Toano Middle School 8th Grade Geometry – Investigating Secants and Tangents

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Context/Grade Level: This lesson is designed for an 8thgrade geometry class at Toano Middle School. This advanced math class contains 16 students, 2 of which are diagnosed with ADD/ADHD. This is an introductory lesson to central angles, inscribed angles, secant lines, and tangent lines.

Objectives:

-The student will define vocabulary associated with circles, such as central angle, inscribed angle, secant, and tangent and make figures to demonstrate examples and non examples of each word.

-The student will measure central angles, inscribed angles, and arcs of circles directly and indirectly, using geometer’s sketchpad.

-The student will generalize the relationship between angle measure and arc measure using geometer’s sketchpad.

-The student will complete problems with central and inscribed angles before the end of class with at least 80% accuracy.

SOL Strand: Circles

SOL G.10 The student will investigate and solve practical problems involving circles, using properties of angles, arcs, chords, tangents, and secants. Problems will include finding arc length and the area of a sector, and may be drawn from applications in architecture.

Materials/Resources:

-Geometer’s sketchpad

- class set of laptops (or computer lab if this is not available)

-anglegs

-paper plates

-Inspiration software (worksheet is attached if this is not available)

Approximate time: 1 hour and 20 minutes

Content and Instructional Strategies:

1. **Pre-Class:** Set up the daily agenda document and display on the screen. It should look like:

**Do Now – Geometry (Date)**

 **-**Find the circumference of a circle with radius 5 cm.

 -Find the diameter of a circle with circumference 26π

-Anne is riding a horse which is tied to a pole with a 3.5 m piece of rope and her friend Laura is riding a horse which is 2 m from the same pole. Calculate the distance travelled by each when they have rotated 10 times around the center.

-Define central angle, major arc, minor arc, inscribed angle, secant, and tangent.

**Aims:**-TLW investigate and solve practical problems involving circles, using properties of angles, arcs, chords, tangents, and secants.

**Procedures:**

-Do Now

-Tangent and secant Activity

-Geometer’s Sketchpad

**Homework:** Read the next section in the book.

1. **Intro (5-7 min).**

 **Activity Type: Practice – Do Computation**

**Technology: problem/daily agenda is set up on the computer**

This lesson is designed for the spring semester, so I do not know if there will be homework from the night before to go over. In the event that this is not the first lesson after a test, the class will begin by going over any homework before going over the do now. The do now for this class activates prior knowledge about radius, diameter, and circumference of a circle, and it asks the students to define vocabulary for the class. The vocabulary can be defined using the textbook or using any prior knowledge of the subject.

Do Now:

**-**Find the circumference of a circle with radius 5 cm.

 -Find the diameter of a circle with circumference 26π in

-Anne is riding a horse which is tied to a pole with a 3.5 m piece of rope and her friend Laura is riding a horse which is 2 m from the same pole. Calculate the distance travelled by each when they have rotated 10 times around the center.

-Define secant, tangent, central angle, and inscribed angle.

Remind the students that drawing pictures is a good strategy for solving problems such as these.

1. **Go Over the Do Now (5 min)**

**Activity Type: Practice – Do Computation**

**Technology: Whiteboard to work out solutions**

Students may come to the board to work out each of the first three problems. The definitions will be used in the manipulatives activity.

* $c= 2π5$

c = 10$ π≈ 31.4159265$cm – let students know that either answer is fine, but that the decimal version is an approximation

* $26π= πd$

d = 26 in

* $a= 2\left(3.5\right)π\*10≈ 219.911486 m$
* $l=2\left(2\right)π\*10≈ 125.663706 m$ - discuss how what seems like a small change in radius turns into a large difference in total distance travelled after ten rotations.
1. **Go over vocbulary (10 – 15 min)**

**Activity Type: Produce – Do a demonstration**

**Technology: Anglegs (manipulatives for creating angles), paper plates, markers**

* Using the definitions for central angle, major arc, minor arc, inscribed angle, secant, and tangent, have students construct an example and a non example of each using paper plates and anglegs. First, construct a central angle, then one student can put their version on the document camera for the class. When the student displays their example, be sure to rotate the circle around to illustrate that there is no one “right” example. Then a student can display their non example. Repeat for all vocabulary
* central angle: and angle whose vertex is the center of a circle



* major arc: an arc that is more than 180 degrees (more than half of the circle)
* minor arc: an arc that is less than half of the circle. (major arc and minor arc can be demonstrated together, using the paper plates and markers. Be sure to point out that arcs at 180 degrees are semicircles.)



* -inscribed angle: angle whose vertex is on a circle and whose sides contain chords of the circle. Example:



* secant: a line that intersects a circle in two points. Example:



-tangent: a line in the plane of a circle that intersects the circle in exactly one point. The point is called the point of tangency. Example:



1. **Introduction to Technology (5-6 min)**

**Activity Type: Consider – Attend to a demonstration**

**Technology: Computer for teacher, laptops for students, Geometer’s Sketchpad, and Inspiration (if available)** Toano definitely has a class set of laptops with Geometer’s Sketchpad, but possibly no Inspiration.

* The teacher should connect their computer so that it is on the large display. Walk through creating a new sketch, creating a circle, putting points on the circle, constructing an arc, measuring an arc, making segments, and switching to the arrow to highlight points. Look through the options on the Construct and Measure menus, since the students will be using these most often.
* The teacher should also show a demonstration of Inspiration for the graphic organizer at the end of class. If the school does not have inspiration, the teacher should demonstrate a graphic organizer on the board. Remind the students that any definitions and big ideas (like the questions on the Geometer’s Sketchpad Activity) are good ideas for the graphic organizer.
1. **Geometer’s Sketchpad Activity and worksheet (20-30 min)** Time depends on class familiarity with Geometer’s Sketchpad.

**Activity Type: Consider – Investigate a concept (Geometer’s Sketchpad)**

**Activity Type: Practice – Do Computation (worksheet of practice problems)**

**Technology: laptops with Geometer’s sketchpad**

* See “Differentiation” below for plans if the technology fails.
* Group students in pairs (or larger groups for a class larger than 16). Every student should have a laptop and an activity sheet (attached). The activity sheet has a section for Central Angles, Inscribe Angles, Tangents, Secants, and Intersecting Circles. The pairs of students should complete the Central Angle instructions, discuss the conclusions they have drawn, and discuss the conclusions with another pair before moving on to the next section. This should be repeated for all sections. Remind students that to label a point, they right click on the point and choose label. Also remind students that when they construct arcs, they need to be sure no points or segments other than the three they are working with are highlighted.
* A teacher copy of figures that the students should be drawing is attached for reference. This shows what students should be seeing. If they are having problems, try to redirect them back to a semblance of the picture on the reference sheet. If the picture has gotten truly muddled, the student may need to start from scratch. Be on the lookout for problems constructing the arc on the circle. If the students have not selected the points in counterclockwise order, starting with the center of the circle, they will not even have the Construct, Arc on circle option.
* Once the students have used Geometer’s Sketchpad to explore central angles, inscribed angles, tangents, secants, and intersecting circles, they should work in their pairs to complete the associated problems to apply their knowledge.
* As students finish the activity and the worksheets, the teacher should check the answers to the problems. Once the students have completed both activities, have them begin a graphic organizer for the day.
1. **Graphic Organizer (10 min)**

**Activity Type: Interpret – Categorize**

**Technology: Inspiration**

* Once the students have finished the activity and their worksheet, and both have been checked with another group and by the teacher, they should create a graphic organizer of the subjects for the day. This can be done using Inspiration if the school laptops have the software installed. A sample inspiration organizer is attached. If the school does not have Inspiration, the students should return their laptops when they have finished with Geometer’s Sketchpad, then complete the worksheet and the graphic organizer. A partial organizer is attached to get the students started. Depending on the way the beginning of the class has gone or how much time is left in the class period, this can be done as a whole-class activity or left up to the pairs to organize their thoughts as desired.
1. **Summary**

**Activity Type: Consider – Attend to a demonstration**

**Technology: none**

* The graphic organizer is the exit ticket for the day—before students leave, the teacher should check to be sure all relevant information has been included. A sample graphic organizer is attached (it was made using Inspiration).
* The teacher should go over the main points from the organizer with the whole class to summarize the lesson for the day.

Evaluation/Assessment:

The in-class worksheet and the graphic organizer will be used as assessment for the day. At the beginning of class, students may need to look up the vocabulary definitions, but by the end of class they should know the definitions and be able to apply those to the class work sheets.

Differentiation/Adaptations:

-Working in pairs lends itself to differentiation. Students can help each other and compare answers with other pairs.

-The in class worksheet can be shorter for students who need differentiation.

-The teacher can offer more structured instructions for creation of the graphic organizer for special education students who need explicit directions.

-If Geometer’s sketchpad is not working, or if the laptops crash, but the teacher’s computer works, the teacher can demonstrate these concepts using one of many online tools with the class, allowing students to come up and move some of the points around, before students complete the worksheet for practice and the graphic organizer. Tools include:

-Math Warehouse: [http://www.mathwarehouse.com/geometry/circle/central-angle-of-circle.php#](http://www.mathwarehouse.com/geometry/circle/central-angle-of-circle.php) for central angles and <http://www.mathwarehouse.com/geometry/circle/inscribed-angle.html#inscribedAngleDemo> for inscribed angles.

-Wolfram Math World: <http://www.mathwarehouse.com/geometry/circle/inscribed-angle.html#inscribedAngleDemo> for secants <http://mathworld.wolfram.com/CircleTangentLine.html> and tangents

-If all computer-based technology fails, the teacher could also begin with the graphic organizer to give an overview of the topics for the day, and use the paper plate, anglegs, and a protractor to follow the same steps as the Geometer’s Sketchpad worksheet in order to demonstrate the different properties, and the students can follow along with their manipulatives. In this case, the teacher should lead a discussion for the questions on the Geometer’s Sketchpad worksheet, and ask the students to collectively form the expressions for the Tangent angle and Secant angle. The teacher (s)he has a reference sheet for the geometer’s sketchpad activity, (s)he can use those pictures as a guide. The notes should follow the form:

1. Type of Angle/Line

a. Definition

b. Picture Example (refer back to the examples and non examples from the vocabulary)

 -Use the anglegs and paper plates to demonstrate

c. General relationship (the questions from the notes—let the students puzzle this out as much as possible, although they may need some assistance with the tangent angle and the secant angle formulas).

This format for notes follows the format that the students are used to seeing, so they will be able to fill this in easily.

1. Central Angle
	1. an angle whose vertex is the center of a circle
	2. (see picture above or Instructor Reference Sheet for Geometer’s Sketchpad)
	3. Measure of the central angle = Measure of the arc it encloses
2. Inscribed Angles
	1. angle whose vertex is on a circle and whose sides contain chords of the circle
	2. (see picture above or Instructor Reference Sheet)
	3. measure of the angle = ½ \*arc angle measure
	4. If chords are an equal length, their corresponding arc lengths are equal.
3. Tangent Lines
	1. a line in the plane of a circle that intersects the circle in exactly one point. The point is called the point of tangency
	2. (see picture above or Instructor Reference Sheet)
	3. tangent lines are perpendicular to radii at the point of tangency.
	4. measure of angle between two tangents = (major arc – minor arc) / 2
4. Secant lines
	1. a line that intersects a circle in two points
	2. (see picture above or Instructor Reference Sheet)
	3. measure of angle between two secant lines = (angle measure of larger arc – angle measure of smaller arc) / 2

After the notes are demonstrated and recorded, have the students complete the practice worksheet to apply their new knowledge, then revisit the graphic organizer at the end of class for a summary.

Reflection

The Lesson Plan is designed to engage students in learning about some of the attributes of circles. Traditionally, central angles, inscribed angles, secants, and tangents are not particularly exciting topics. As interesting as I find inscribed angles now, and as often as I have used them for proofs during the past three years of college, as an eighth grade student, they were not even a blip on my radar. The lesson starts with activating background knowledge with the warm up problems. This is part of the recognition network in the UDL model. The vocabulary definitions will be new for the students, so by taking the time and using the manipulatives to go over each vocabulary term, I am clarifying vocabulary, another one of the options for the recognition networks as described in the UDL model. Also, asking the students to create a non example takes the understanding to the next level. For this, students have to demonstrate that they can do more than memorize a definition; they need to apply the definition to create two products.

Once the students have an understanding of the vocabulary for the day, the class begins the meat of the technology for the lesson. Geometer’s Sketchpad allows students to create their own examples of properties, and move points and lines around to see how a property holds. I felt that this would be more enjoyable for students than a lecture on the topic. What students aren’t excited to use laptops for the day? I combined the technology with cooperative learning –the students complete a section of the guided worksheet, talk about the answers with their partner, and discuss the partner findings with those of another pair. Through this process, the kids have created a property, tested it, named it, and discussed it twice. On top of all of that, they have (hopefully) written something down about the property, as well. This is offering benefits for students who learn best visually, orally, or by writing, and supporting affective learning of the UDL model. The students then apply their new knowledge to a series of practice problems with their partner. Finally, if the technology is available for the graphic organizer, then that activity is a quick, easy way to collect all of the thoughts of the day together. It requires the students to think like a teacher and pick out the important parts to include on the web in an appropriate place. This ties the vocabulary, the activity, and the important math parts of the lesson together in a very neat package that the students can use to study for the rest of the year or semester.

Since the Geometer’s Sketchpad technology is designed specifically for math, all aspects of it are very technically correct. For example, when anyone writes answers to angle measures, they skip the “m” at the beginning. Technically speaking, it is not angle ABC that equals 50 degrees, but the measure of angle ABC that equals 50 degrees. Using Geometer’s Sketchpad, the students go through the process of selecting the “Measure” tab, before they choose what to measure, which cements that fact in their heads. After going through the activity a few times, I will never forget the “measure” part of the process again. The software also does a great job of using different colors to highlight different parts of the circle, from the points to the arcs to the lines. Also, the exact angle and line measures are incredibly convenient. With a hands-on version of the same activity, the students would need to physically measure everything. This would also help them remember the “Measure” step, but it would also make them hate it, because it would be very time consuming and tedious. The focus of the lesson is not how to use a protractor, but on the properties of the circle attributes. On the content side, Inspiration is not as connected, because it is hard to write out math expressions. However, the convenience of creating the graphic organizer so easily makes it worth it to use. If the students wanted, they could create an empty circle for the expressions and fill those in by hand.

All together, the math, the technology, and the cooperative learning fit together like a puzzle. I have not tried this lesson with a real, live class yet, and I’m sure that there will be some difficulties in that event. However, since the kids are learning by seeing, doing, talking, and writing, I think the technology is well worth using. This lesson was designed around the use of Sketchpad, so the backup for the technology is much more one-dimensional. With the technology, the students are learning technically correct math, and they are getting a chance to move the points, move the lines, and explore the topic in depth. Also, for this class at Toano in particular, the students are multiple grade levels ahead in mathematics, and their day-to-day curriculum is not student-centered. I believe that this type of student-centered lesson will give the students a chance to shine and to see what they can do while using age-appropriate technology. Even though it was time consuming to teach myself some of the nuances of Geometer’s Sketchpad, I believe that the opportunities this lesson provides greatly outweigh any inconvenience on my part.