process begins with the liftoff from the launch pad. Propellant is being burned from the Solid Rocket Boosters, or SRB, and the external tank, or ET, causing the space shuttle to accelerate very quickly. This high-rate of acceleration as the space shuttle launches through the Earth's atmosphere causes a rapid increase in dynamic pressure, known as  $Q$  in aeronautics. The structure of the space shuttle can only withstand a certain level of dynamic pressure (critical  $Q$ ) before it suffers damage. Before this critical level is reached, the engines of the space shuttle are throttled down to about 70% of full power. At about one minute after launch the dynamic pressure reaches its maximum level (max  $Q$ ). The air density then drops rapidly due to the thinning atmosphere, and the space shuttle can be throttled to full power without fear of structural damage.

At about 2 minutes after launch, the atmosphere is so thin that the dynamic pressure drops down to zero. The SRB, having used their propellant, are commanded by the space shuttle's onboard computer



to separate from the external tank. The jettison of the booster rockets marks the end of the first ascent stage and the beginning of the second.

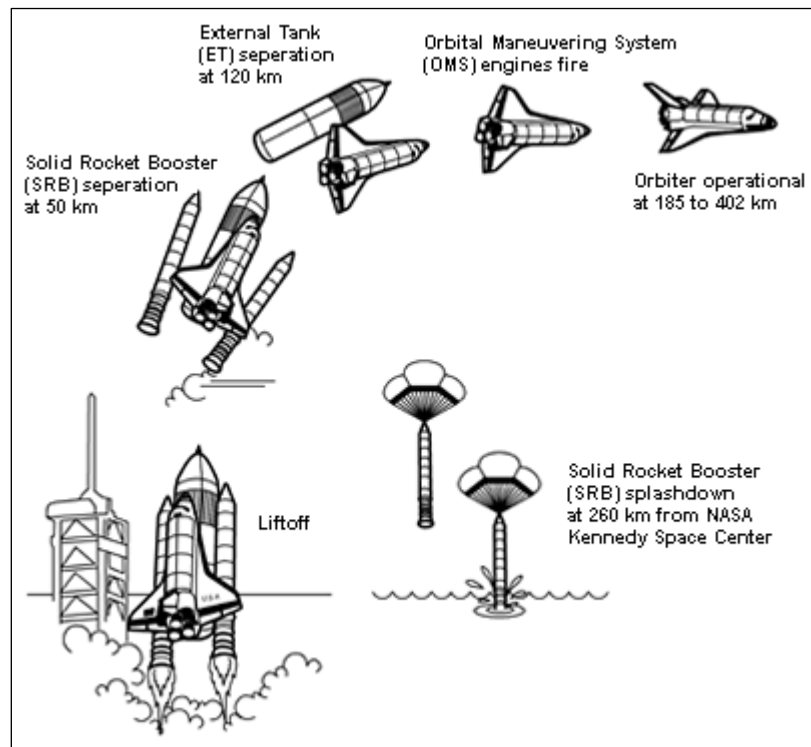


Figure 2: Space Shuttle Ascent Process

The second stage of ascent lasts about six and a half minutes. The space shuttle gains more altitude above Earth and the speed increases to the nearly 7,850 m/s (17,500 mph) required to achieve orbit. The main engines are commanded by the onboard computer to reduce power, ensuring that acceleration of the space shuttle does not exceed  $29.4 \text{ m/s}^2$  (3 g). This is 3 times the force of gravity that we feel while standing on the Earth's surface. Within thirty seconds the space shuttle reaches Main Engine Cut-Off, or MECO. For the next eleven seconds, the shuttle coasts through space. At nine minutes, the command to jettison the nearly empty external tank is given by the onboard computer leaving the shuttle's maneuvering engines to control any movement needed to achieve final orbit.

As we look to the future of space exploration, the ascent stage will remain a critical part of any successful mission.

For more information about space shuttle ascent and the U.S. Space Exploration policy, visit [www.nasa.gov](http://www.nasa.gov).



## NCTM Principles and Standards

### Number and Operations

- Develop a deeper understanding of very large and very small numbers and of various representations of them.

### Algebra

- Understand relations and functions and select, convert flexibly among, and use various representations for them.
- Analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior.
- Use symbolic algebra to represent and explain mathematical relationships.
- Identify essential quantitative relationships in a situation and determine the class or classes of functions that might model the relationships.
- Approximate and interpret rates of change from graphical and numerical data.

### Data Analysis and Probability

- For bivariate measurement data, be able to display a scatterplot, describe its shape, and determine regression coefficients, regression equations, and correlation coefficients using technological tools.
- Identify trends in bivariate data and find functions that model the data or transform the data so that they can be modeled.

### Problem Solving

- Solve problems that arise in mathematics and in other contexts.
- Apply and adapt a variety of appropriate strategies to solve problems.

### Communication

- Use the language of mathematics to express mathematical ideas precisely.

### Connections

- Recognize and apply mathematics in contexts outside of mathematics.

## Problem

On July 4, 2006 Space Shuttle Discovery launched from Kennedy Space Center on mission STS-121, to begin a rendezvous with the International Space Station, or ISS. Before each mission, projected data are compiled to assist in the launch of the space shuttle to ensure safety and success during the ascent. To complete these data, flight design specialists take into consideration a multitude of factors such as space shuttle mass, propellant used, mass of payload being carried to space and of that returning. They must also factor in atmospheric density, which is changing throughout the year. After running multiple tests, information is compiled in a table showing exactly what should happen each second of the ascent.

The data for mission STS-121, showing the mass and altitude of the space shuttle every 10 seconds from liftoff to Solid Rocket Boosters, or SRB, separation, are displayed in Table 1.

It is during the first stage of the ascent, that the space shuttle is burning the greatest amount of propellant. You can see in the table that the space shuttle has a mass of an amazing 2,051,113 kg.



After 2 minutes its mass is 880,377 kg. The burning of this vast amount of propellant is needed to get the space shuttle through Earth's atmosphere and into orbit. It is amazing to see how quickly the space shuttle can cover so much distance.

Table 1: STS-121 Ascent Data

Time (s)	Space Shuttle Mass (kg)	Altitude (m)
0	2,051,113	-8
10	1,935,155	241
20	1,799,290	1,244
30	1,681,120	2,872
40	1,567,611	5,377
50	1,475,282	8,130
60	1,376,301	11,617
70	1,277,921	15,380
80	1,177,704	19,872
90	1,075,683	25,608
100	991,872	31,412
110	913,254	38,309
120	880,377	44,726

Students will be asked to create scatterplots that show the relationship between mass and time, and also between altitude and time. They will be asked to describe and explain the respective correlations in terms of the problem, and then find the regression equations for these scatterplots. The first one being linear, the students will evaluate the meaning of slope and y-intercept. For the second scatterplot, students will identify the data as being quadratic in order to determine the regression. They will explain how the graph is quadratic rather than linear.

As an option, students may use a spreadsheet application to insert trend lines on the graphs.

## Lesson Development

Students will work in pairs or small groups to answer the questions. Encourage students to help each other understand and master the different uses of the graphing calculator and/or the spreadsheet application. You may need to review some of the steps on entering tables and finding regressions with the graphing calculator and/or the spreadsheet application.

## Wrap-Up

When students have completed the activity discuss some of the information as a class. Discuss the mass of the space shuttle and the great amount of propellant being burned per second. Discuss space shuttle launches that students may have seen in person or on television. Let students share their thoughts on what they have seen. If possible view a space shuttle launch with the class and discuss some of the terminology that students should now be more familiar with. Launches from many different missions can be found at [www.nasa.gov](http://www.nasa.gov).



## Extensions

Have students determine the mass of a typical fuel truck when it is fully loaded by researching online or by calling local companies. Have them then determine how many trucks it would take to approximate the same mass of propellant being burned during the 2-minute period and per second. Doing this will help students better relate to the magnitude of the problem.

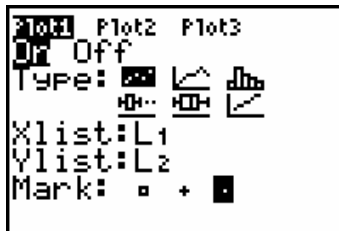
*Note: a Boeing 767 burns about 34,060 liters (9000 gallons) of fuel flying from NY to LA. This is equivalent to a mass of about 24,500 kg. The space shuttle burns that same amount of propellant in 2.5 seconds. This is also the approximate mass of a typical fuel tank fully loaded.*

## Solution Key (One approach)

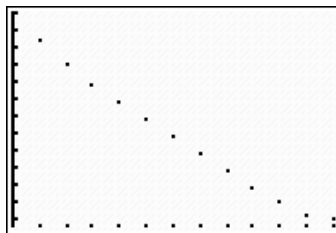
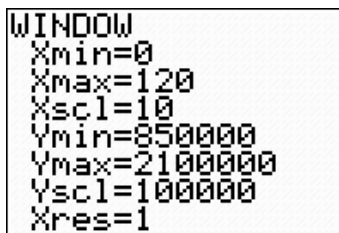
1. Enter the information from flight STS-121 into your graphing calculator; enter time in L1, mass in L2, and altitude in L3. Create a scatterplot of the mass vs. time. You may need to adjust your viewing window. What is the correlation of the data (positive, negative, constant, or no correlation)? Explain this in terms of the problem.

**Step 1:** Press the STAT button; select option 1: Edit; enter in information into L1, L2, and L3.

**Step 2:** Go to STAT PLOT (2<sup>nd</sup>, Y=); Select Plot 1 and set it as shown below.



**Step 3:** Adjust the viewing window and graph.



The graph shows a negative correlation. This is because as time passes propellant is being burned causing the mass of the space shuttle to decrease.

2. Use your calculator to find the linear regression equation for the function of mass vs. time. Make sure you graph the equation with the scatterplot to verify that it represents the data. Using function notation, what is the function of mass in relation to time,  $t$ ? Round coefficients and constants to the nearest whole number.

**Step 1:** Push the STAT key, go to the CALC menu, select option 4: LinReg, and press ENTER.

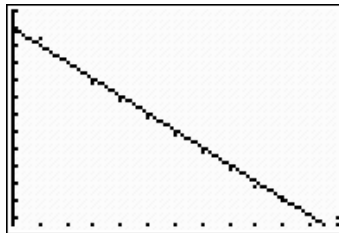




```
LinReg
y=ax+b
a=-9976.417033
b=1998791.407
```

**Step 2:** Students can either enter it manually into  $Y_1$  or the calculator will enter it automatically by going to  $Y_1$ , pressing the VARS key, selecting option 5: Statistics, then selecting option 1: RegEQ under the EQ menu. By graphing it they can see it fits the data.

```
Plot1 Plot2 Plot3
Y1=-9976.417032
967X+1998791.406
5934
Y2=
Y3=
Y4=
Y5=
```



Rounding coefficients and using function notation, the function is:

$$f(t) = -9,976t + 1,998,791$$

- What is the slope of the equation found in question 2? Explain what this represents in relation to the space shuttle.

The slope is  $-9,976$ . This means that the shuttle is burning 9,976 kg of propellant per second.

- What is the y-intercept of the equation found in question 2? Explain what this represents in relation to the space shuttle. How close is this to the data found in Table 1? Express this as a percent error rounded to the nearest tenth.

The y-intercept is 1,998,791. This represents the mass in kg of the shuttle at liftoff.

Note: this is approximately 4.4 million lbs, almost ten times the mass of the Statue of Liberty or the combined mass of 220 large school buses.

Table 1 shows the mass at liftoff is 2,051,113 kg. The percent error is found as follows.

$$\% \text{ error} = \left| \frac{\text{actual value} - \text{estimated value}}{\text{actual value}} \right| \cdot 100$$

$$\% \text{ error} = \left| \frac{2,051,113 - 1,998,791}{2,051,113} \right| \cdot 100$$

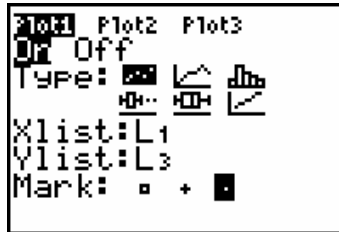
$$\% \text{ error} = 2.6\%$$

- Create a scatterplot of the altitude versus time. Again, you will need to adjust your viewing window. What is the correlation of the data? Explain this in terms of the problem.

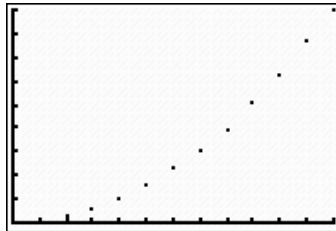
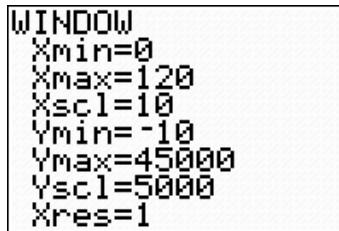


*Note: Students may ask why the altitude is negative at liftoff. Zero altitude can be described as a specific distance from the center of the Earth. Since the Earth is not perfectly spherical the location of the launch just happens to be below this specified point. Also, because this is a calculated number, some degree of error may be present.*

**Step 1:** Go to STATPLOT (2<sup>nd</sup> Y=) and change the Y list under Plot1 to L3 as shown.



**Step 2:** Clear the equation from Y<sub>1</sub>, change your viewing window, and graph.



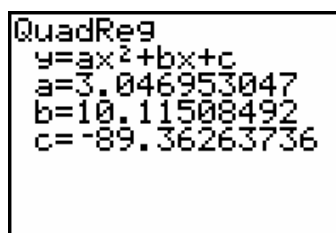
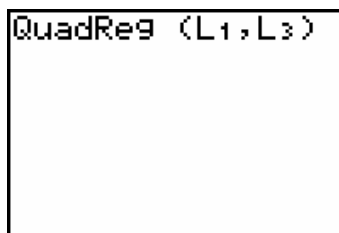
The graph shows a positive correlation. The shuttles altitude is increasing as time passes.

6. What kind of function would best describe the change in altitude vs. time? Explain your reasoning.

A quadratic equation would fit the data. It is not in a straight line but looks parabolic instead. As time passes the altitude is increasing at a greater rate.

7. Find the regression equation for the function of altitude vs. time. After selecting the regression option also input (L1, L3) before pushing enter. Otherwise the calculator will automatically use L1 and L2. Once again, be sure to graph the equation with the scatterplot to verify it represents the data. Using function notation, what is the function of altitude in relation to time,  $t$ ? Round coefficients and constants to the nearest whole number.

**Step 1:** Press the STAT key and select option 5: QuadReg under the CALC menu. Input (L1, L3) and press ENTER.

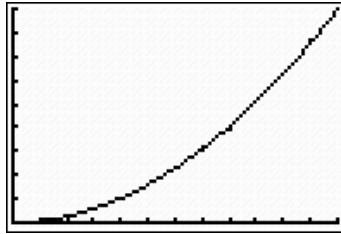






**Step 2:** Enter the regression equation in  $Y_1$  and graph.

```
Plot1 Plot2 Plot3
Y1=3.0469530469
529X^2+10.115084
9151X+-89.362637
363
Y2=
Y3=
Y4=
```



Rounding coefficients and using function notation, the function is:

$$a(t) = 3t^2 + 10t - 89$$



## Contributors

Thanks to the subject matter experts for their contributions in developing this problem:

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## Exploring Space through Algebra

### Space Shuttle Ascent

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Or type your responses in an email and send to: [Monica.Trevathan-1@nasa.gov](mailto:Monica.Trevathan-1@nasa.gov)

*Please circle the appropriate response.*

1. This problem was useful in my classroom.      YES      NO
2. The problem successfully accomplished the stated Instructional Objectives.      YES      NO
3. I will use this problem again.      YES      NO

4. Please provide suggestions for improvement of this problem and associated material:

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5. Please provide suggestions for future Algebra problems, based on NASA topics, that you would like to see developed:

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Thank you for your participation.

Please fax this completed form to Monica Trevathan at (281) 461-9350.