**2.1 Lesson Guide: How Do You Make a Mural?**

**Context:**

Students should already have learned how to write an equation for a linear scenario with a slope and y-intercept. In this lesson, they will be introduced to proportionality, how to recognize when it occurs, and how to use it to solve problems when it does occur.

**Big Ideas:**

* To enlarge something without distorting it, we have to multiply all the dimensions by a constant number. If instead you add the constant number to all the dimensions, the image gets distorted.
* The words “proportional” or “in proportion” describe shapes that have been enlarged or shrunk without being distorted.
* Sometimes we find an unknown amount by assuming that the ratios of two quantities must be equal. Then we can solve using the method of equivalent fractions. This only works when doubling or tripling quantity A would result in a doubling or tripling of quantity B.

**Materials needed:**

* Each group will need 1 copy of this activity packet.
* However, it’s a good idea to have spare copies of the 2 grid pages for enlarging the dog picture, because if students make mistakes when drawing the pictures, they may ask for fresh copies so they can start over.

**Procedure:**

* #8 (Drawing the mural with multiplying). If possible, emphasize that students should be expanding the negative (empty) spaces by a factor of 3 as well—for example, instead of leaving 1 space between each leg, they should leave 3 spaces.
* If a group is ready for this extension, just give them a copy of the activity. It’s pretty self-explanatory.
* It’s a good idea to check groups’ answers to #4 before they move on to drawing the dog. If they have the wrong measurements, their pictures will definitely come out wrong.
* An answer key is attached at the end of this document.
* If they don’t finish this by the time you move the class on to Lesson 2.3, they can keep it and finish it as an extension activity after they finish 2.3.

 **Answer Key: “How Do You Make a Mural?”**

1). Answers will vary.

2).

Tail:

 Height: **4 units**

 Width: **1 unit**

Body:

 Height: **4 units**

 Width: **10 units**

Head:

 Height: 3 units

 Width: **3 units**

Nose:

 Height: **1 unit**

 Width: **1 unit**

**Example:**

Legs:

 Height: **4 units**

 Width: **1 unit**

3). It doesn’t matter whether they write “adding” or “multiplying” here. They’re just predicting.

4).

|  |  |  |  |
| --- | --- | --- | --- |
| Part of Dog | Original Measurements (from last page) | ADD 3 to the original measurements | MULTIPLY the original measurements by 3 |
| Head | Height: **3 units**Width: **3 units** | Height: 3+3 = 6 unitsWidth: **6** | Height: (3)(3) = 9 unitsWidth: **9** |
| Body | Height: **4** Width: **10**  | Height: **7** Width: **13**  | Height: **12** Width: **30** |
| Tail | Height: **4**Width: **1**  | Height: **7**Width: **4**  | Height: **12**Width: **3** |
| Legs | Height: **4** Width: **1**  | Height: **7** Width: **4**  | Height: **12** Width: **3**  |
| Nose | Height: **1** Width: **1**  | Height: **4** Width: **4** | Height: **3** Width: **3** |

**5). Picture made by ADDING 3:**

Note: because of the distortion, it’s actually impossible to get the legs to fit on the body properly.

6). The dog looks all messed up. His tail is too fat, and so are his legs. And his nose is way too big. His body is too small.

7). Adding 3 to all the measurements was not a good way to do it. The picture doesn’t look proportional any more.

**8). Picture made by MULTIPLYING by 3:**

**You need to turn the paper sideways to fit the entire picture on the page.**

#8 (Drawing the mural with multiplying). If possible, emphasize that students should be expanding the negative (empty) spaces by a factor of 3 as well—for example, instead of leaving 1 space between each leg, they should leave 3 spaces.

9). The dog looks fine, just larger. But it definitely looks proportional.

10). Multiplying by 3 is a good way to enlarge the dog without distorting it.

Vocabulary Moment The terms “ \_**in proportion**\_” and “\_\_\_**proportional**\_\_\_\_” describe a shape whose parts are all the correct size in comparison to each other.

11). Use one of those terms in a complete sentence to describe this picture of George Washington.

Be as specific as possible.

**He is not proportional. His head is out of proportion with to the rest of his body.**

Practice and extend

12). Bryan wants to climb the palm tree to get the coconut, but it looks pretty tall. He can measure the shadows, since they’re on the ground. Based on the shadows shown, how tall is the tree?

6 ft.

8 ft.

32 ft.

*x*

*Answer: x = 24 ft.*

**× 4**

**× 4**

**Here I plan NOT to explain this as a proportion, since that line of reasoning does not mimic what was discovered with the mural activity. Instead, I will point out that the triangles look similar—in the language of the mural investigation, that the palm tree’s triangle is looks enlarged from the stick figure’s triangle without being distorted. If that’s true, it must have been enlarged by multiplying a constant amount by all the measurements. What was it multiplied by? By 4, since 8\*4 = 32. So x is 6\*4.**

**If a student does explain it with fractions, I will respond by breaking down what they’ve said into 2 pieces: “It looks like you did 2 things. You realized that you should multiply by 4 to make the comparison, and you set up the problem as 2 equal fractions. Multiplying by 4 is an important point, let’s see if everyone understands that… Now why did you set it up with fractions—how does enlarging shapes relate to fractions?” Whether or not the student can explain, I’ll say “Huh, that’s interesting food for thought. We’ll come back to that a little later.”**

13). At the movies, 3 tickets cost $36.48. How much would 21 tickets cost?

 **Answer: (36.48)\*(7) = 255.36**

**Discussion: This should be pretty easy at this point. The purpose is to introduce a table to organize the comparisons. If you ask the class how to solve the problem, it’s likely that a student will explain that since 3\*7=21, you need to multiply $36.48 by 7. If I am at the board, I’ll try to represent the student’s explanation with a table, but without introducing a fraction bar yet. For example:**

|  |  |  |  |
| --- | --- | --- | --- |
| **3 tickets** | **× 7** | **= 21 tickets** |  |
| **$36.48** | **× 7** | **= ??**  | **🡨Must be = $255.36** |

**I will follow up by asking: “How else could we arrange the numbers in the table and still have it make sense? Is there any way you couldn’t arrange the numbers?” The point is that you could set it up any way the units are consistent, such as this:**

|  |  |
| --- | --- |
| **3 tickets** | **= $36.58** |
| **21 tickets** | **= ??**  |

**But not this:**

|  |  |
| --- | --- |
| **3 tickets** | **= $36.58** |
| **??** | **21 tickets** |

**The fact that this is behaving like two equal fractions is not yet needed. For students who lack a deep understanding of fractions, introducing fractions at this point may cause them to begin blindly following computational procedures, when we are trying to get them thinking about the topic first before addressing the computational aspects.**

Check-in: what have we learned so far?

**The following points need to be drawn out of the class in the discussion. After discussion, the class uses the lined spaces to record their versions of points 1-3 below:**

1. You should compare by multiplying, not by adding
	1. If this point doesn’t come out, prime the pump by asking “what did we learn with the dog picture?”
2. Labels need to be in a consistent order
3. Add a short discussion that if multiplication works, dividing would work, too (the original and final pictures would just be reversed).

**Discussion prompt:** Ask if comparing by multiplying is always right, or just in certain cases. It’s not important what conclusion the class reaches on this; it’s just to get them thinking about it, and to lead them into the next question.

Practice and extend

14). Some pizzas are $20 without delivery, and $24 with delivery. If a larger order of pizzas is $40 without delivery, how much would it be with delivery?

**Answer: $44.**

|  |  |  |  |
| --- | --- | --- | --- |
| **$20 no delivery**  | **+ 4** | **= 24 w/ delivery** |  |
| **$40 no delivery** | **+4** | **= ??**  | **🡨Must be $44** |

* **Elicit both ideas of comparing by + and comparing by \*. Make 2 comparison tables, to show each method. Discuss.**
* **Must make students explicitly state that the correct method is to compare by adding/subtracting here. The reason is that pizza delivery is a flat fee that doesn’t increase when the order size increases.**
* **Pose question at end: so when can we compare by multiplying/dividing and when not? Not too important what response students give—the question at this point is just to get them thinking about the issue. This leads into the next question.**

15). Mr. Hall is not great at shooting free-throws. Out of 6 shots, he makes 4. Based on that, how many would he probably make out of 18 shots?

* **First I’ll walk around the room and look at students’ ideas as they write their answers down. I want to see if almost everyone agrees that you should compare by multiplying, or compare by adding. I’ll also notice whose written work suggests they’d be good students to call on in the discussion.**
* **The first question I’ll ask is: “Does it make sense to compare by adding or multiplying in this situation?” I’ll let a few students speak.**
* **To clarify what comparing by adding would mean, I’ll ask someone to demonstrate how it would work in this problem: 3 is 2 less than 5. What is 2 less than 30? 28. So Mr. Hall would make 28/30 free throws if you use the addition comparison. (Student could explain in words or in a table).**
* **To clarify what comparing by multiplying would mean, I’ll ask someone explain that approach: 6\*3 is 18, and 4\*3 is 12, so the answer is 12.**
* **If necessary, I’ll ask a few more students to explain why multiplying makes more sense here, until someone gives a good explanation involving the fact that Mr. Hall would be expected to make 4 out of every 6 shots, so he would make 8 out of 12, and therefore 12 out of 16. I’ll ask permission to translate that reasoning into a picture and ask the students to copy the picture, too.**

4 shots made

6 total shots taken

4

6

Made

Taken

4

6

4

6

Then I’ll ask students to go back and draw a similar picture for the pizza-delivery example, for contrast. (Lots of extra space was left to allow for that) Answer:

$24 with delivery

 $20

 $24

$20 no delivery made

$4

You only pay for delivery once, so the full $24 does not repeat for the 2nd pizza.

No delivery

Delivery

 $20

 $20

1st pizza

2nd pizza

**Conclusion:** What’s the difference between the pictures for when you can compare by multiplying works and the pictures when you can’t?

**When comparing by multiplying doesn’t work, it’s because one of the bars changes its amount when it repeats. When comparing by multiplying does work, then both bars stay the same when they repeat.**

16). When you get a large frozen yogurt from Pinkberry with 2 servings of coconut topping, you get a total of 380.6 calories. If you got 4 servings of coconut instead, how many calories would that have?

 Source: <http://www.pinkberry.com/images/pdf/Nutrition.pdf> accessed 4/28/12

**Jose’s work**

|  |  |
| --- | --- |
| 2 servings coconut | 4 servings coconut |
| 380.6 calories | ?? |

× 2

× 2

(380.6)(2) =

 761.2 calories

**Use a bar picture (like in #14-15) to explain whether or not you agree with Jose.**

380.6

 2

Servings of coconut

Total calories

380.6

 2

Servings of coconut

Total calories

few

 2

Most of these calories come from the frozen yogurt, not the coconut topping.

Jose is incorrect, because the bar for total calories changes when you repeat it, so you can’t compare by multiplying.

Small-group Practice:

For #17-21, explain in a complete sentence whether you agree or disagree with the work shown. Be sure to give your reasons.

**I plan to let students work on these on their own. We’ll discuss them as a class once groups have had enough time to try each question.**

17). A baby weighs 9.4 lbs on the second day after it was born. How much will it weigh on the sixth day?

|  |  |
| --- | --- |
| 2 days old | 6 days old |
| 9.4 lbs | ?? |

× 3

× 3

Zack’s work

 Do you agree or disagree with Zack’s decision to compare by multiplying? Why?

**Disagree. The reason is that each time you repeat 2 more days, you do not add 9.4 lbs. Most of that weight was added before the baby was born, not in its first 2 days.**

**During the discussion, some groups may say they disagree because common sense tells them that (9.4)(3)=28.2lbs is too heavy to be the weight of a 6-day old baby. I’ll praise them for their common sense but press them for why comparing by multiplying gives an impossible answer.**

18). When you work 5 hours, you get paid $45. When you work 40 hours, how much do you get paid?

|  |  |
| --- | --- |
| 5 hrs worked | 40 hrs worked |
| $45  | ?? |

× 8

× 8

**David's work:**

**Question:** Do you agree or disagree with David’s decision to compare by multiplying? Why?

**David’s work is correct, because every time you add another 5 hours, you will add another $45. Each bar will repeat without changing if you draw the picture.**

19). You pay 22% taxes. How much tax would you pay if you earned $400?

(Remember, 22% means 22 out of \_\_\_\_\_).

 100

 22

Taxes paid

Dollars earned

Taxes paid

Dollars earned

 100

 22

 100

 22

 100

 22

 100

 22

So your taxes would be (22)(4) =

**Milena’s work**

 $88

 **Question:** Does Milena’s picture make sense? Why or why not? Do you agree with her answer?

**Milena’s work makes sense. Because 22% means 22 out of every 100, you would repeat the 22 every time you repeat the 100.**

20). The Miami Heat basketball team has a score of 58 points when it’s 3 minutes after halftime. What will be their score when it’s 6 minutes after halftime?

|  |  |
| --- | --- |
| 3 minutes | 6 minutes |
| 58 score | ?? (score) |

× 2

× 2

**Diana’s work:**

So their score will be (58)(2) =

 116 pts.

**Question:** The answer is too high. Why doesn’t comparing by multiplying work? Be as specific as possible.

**They did not score all 58 points in those 3 minutes. A lot would have been scored in the first half. So repeating another 3 minutes would not repeat another 58 points.**

What does this have to do with fractions?

**Here I plan to ask “What’s another fraction that’s equal to ½?” If nobody answers, I’ll prompt them by writing, . I’ll have the class build up a string of equivalent fractions. Then we’ll quickly check that they are equivalent by dividing them on the calculator to get 0.5 for each one.**

Vocabulary moment:

* A \_\_\_**ratio**\_\_\_ is a comparison of 2 numbers using division.

Examples:

 **(2 shots made)/(5 shots taken)**

 **(2 shots made) : (5 shots taken)**

* A \_\_\_\_\_**proportion**\_\_\_\_ is an equation that says 2 ratios are equal.

 Examples:

21). Use your calculator to check whether these proportions are correct. Write “Correct” or “Incorrect.”

 a). b). , with c).

 **Correct. Correct Incorrect**

small-group practice:

Set up and solve each problem with a proportion. Write each answer in a complete sentence. If you don’t think a situation would be proportional, just write “not proportional,” and skip the problem.

22). Trayon took 25 total shots, and made 18 of them. How many would he make out of 100?

 . X =72. **He would make 72 shots out of 100.**

**Discussion note: this was deliberately set up to yield a percentage (out of 100), so in the discussion you can bring out the other way of doing it (simply divide 18/25 and move the decimal 2 places).**

23). You are skateboarding down a hill (without pushing with your legs). After 2 seconds, you have gone 11feet. How far will you have gone after 8 seconds?

**Not proportional, because it’s not true that for every 2 seconds, you’ll go another 11 feet. You speed up as you keep going.**

24). You figure out that you like about 4 out of every 9 songs that you hear on your 97.1 FM. At that rate, if you hear 54 songs, how many will you like?

 . **You will like 36 songs out of 54.**

25). You turn off the oven to cool it down. About 4 minutes later, it’s 300oF. How hot will it be 8 minutes after you turned it off?

**Not proportional. It’s not true that every time you add 4 minutes to the time, you add 300 degrees to the temperature (it’s cooling down!).**

26). About of the calories in peanut butter come from fat (that’s true). If you eat 128 calories’ worth of peanut butter, how many calories come from fat?

 . **You will get 96 calories from fat.**

27). Students are arguing about how to solve the proportion . Which methods are incorrect, and why?

+ 10

+ 10

So *x* = 9 + 10

 *x* = 19

• 3

• 3

So *x* = 9 • 3

 *x* = 27

– 10

So *x* = 19, because

 19 – 10 = 9

– 10

÷ 3

So *x* = 27, because

 27 ÷ 3 = 9

÷ 3

Method A

Method B

Method C

Method D

**Methods A and C are incorrect. You can’t compare fractions by adding or subtracting.**

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