

Lesson 37 Operations on Algebraic Fractions

Products of Fractions

To multiply two fractions together, we multiply their numerators together and then multiply their denominators together.

Product of Fractions

If $b \neq 0$ and $d \neq 0$, then

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

If a common factor occurs in a numerator and a denominator of either fraction, we can divide it out either before or after multiplying. Thus,

$$\frac{3a}{4} \cdot \frac{5}{6a^2} = \frac{\cancel{3} \cdot a}{4} \cdot \frac{5}{\cancel{3} \cdot 2 \cdot \cancel{a} \cdot a} = \frac{5}{8a}$$

or

$$\frac{3a}{4} \cdot \frac{5}{6a^2} = \frac{15a}{24a^2} = \frac{\cancel{3} \cdot 5 \cdot \cancel{a}}{\cancel{3} \cdot 8 \cdot \cancel{a} \cdot a} = \frac{5}{8a}$$

It is usually easier to cancel any common factors before multiplying.

Example 1 Multiply $\frac{2x-4}{3x+6} \cdot \frac{6x+9}{x-2}$

Solution First, we factor each numerator and denominator. Then we divide numerator and denominator by any common factors.

$$\begin{aligned} \frac{2x-4}{3x+6} \cdot \frac{6x+9}{x-2} &= \frac{2(x-2)}{\cancel{3}(x+2)} \cdot \frac{\cancel{3}(2x+3)}{x-2} && \text{Multiply remaining factors of} \\ &= \frac{2(2x+3)}{x+2} \quad \text{or} \quad \frac{4x+6}{x+2} && \text{numerators and of denominators.} \end{aligned}$$

Exercise 1 Find the product $\frac{4y^2-1}{4-y^2} \cdot \frac{y^2-2y}{4y+2}$

To multiply a fraction by a whole number, we write the whole number with 1 as denominator.

$$\frac{2}{3} \cdot 4 = \frac{2}{3} \cdot \frac{4}{1} = \frac{8}{3} \quad \text{Write 4 as } \frac{4}{1}, \text{ then multiply.}$$

The same applies to the product of an algebraic fraction and any nonfractional expression. For example,

$$6x \left(\frac{2}{x^2-x} \right) = \frac{6x}{1} \cdot \frac{2}{x^2-x} = \frac{\cancel{6}x}{1} \cdot \frac{2}{\cancel{x}(x-1)} = \frac{12}{x-1}$$

We summarize the procedure for multiplying algebraic fractions as follows.

To Multiply Algebraic Fractions:

1. Factor each numerator and denominator.
2. If any factor appears in both a numerator and a denominator, divide out that factor.
3. Multiply the remaining factors of the numerator and the remaining factors of the denominator.
4. Reduce the product if necessary.

Quotients of Fractions

To divide one fraction by another, we multiply the first fraction by the **reciprocal** of the second fraction. For example,

$$\frac{m}{2} \div \frac{2p}{3} = \frac{m}{2} \cdot \frac{3}{2p} = \frac{3m}{4p}$$

We express this rule in symbols as follows.

Quotient of Fractions

If $b, c, d \neq 0$, then

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$$

Thus, to divide two algebraic fractions, we take the reciprocal of the divisor and then follow the rules for multiplying algebraic fractions.

Example 2 Divide: $\frac{a-2}{6a^2} \div \frac{4-a^2}{4a^2-2a}$

Solution First, we change the operation to multiplication by taking the reciprocal of the divisor.

$$\frac{a-2}{6a^2} \div \frac{4a^2-2a}{4-a^2}$$

Now we follow the rules for multiplication: First, we factor each numerator and denominator.

$$\frac{a-2}{6a^2} \div \frac{4a^2-2a}{4-a^2} = \frac{a-2}{3 \cdot 2 \cdot a \cdot a} \cdot \frac{2a(2a-1)}{(2-a)(2+a)} \quad \begin{array}{l} \text{Divide out common factors;} \\ \text{Note that } a-2 = -1(2-a). \end{array}$$

$$= \frac{-1(2-a)}{3 \cdot \cancel{2} \cdot \cancel{a} \cdot a} \cdot \frac{\cancel{2}a(2a-1)}{(2-a)(2+a)} \quad \text{Multiply.}$$

$$= \frac{-1(2a-1)}{3a(2+a)} = \frac{1-2a}{3a(2+a)} \quad \text{We often leave the denominators of algebraic fractions in factored form.}$$

RQ1a. The first step in multiplying fractions is to _____.

b. It is usually easier to cancel common factors _____ multiplying.

c. To divide two fractions, we multiply the first fraction by the _____ of the second fraction.

d. We can write a whole number as a fraction with denominator _____.

We summarize the procedure for dividing algebraic fractions as follows.

To Divide Algebraic Fractions:

1. Take the reciprocal of the second fraction and change the operation to multiplication.
2. Follow the rules for multiplication of fractions.

Exercise 2 Divide: $\frac{x^2 - 4y^2}{4y^2} \div \frac{x^2 - 3xy + 2y^2}{3xy}$

Adding Like Fractions

Fractions with the same denominator are called **like** fractions. Thus,

$$\frac{5}{8} \text{ and } \frac{9}{8}, \quad \frac{4}{5x} \text{ and } \frac{3}{5x}, \quad \frac{1}{a-2} \text{ and } \frac{a}{a-2}$$

are like fractions, while

$$\frac{2}{3} \text{ and } \frac{2}{5}, \quad \frac{5}{x+1} \text{ and } \frac{2x}{x-1}$$

are unlike fractions. We can add or subtract like fractions for the same reason that we can add like terms. Just as

$$3x + 4x = 7x$$

we can think of the sum $\frac{3}{5} + \frac{4}{5}$ as

$$3\left(\frac{1}{5}\right) + 4\left(\frac{1}{5}\right) = 7\left(\frac{1}{5}\right)$$

or $\frac{7}{5}$. The denominators of the terms must be the same because they tell us what kind of quantity we are adding. We can add two quantities if they are the same kind.

When we add like fractions, we add their numerators and keep their denominators the same. For example,

$$\frac{10}{3x} + \frac{4}{3x} = \frac{10+4}{3x} = \frac{14}{3x}$$

Add the numerators.
Keep the same denominator.

The same holds true for all algebraic fractions.

Sum or Difference of Like Fractions

If $c \neq 0$, then

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\frac{a}{c} - \frac{b}{c} = \frac{a-b}{c}$$

Example 3 Add.

a. $\frac{2x}{9z^2} + \frac{5x}{9z^2}$

b. $\frac{2x-5}{x+2} + \frac{x+4}{x+2}$

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Solutions a. Because these are like fractions, we add their numerators and keep the same denominator.

$$\frac{2x}{9z^2} + \frac{5x}{9z^2} = \frac{2x + 5x}{9z^2} = \frac{7x}{9z^2}$$

b. We combine the numerators over a single denominator.

$$\begin{aligned}\frac{2x-5}{x+2} + \frac{x+4}{x+2} &= \frac{(2x-5) + (x+4)}{x+2} && \text{Add like terms} \\ &= \frac{3x-1}{x+2} && \text{in the numerator.}\end{aligned}$$

We summarize our procedure for adding or subtracting like fractions as follows.

To Add or Subtract Like Fractions

1. Add or subtract the numerators.
2. Keep the same denominator.
3. Reduce the sum or difference if necessary.

Exercise 3 Add: $\frac{2n}{n-3} + \frac{n+2}{n-3}$

Subtracting Like Fractions

We must be careful when subtracting algebraic fractions: A subtraction sign in front of a fraction applies to the *entire* numerator.

Example 4 Subtract: $\frac{x-3}{x-1} - \frac{3x-5}{x-1}$

Solution We combine the numerators over a single denominator. We use parentheses around $3x-5$ to show that the subtraction applies to the entire numerator.

$$\begin{aligned}\frac{x-3}{x-1} - \frac{3x-5}{x-1} &= \frac{(x-3) - (3x-5)}{x-1} && \text{Remove parentheses;} \\ &= \frac{x-3-3x+5}{x-1} && \text{distribute negative sign.} \\ &= \frac{-2x+2}{x-1} && \text{Combine like terms} \\ & && \text{in the numerator.}\end{aligned}$$

We should always check to see whether the fraction can be reduced. Factor the numerator to find

$$\frac{-2x+2}{x-1} = \frac{-2(x-1)}{x-1} = -2$$

Caution! In Example 4, the subtraction symbol in the numerator $(x-3) - (3x-5)$ applies to *both* terms of $(3x-5)$. That is why

$$(x-3) - (3x-5) = x-3-3x+5$$

RQ2 True or False.

- Like fractions have the same denominator.
- To add like fractions, we add their numerators and add their denominators.
- A negative sign in front of a fraction changes the sign of both numerator and denominator.
- When we subtract fractions, the subtraction symbol applies only to the first term of the numerator.

Exercise 4 Subtract: $\frac{3}{x^2 + 2x + 1} - \frac{2 - x}{x^2 + 2x + 1}$

Unlike Fractions

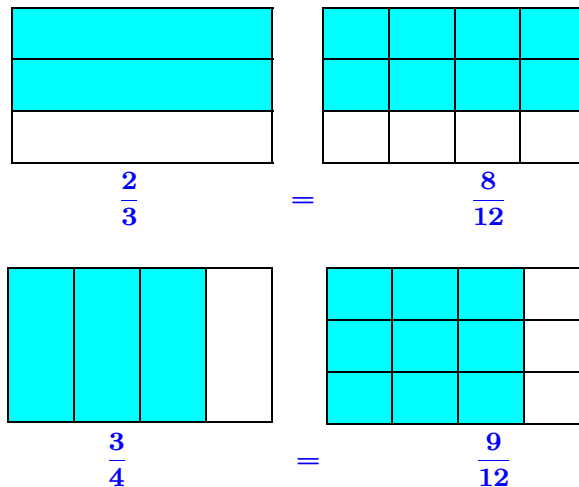
Let's review the steps for adding or subtracting unlike fractions. For example, to compute the sum

$$\frac{2}{3} + \frac{3}{4}$$

we first find the lowest common denominator, or LCD, for the two fractions: The smallest number that is a multiple of both 3 and 4 is 12. Next we use the fundamental principle of fractions to build each fraction to an equivalent one with denominator 12:

$$\begin{aligned} \frac{2}{3} &= \frac{2 \cdot 4}{3 \cdot 4} = \frac{8}{12} \\ \frac{3}{4} &= \frac{3 \cdot 3}{4 \cdot 3} = \frac{9}{12} \end{aligned}$$

There is nothing mysterious about the building process; we are breaking up each fraction into smaller pieces of the same size so that we can add them together.



The new fractions are like fractions, and we can add them by combining their numerators

$$\frac{2}{3} + \frac{3}{4} = \frac{8}{12} + \frac{9}{12} = \frac{17}{12}$$

We add or subtract algebraic fractions in the same way.

To Add or Subtract Algebraic Fractions

1. Find the lowest common denominator (LCD) for the fractions.
2. Build each fraction to an equivalent one with the same denominator.
3. Add or subtract the resulting like fractions: Add or subtract their numerators, and keep the same denominator.
4. Reduce the sum or difference if necessary.

Example 5 Subtract: $\frac{x+2}{6} - \frac{x-1}{15}$

Solution Step 1 We find the LCD: The smallest multiple of both 6 and 15 is 30.

Step 2 We build each fraction to an equivalent one with denominator 30. The building factor for the first fraction is **5**, and for the second fraction the building factor is **2**.

$$\frac{x+2}{6} = \frac{(x+2) \cdot 5}{6 \cdot 5} = \frac{5x+10}{30}$$

$$\frac{x-1}{15} = \frac{(x-1) \cdot 2}{15 \cdot 2} = \frac{2x-2}{30}$$

Step 3 We subtract the resulting like fractions to obtain

$$\begin{aligned} \frac{x+2}{6} - \frac{x-1}{15} &= \frac{5x+10}{30} - \frac{2x-2}{30} && \text{Combine the numerators;} \\ &= \frac{(5x+10) - (2x-2)}{30} && \text{keep the same denominator.} \\ &= \frac{5x+10-2x+2}{30} = \frac{3x+12}{30} && \text{Simplify the numerator.} \end{aligned}$$

Step 4 Finally, we reduce the fraction.

$$\frac{3x+12}{30} = \frac{\cancel{3}(x+4)}{\cancel{3} \cdot 10} = \frac{x+4}{10}$$

Exercise 5 Add: $\frac{5x}{6} + \frac{3-2x}{4}$

RQ3 Complete each statement.

- a. To add unlike fractions, we must first convert them into ____ fractions.
- b. Building a fraction is an application of the _____.
- c. The LCD is the smallest _____ of each denominator.
- d. The last step in adding fractions is to _____.

The Lowest Common Denominator

The **lowest common denominator** for two or more algebraic fractions is the simplest algebraic expression that is a multiple of each denominator. If neither denominator can be factored, then their LCD is just the product of the two expressions.

Example 6 Add: $\frac{6}{x} + \frac{x}{x-2}$

Solution Step 1 The LCD for these fractions is just the product of their denominators, $x(x-2)$.

Step 2 We build each fraction to an equivalent one with denominator $x(x-2)$. The building factor for $\frac{6}{x}$ is $(x-2)$, and the building factor for $\frac{x}{x-2}$ is x . We multiply numerator and denominator of each fraction by its building factor.

$$\begin{aligned}\frac{6}{x} &= \frac{6(x-2)}{x(x-2)} = \frac{6x-12}{x(x-2)} \\ \frac{x}{x-2} &= \frac{x(x)}{(x-2)(x)} = \frac{x^2}{x(x-2)}\end{aligned}$$

Step 3 We combine the resulting like fractions.

$$\begin{aligned}\frac{6}{x} + \frac{x}{x-2} &= \frac{6x-12}{x(x-2)} + \frac{x^2}{x(x-2)} && \text{Combine the numerators;} \\ &= \frac{x^2 + 6x - 12}{x(x-2)} && \text{keep the same denominator.}\end{aligned}$$

Step 4 The numerator of this fraction cannot be factored, so the sum cannot be reduced. ■

Exercise 6 Write as a single fraction in reduced form: $\frac{2}{x+2} - \frac{3}{x-2}$

Finding the Lowest Common Denominator

If the denominators contain any common factors, the LCD is not simply their product. For example, the LCD for

$$\frac{5}{12} + \frac{7}{18}$$

is not $12(18) = 216$. It is true that 216 is a multiple of both 12 and 18, but it is not the smallest one! We can find a smaller common denominator by factoring each denominator.

$$\begin{aligned}12 &= 2 \cdot 2 \cdot 3 && \text{This denominator has the most factors of 2.} \\ 18 &= 2 \cdot 3 \cdot 3 && \text{This denominator has the most factors of 3.}\end{aligned}$$

To find a number that both 12 and 18 divide into evenly, we need only enough factors to cover each of them. In this case two 2's and two 3's are sufficient, so the LCD is

$$\text{LCD} = 2 \cdot 2 \cdot 3 \cdot 3 = 36$$

You can check that both 12 and 18 divide evenly into 36.

In general, we can find the LCD in the following way.

To Find the LCD:

1. Factor each denominator completely.
2. Include each different factor in the LCD as many times as it occurs in any *one* of the given denominators.

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Example 7 Find the LCD for the fractions $\frac{2x}{x^2 - 1}$ and $\frac{x + 3}{x^2 + x}$

Solution We factor the denominators of each of the given fractions.

$$x^2 - 1 = (x - 1)(x + 1) \quad \text{and} \quad x^2 + x = x(x + 1)$$

The factor $(x - 1)$ occurs once in the first denominator, the factor x occurs once in the second denominator, and the factor $(x + 1)$ occurs once in each denominator. Therefore we include in our LCD one copy of each of these factors. The LCD is $x(x + 1)(x - 1)$. ■

Caution! In Example 7, we do not include two factors of $(x + 1)$ in the LCD. We need only one factor of $(x + 1)$, because $(x + 1)$ occurs only once in each denominator. You can check that each original denominator divides evenly into our LCD, $x(x + 1)(x - 1)$. ■

Exercise 7 Find the LCD for the fractions $\frac{1}{2x^3(x - 1)^2}$ and $\frac{3}{4x^2 - 4x}$

Adding Unlike Fractions

After finding the LCD, we **build** each fraction to an equivalent one with the LCD as its denominator. The new fractions will be like fractions, so we can combine their numerators.

Building a fraction is an application of the fundamental principle of fractions,

$$\frac{a}{b} = \frac{a \cdot c}{b \cdot c}, \quad \text{if } c \neq 0$$

It is the opposite of reducing a fraction, because we multiply, rather than divide, the numerator and denominator by an appropriate factor. To find the **building factor**, we compare the factors of the original denominator with those of the desired common denominator.

Example 8 Add: $\frac{x - 4}{x^2 - 2x} + \frac{4}{x^2 - 4}$

Solution Step 1 Find the LCD. We factor each denominator completely.

$$\begin{aligned}x^2 - 2x &= x(x - 2) \\x^2 - 4 &= (x - 2)(x + 2)\end{aligned}$$

The LCD is $x(x - 2)(x + 2)$.

Step 2 We build each fraction to an equivalent one with the LCD as its denominator.

The first fraction, $\frac{x - 4}{x(x - 2)}$, needs a factor of $(x + 2)$ in its denominator, so $(x + 2)$ is its building factor. We multiply the numerator and denominator of the first fraction by $(x + 2)$:

$$\frac{x - 4}{x(x - 2)} = \frac{(x - 4)(x + 2)}{x(x - 2)(x + 2)} = \frac{x^2 - 2x - 8}{x(x - 2)(x + 2)}$$

The building factor for the second fraction, $\frac{4}{(x - 2)(x + 2)}$, is x .

$$\frac{4}{(x - 2)(x + 2)} = \frac{4x}{(x - 2)(x + 2)x}$$

Step 3 The fractions are now like fractions, so we add them by combining their numerators.

$$\begin{aligned}\frac{x-4}{x^2-2x} + \frac{4}{x^2-4} &= \frac{x^2-2x-8}{x(x-2)(x+2)} + \frac{4x}{x(x-2)(x+2)} \\ &= \frac{x^2+2x-8}{x(x-2)(x+2)}\end{aligned}$$

Step 4 Finally, we reduce the fraction.

$$\frac{x^2+2x-8}{x(x-2)(x+2)} = \frac{(x+4)(x-2)}{x(x-2)(x+2)} = \frac{x+4}{x(x+2)}$$

Caution! Do not reduce the built-up fractions in Step 3 -- you will just get back to the original problem. When adding fractions, we have to make the fractions "harder" before we can combine them. Don't reduce until the last step of the problem.

RQ4 True or False.

- We can always find the LCD by multiplying together the denominators of the fractions.
- If each denominator includes one factor of $x - 3$, we include two factors of $x - 3$ in the LCD.
- We find the building factor for each fraction by comparing its denominator to the LCD.
- When adding fractions, it is best to reduce the fractions at each step.

Exercise 8 Add $\frac{2}{x^2-x-2} + \frac{2}{x^2+2x+1}$

Applications

Recall that the formulas for rational functions are algebraic fractions. It is often useful to simplify the formula for a function before using it.

Example 9 When estimating their travel time, pilots must take into account the prevailing winds. A tail wind adds to the plane's ground speed, while a head wind decreases the ground speed. Skyhigh Airlines is setting up a shuttle service from Dallas to Phoenix, a distance of 800 miles.

- Express the time needed for a one-way trip, without wind, as a function of the speed of the plane.
- Suppose there is a prevailing wind of 30 miles per hour blowing from the west. Write expressions for the flying time from Dallas to Phoenix and from Phoenix to Dallas.
- Write an expression for the round trip flying time, excluding stops, with a 30-mile-per-hour wind from the west, as a function of the plane's speed. Simplify your expression.

Solutions a. Recall that $\text{time} = \frac{\text{distance}}{\text{rate}}$. If we let r represent the speed of the plane in still air, then the time required for a one-way trip is

$$f(r) = \frac{800}{r}$$

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b. On the trip from Dallas to Phoenix the plane encounters a head wind of 30 miles per hour, so its actual ground speed is $r - 30$. On the return trip the plane enjoys a tail wind of 30 miles per hour, so its actual ground speed is $r + 30$. Therefore, the flying times are

$$\begin{array}{l} \text{Dallas to Phoenix: } \frac{800}{r - 30} \\ \text{Phoenix to Dallas: } \frac{800}{r + 30} \end{array}$$

c. The round-trip flying time from Dallas to Phoenix and back is

$$F(r) = \frac{800}{r - 30} + \frac{800}{r + 30}$$

The LCD for these fractions is $(r - 30)(r + 30)$. Thus,

$$\begin{aligned} \frac{800}{r - 30} + \frac{800}{r + 30} &= \frac{800(\mathbf{r + 30})}{(r - 30)(\mathbf{r + 30})} + \frac{800(\mathbf{r - 30})}{(r + 30)(\mathbf{r - 30})} \\ &= \frac{(800r + 24000) + (800r - 24000)}{(r + 30)(r - 30)} = \frac{1600r}{r^2 - 900} \quad \blacksquare \end{aligned}$$

Exercise 9 A rowing team can maintain a speed of 15 miles per hour in still water. The team's daily training session includes a 5-mile run up the Red Cedar River and the return downstream.

- Express the team's time on the upstream leg as a function of the speed of the current.
- Write a function for the team's time on the downstream leg.
- Write and simplify an expression for the total time for the training run as a function of the current's speed.

Answers to Exercises

- $\frac{-y(2y - 1)}{2(y + 2)}$
- $\frac{3x(x + 2y)}{4y(x - y)}$
- $\frac{3n + 2}{n - 3}$
- $\frac{1}{x + 1}$
- $\frac{4x + 9}{12}$
- $\frac{-x - 10}{x^2 - 4}$
- $4x^3(x - 2)^2$
- $\frac{2(2x - 1)}{(x + 1)^2(x - 2)}$
- a. $\frac{5}{15 - c}$
- b. $\frac{5}{15 + c}$
- c. $\frac{150}{225 - c^2}$

Now go to WeBWork and do the Reading Assignment for this Lesson.

Homework 37

1. Multiply.

a. $\frac{-24a}{5b} \cdot \frac{15ab}{14}$

b. $\frac{2b}{3} \cdot \frac{4}{b + 1}$

c. $\frac{-v}{v + 1} \cdot \frac{v}{v - 1}$

2. Write each product as a fraction.

a. $\frac{3}{4}(a - b)$

b. $4a^2b \left(\frac{2a + b}{6ab} \right)$

c. $15x^2y \cdot \frac{3}{35xy^2}$

□ Multiply.

3. $\frac{4V}{D} \cdot \frac{LR}{DV}$

$\frac{1}{2}MR^2 \cdot \frac{a}{R}$

A 5. $\frac{2L}{c} \left(1 + \frac{V^2}{c^2}\right)$

6. $\frac{4\pi}{c^2} \left(\frac{c}{4\pi}H + cM\right)$

7. $\frac{q}{8\pi} \left(\frac{3}{R} - \frac{a^2}{R^3}\right)$

8. $\frac{a^2}{d} \left(1 - \frac{at}{2d}\right)$

□ Multiply.

A 9. $\frac{-2}{t^2} \left(4t^3 - \frac{t^2}{8} + \frac{3t}{2}\right)$

10. $\frac{4}{3}v \left(\frac{2}{3}v - \frac{6}{v^2} - \frac{3}{4v}\right)$

□ Divide.

11. $\frac{12a^4}{21b^4} \div \frac{24ab^2}{27b^3}$

12. $6x^2y \div \frac{3x}{-2y^3}$

13. $\frac{a^2 - ab}{ab} \div \frac{2a - 2b}{3ab}$

14. $\frac{x^2 + 3x}{2y} \div 3x$

15. $\frac{3xy + x}{y^2 - y} \div \frac{3y + 1}{xy}$

16. $\frac{6a^2 - 12a}{3a + 9} \div \frac{8a^2 - 4a^3}{15 + 5a}$

□ Write an algebraic expression for each phrase, and simplify.

17 a. One-half of x .

b. x divided by one-half.

c. One-half divided by x .

18 a. Two-thirds of y .

b. y divided by two-thirds

c. Two-thirds divided by y .

A 19 a. The reciprocal of $a + b$.

b. Three-fourths of the reciprocal of $a + b$.

c. The reciprocal of $a + b$ divided by three-fourths.

20. Simplify.

a. $\left(\frac{1}{c} \cdot \frac{1}{5}\right) \div \frac{2}{5}$

b. $\frac{1}{c} \cdot \left(\frac{1}{5} \div \frac{2}{5}\right)$

c. $\frac{1}{c} \div \left(\frac{1}{5} \div \frac{2}{5}\right)$

d. $\left(\frac{1}{c} \div \frac{1}{5}\right) \div \frac{2}{5}$

□ Add or subtract.

21. $\frac{x - 2y}{3x} + \frac{x + 3y}{3x}$

22. $\frac{m^2 + 1}{m - 1} - \frac{2m}{m - 1}$

23. $\frac{z^2 - 2}{z + 2} - \frac{z + 4}{2 + z}$

24. $\frac{b + 1}{b^2 - 2b + 1} - \frac{5 - 3b}{b^2 - 2b + 1}$

□ a. Find the lowest common denominator for the fractions.

b. Add or subtract.

25. $\frac{5}{2x} + \frac{3}{4x^2}$

26. $\frac{2z - 3}{8z} + \frac{z - 2}{6z}$

27. $\frac{3}{2a - b} + \frac{1}{8a - 4b}$

28. $\frac{3}{x} - \frac{2}{x + 1}$

29. $h - \frac{3}{h + 2}$

30. $\frac{3}{n + 3} + \frac{4n}{n - 3}$

A 31. $\frac{2}{x} + \frac{x}{x - 2} - 2$

32. $\frac{1}{x} + \frac{x}{y} + \frac{x}{z}$

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□ Add or subtract.

33. $\frac{-H}{RT} + \frac{S}{R}$

34. $\frac{q}{4\pi r} + \frac{qa}{2\pi r^2}$

35. $\frac{1}{LC} - \left(\frac{R}{2L}\right)^2$

36. $\frac{L}{c-V} + \frac{L}{c+V}$

37. $\frac{2r^2}{a^2} + \frac{2r}{a} + 1$

38. $\frac{q}{r-a} - \frac{2q}{r} + \frac{q}{r+a}$

□ Write algebraic fractions in simplest form for each problem.

39. The dimensions of a rectangular rug are $\frac{12}{x}$ feet and $\frac{12}{x-2}$ feet.

- Write and simplify an expression for the area of the rug.
- Write and simplify an expression for the perimeter of the rug.

A 40. Colonial Airline has a commuter flight between Richmond and Washington, a distance of 100 miles. The plane flies at x miles per hour in still air. Today there is a steady wind from the north at 10 miles per hour.

- How long will the flight from Richmond to Washington take?
- How long will the flight from Washington to Richmond take?
- How long will a round trip take?
- Evaluate your fractions in parts (a)-(c) for $x = 150$.

41. Two pilots for the Flying Express parcel service receive packages simultaneously. Orville leaves Boston for Chicago at the same time Wilbur leaves Chicago for Boston. Each selects an air speed of 400 miles per hour for the 900-mile trip. The prevailing winds blow from east to west.

- Express Orville's flying time as a function of the wind speed.
- Write a function for Wilbur's flying time.
- Who reaches his destination first? By how much time (in terms of wind speed)?

42. Francine's cocker spaniel eats a large bag of dog food in d days, and Delbert's sheep dog takes 5 days less to eat the same size bag.

- What fraction of a bag of dog food does Francine's cocker spaniel eat in one day?
- What fraction of a bag of dog food does Delbert's sheep dog eat in one day?
- If Delbert and Francine get married, what fraction of a bag of dog food will their dogs eat in one day?
- If $d = 25$, how soon will Delbert and Francine have to buy more dog food?

**Now go to WeBWork and do the Skills
Practice for this Homework.**