Unit 5: Quadratic Functions and Modeling
The eight lessons (5.1-5.8) provide the instruction and practice that supports the culminating activity in the final unit project. The lessons in this unit focus on working with quadratic relationships.

Lesson 5.6: Relationship between two Quantities
In this lesson, students will learn about quadratic functions that model a relationship between two quantities. They will apply this function to graphing.

Common Core State Standards by Cluster:

<table>
<thead>
<tr>
<th>Grade Level</th>
<th>Cluster</th>
<th>CCS Standard</th>
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</thead>
<tbody>
<tr>
<td>8</td>
<td>Solve equations in one variable</td>
<td>8.EE.7</td>
</tr>
<tr>
<td>9</td>
<td>Interpret functions that arise in applications in terms of a context</td>
<td>F.IF.6, F.IF.9</td>
</tr>
<tr>
<td>9</td>
<td>Build a function that models a relationship between two quantities</td>
<td>F.BF.1 a and b</td>
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Lesson Preparation and Resources for Teachers:
- Discovering Relationships Among Variables (Extension Activities on Curriki)
- Quadratic Inequalities (Visual Explanation) video
- Solving quadratic equations (Videos by Sal Khan on Curriki)
- Discovering Relationships Among Variables (Extension Activity)
- TE_11.07 Transformations of Quadratic equations
- 11.03 Solving quadratic equations by graphing
- TE_Vocabulary Activity Sheet
- TE_Quadratic Equations Assessment (Questions with solutions from Learn Without Limits on Curriki)
- TE_5.6 Introduction to Quadratic Modeling (PowerPoint by Kevin Hall in Curriki)
- Solving quadratic equations (Series of 9 videos by Sal Khan in Curriki)
- TE_5.3 Solving Quadratic Equations by Graphing Worksheet (Learn Without Limits in Curriki)
- Parabola in Real Life (Math Warehouse Internet Activity page)
- Geogebra
- FOSSWeb
- Functions from Winpossible
Instructional Materials for Students: (print one copy for each student)

SE_Introduction to Quadratic Modeling Handout (To accompany the PowerPoint by Kevin Hall)
SE_Transformations of Quadratic Equations Worksheet #1
SE_Vocabulary Activity Sheet
Graph paper
SE_5.6 Quadratic Equations Assessment (Student assessment worksheet)
Parabola in Real Life (Web Page)
Functions from Winpossible (Additional practice with quadratic functions in Curriki)
SE_Solving Quadratic Equations by Graphing Worksheet #2

Time: 50-minute session

Lesson Objectives:
Students will be able to:
• identify functions as linear or nonlinear and contrast their properties from tables, graphs, or equations
• Determine an explicit expression, a recursive process, or steps for calculation from a context.
• Combine standard function types using arithmetic operations
• model and solve contextualized problems using various representations, such as graphs, tables, and equations
• use graphs to analyze the nature of changes in quantities in quadratic relationships
• use geometric models to represent and explain numerical and algebraic relationships
• recognize and apply mathematics in contexts outside of mathematics

Lesson Content:
1. Background Building Activity for Students (10 minutes)


   Students will create Vocabulary Boxes for Mathematics. Distribute a copy of the Lesson 5.6 Vocabulary Activity Sheet for each student

   Write the following words on the board:
   Quadratic Function
   Constant
   Quadratic Modeling
   Linear functions
Non-linear functions

Ask students to work in pairs and fill out the boxes for each word given. They must put the word in the top right square, the root of the word in the top left square, the short definition in the bottom left square and a visual representation (graph, drawing, equation or other) in the bottom right square. Their work must be brief and focused in order to fit in the squares.

b. Warm Up Problem
Show the class the Quadratic Inequalities (Visual Explanation) video by Sal Khan (on Curriki). This will start the discussion about how the relationship between two variables can function. This video shows the quadratic relationship between \( f(x) \) and x-axis. When plotted on a graph, the formula he gives becomes a parabola.

If students need more background and practice in solving quadratic equations, there are nine videos by Sal Khan that review how to complete quadratic equations. This page is found on Curriki. Ask students to work in pairs on computers to review this process.

2. Focus Question based on today’s lesson (25 minutes)

Concept Question: Why do non-linear functions not make straight lines?
Answer: They do not make straight lines because they do not change at a constant rate.

Focus Question: Some friends form a Facebook group and start posting on each other’s walls. If everyone posts twice per day on everyone’s wall (including their own), how many total posts will be made per day?

a. Whole Class Activity:

Give each student a copy of the SE_5.6 Introduction to Quadratic Modeling Student Handout. Ask students to use this as a guide and fill it out as you lead the discussion using the PowerPoint.

Show Kevin Hall’s PowerPoint: Introduction to Quadratic Modeling to the whole class. In the slide show, he discusses the different families of functions, linear and non-linear. He introduces a real life example that illustrates non-linear functions, including rates of change.

The first five slides introduce the difference between linear and non-linear functions.
Slide 6 begins the focus question. The example shows what happens when people who form a Facebook group start posting on each other’s walls. Show slides 6 and 7. Ask students to turn to a partner and recreate the problem with different numbers of people. They can use a real-life Facebook group if they like.

Student pairs share their problems and review their solutions with the whole group. Ask the class to discuss the pattern in the solutions.

Show slides 8 and 9 to introduce quadratic functions to the whole group. Ask them to discuss how this relates to what they just did with the Facebook example.

Show slides 10 and 11 to introduce rates of change and how this would affect a graph of the data.

Slides 12 – 15 illustrate the shape of quadratic function graphs. Review these slides.

Slide 16 introduces the small group practice activity.

b. Small Group Practice activity
Ask students to work in small groups of four or five to discuss other real-life examples of quadratic functions.

Show slide 17. Students should answer the first question: Where in daily life do you see something that looks like a quadratic graph? (Answer: Anything that looks like a parabola i.e., roller coasters, other amusement park rides, leaves and other items in nature, suspension bridges, antenna of a radio telescope, flashlights, headlights, the path of a basketball then someone makes a free shot, and the path of a projectile--think Angry Bird!).

Groups can access the Internet and solve the problems about real world applications on the Parabola in Real Life page. Ask the groups to discuss the solutions with the whole class.

Ask the groups to draw a rainbow (parabola) on graph paper and describe the maxima of their graph. Students should next create the equation of their graph, and compare to another students work. Students should discuss how the maxima differ depending on the height of the rainbow (parabola). Describe how the equations also differ. Next, ask students to make a prediction about a parabola with a minimum, such as a letter U. Ask the student groups to write a function that describes the relationship between two quantities that would produce a quadratic graph in the shape of a parabola. Ask them to create the graph.
3. **Whole Class Discussion (10 minutes)**
   Student groups share the answers to the problems found on the Parabola in Real Life practice page. Then they will share their own real life functions. The class will discuss these functions and help improve upon them.

4. **Assessment**
   Give each student a copy of the SE_5.6 Assessment Sheet and graph paper. Ask them to first solve the problem, then graph it. Students should write a brief explanation about the quadratic relationship between x and y in each problem. They can check their work by going to Geogebra, located at [www.geogebra.org](http://www.geogebra.org/).

   Ask students to share their solutions, graphs, and explanations while the results are checked in class. This will reinforce student learning.

5. **Extension Activity**
   **Discovering Relationships Among Variables**
   Students can use their intuitive definitions of patterns, the infinite, and variables, while exploring the properties and relationships of objects in a variety of contexts. Through this guided discovery, students will move between the four learning stations while exploring the relationship between objects and events. The following activities were taken from FOSS (Full Option Science Solutions) kits. Please visit the FOSSWeb site for informative video segments that detail the activities below.

   **Functions from Winpossible**
   This is a series of on-line mini-lessons and activities on Curriki from Winpossible that focus upon the quadratic relationship between quantities. It will extend practice with what students learned in this lesson.

6. **Homework assignment for additional independent practice** (Note: This homework assignment can be done during a subsequent class period if you have the time.) Give students the following homework sheets for additional practice with quadratic equations and relationships between two quantities.

   SE_Solving Quadratic Equations by Graphing Worksheet #1
   SE_Transformations of Quadratic Equations. Worksheet #2

   The teacher answer keys are located here:
   TE_11.07 Transformations of Quadratic equations Worksheet #1
   11.03 Solving quadratic equations by graphing Worksheet #2