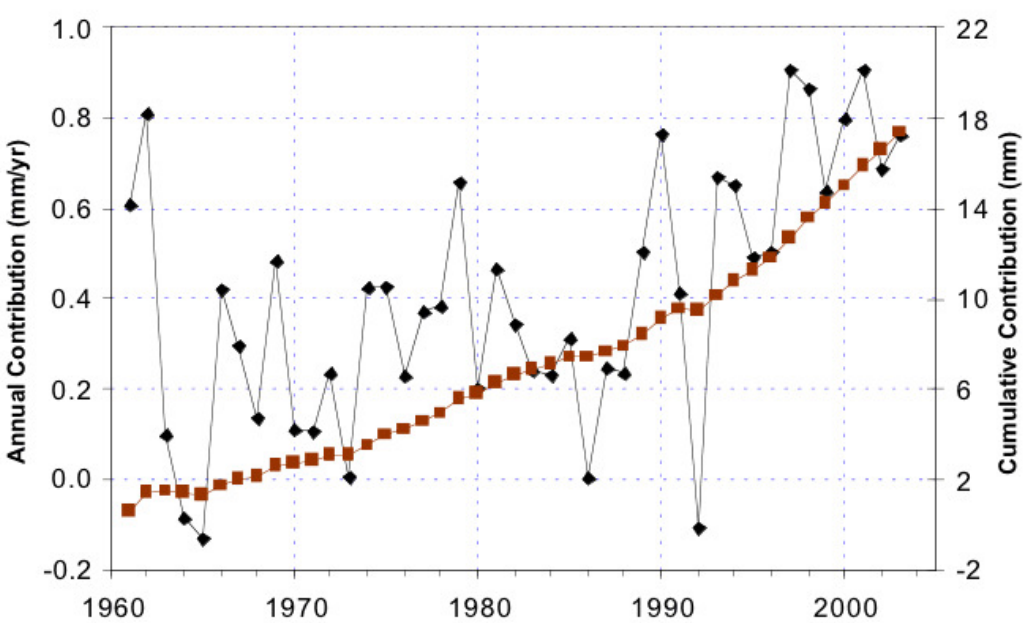
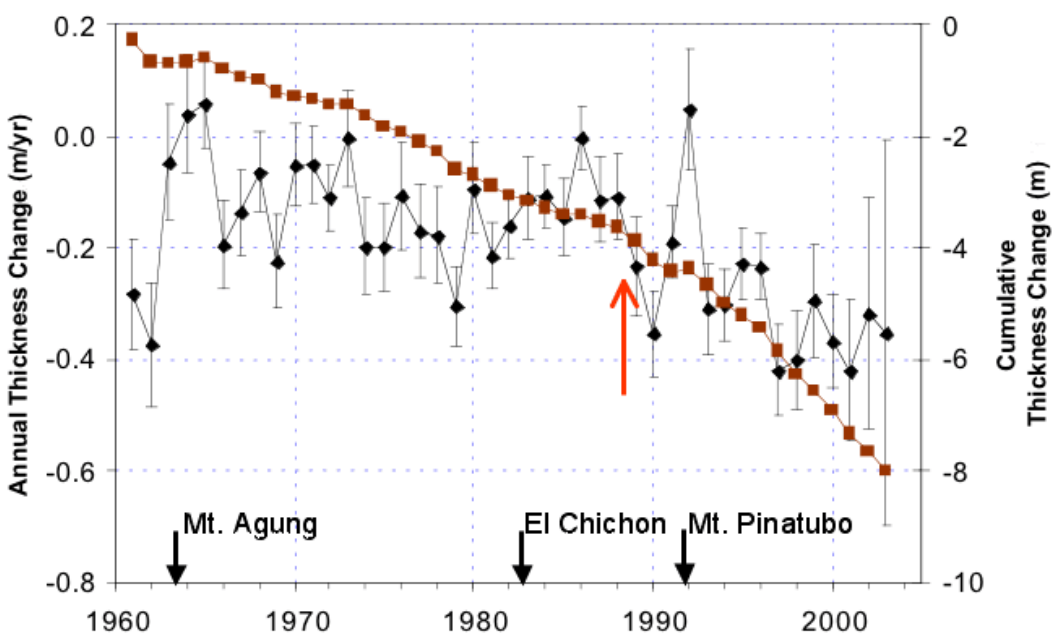


Lesson 4 Slope

Glacier Melt

Glaciers around the world are retreating at accelerating rates, and many lower-latitude mountain glaciers may disappear entirely within decades. The melt water from these smaller glaciers contributed as much as 40% to the total rise in sea level over the 1990's. By measuring the change in ice and snow height at fixed points and multiplying by the surface area of the glacier, scientists calculate the total volume of water lost from land-based glaciers. Dividing this volume by the surface area of the world's oceans gives the resulting change in sea level.

The graphs below show the change in thickness of the land-based glaciers over the past 5 years, and the rise in sea level attributed to their melting.



Rate of Change

A **rate** is a type of ratio that compares two quantities with different units.

RQ1 A ratio that compares two quantities with different units is called a _____.

In the example below, notice that each rate has units of the form something/something else, which we read as "something per something else."

Example 1 Caryn bought 7 used paperback novels for a total \$2.45. Use a ratio to calculate the price per book.

$$\frac{\text{total cost in dollars}}{\text{number of books}} = \frac{2.45 \text{ dollars}}{7 \text{ books}} = 0.35 \text{ dollars/book}$$

The novels were priced at a rate of \$0.35 per book. ■

Exercise 1 Delbert biked 34 miles in 5 hours. Use a ratio to calculate his average speed.

$$\frac{\text{distance in miles}}{\text{time in hours}} =$$

Delbert biked at a rate of _____.

A **rate of change** is a special kind of ratio that compares the *change* in two quantities or variables.

Example 2 Gregor is driving across Montana. At 1 pm his trip odometer read 189 miles, and at 4 pm it read 360 miles. Calculate Gregor's average speed as a rate of change.

Solution We have two variables, time, t , and distance, d , and the following data points:

t	d
1	189
4	360

Gregor's speed is the ratio of the distance he traveled to the time it took. The distance he traveled is the *change* in his odometer reading (from 189 miles to 360 miles), and the time it took is the *change* in the clock reading (from 1 pm to 4 pm). The units of this ratio are miles per hour.

In mathematics, we use the symbol Δ (delta) for *change in*. Thus

$$\text{distance traveled} = \Delta d = 360 - 189 = 171 \text{ miles}$$

$$\text{time elapsed} = \Delta t = 4 - 1 = 3 \text{ hours}$$

Gregor's average speed is the ratio $\frac{\text{distance traveled}}{\text{time elapsed}} = \frac{\Delta d}{\Delta t}$, so

$$\text{speed} = \frac{\Delta d}{\Delta t} = \frac{171 \text{ miles}}{3 \text{ hours}} = 57 \text{ miles/hour} \quad \blacksquare$$

RQ2 What does the symbol Δ stand for?

- a. A triangle b. Speed
c. Change in d. Coordinate

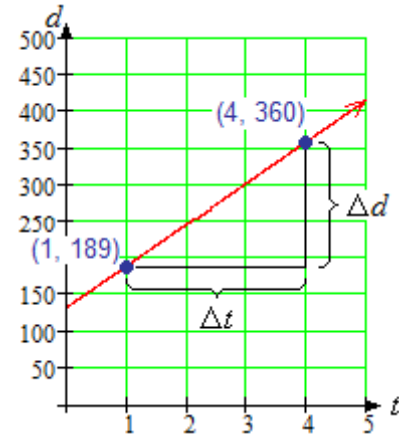
Exercise 2 Nelson is a long-distance truck driver. On a recent trip through the Midwest, he noted these odometer readings:

4 am	127 miles
10 am	421 miles

What was Nelson's average speed?

How do we see the rate of change on a graph? Let's consider the example above. The graph shows Gregor's distance, d , at time t . We plot the two data points, $(1, 189)$ and $(4, 360)$, and draw a straight line joining them. We can illustrate Δd and Δt by vertical and horizontal line segments, as shown on the graph.

The rate of change of distance with respect to time, or speed, is the ratio of Δd to Δt . It measures how much d changes for each unit increase in t , or how far Gregor travels in each hour. This quantity, the ratio $\frac{\Delta d}{\Delta t}$, is the slope of the line.



RQ3 What feature of a graph illustrates rate of change?

- a. The y -axis
- b. Slope
- c. Scales on the axes
- d. The coordinates of a point

Review of Slope

The **slope** of a line is a rate of change that measures the steepness of the line.

The slope tells us how much the y -coordinate changes for each unit of increase in the x -coordinate, as we move from one point to another along the line.

RQ4 The slope of a line tells us how much the y -coordinate changes _____.

- a. From one end of the graph to the other
- b. As we move up the y -axis
- c. From one point to the next point
- d. For each unit of increase in the x -coordinate

We describe slope as follows.

Slope of a Line

$$\text{slope} = \frac{\text{change in } y\text{-coordinate}}{\text{change in } x\text{-coordinate}}$$

or, in symbols,

$$m = \frac{\Delta y}{\Delta x}$$

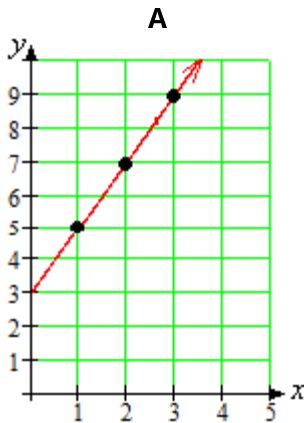
4 Unit 1 Linear Models

In the definition of slope,

$$\Delta x \text{ is } \begin{cases} \text{positive if } x \text{ increases (we move to the right)} \\ \text{negative if } x \text{ decreases (we move to the left)} \end{cases}$$

$$\Delta y \text{ is } \begin{cases} \text{positive if } y \text{ increases (we move up)} \\ \text{negative if } y \text{ decreases (we move down)} \end{cases}$$

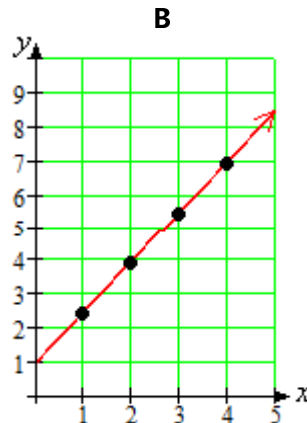
How does slope measure the steepness of a line? Study the three examples below:



For each 1 unit increase in x :

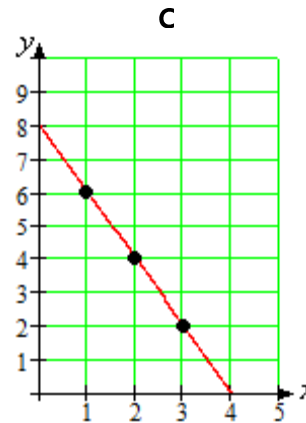
y increases by 2 units

$$m = \frac{\Delta y}{\Delta x} = \frac{2}{1} = 2$$



y increases by $\frac{3}{2}$ units

$$m = \frac{\Delta y}{\Delta x} = \frac{3}{2}$$



y decreases by 2 units

$$m = \frac{\Delta y}{\Delta x} = \frac{-2}{1} = -2$$

From the examples above, we can make the following observations:

1. For positive slopes, the larger the value of m , the more the y -value increases for each unit increase in x , and the more we climb *up* as our location changes from left to right. (So graph A is steeper than graph B.)
2. If y *decreases* as we move from left to right, then Δy is negative when Δx is positive, so their ratio (the slope) is negative. (See graph C.)

Example 3a. Compute the slope of the line.

b. Illustrate the slope on the graph by drawing a vertical segment of length Δy and a horizontal segment of length Δx .

c. If $\Delta x = 1$, what is the length of the vertical segment?

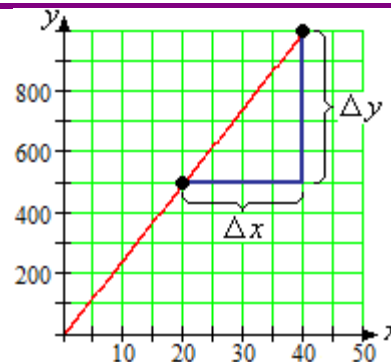
Solutions a. The points (20, 500) and (40, 1000) lie on the graph. As we move from the first point to the second point, x increases by 20

units, so $\Delta x = 20$, and y increases by 500 units, so $\Delta y = 500$. Thus,

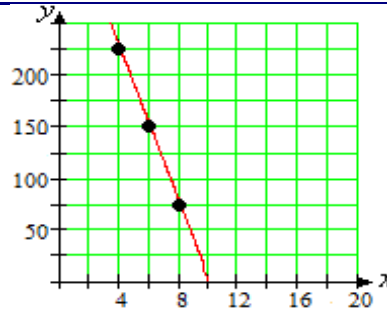
$$\frac{\Delta y}{\Delta x} = \frac{500}{20} = 25$$

b. The segments are shown on the graph.

c. The slope is 25, which means that y increases 25 units for each 1-unit increase in x . So, if $\Delta x = 1$, then $\Delta y = 25$.



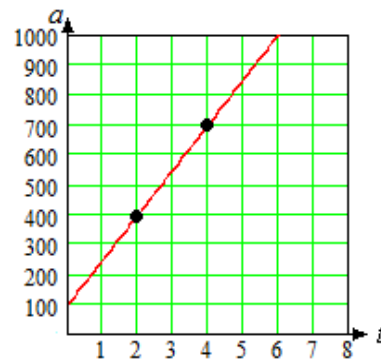
- Exercise 3a.** Compute the slope of the line.
b. Illustrate the slope on the graph by drawing a vertical segment of length Δy and a horizontal segment of length Δx .
c. If $\Delta x = 1$, what is the length of the vertical segment?



Interpreting Slope as a Rate

The slope of a line measures the rate of change of y with respect to x .

- Example 4** The graph shows the altitude a , in feet, of a skier t minutes after boarding the ski lift.
a. Choose two points on the graph and compute the slope, including units.
b. Explain what the slope measures in this problem.
c. Write a linear model for a in terms of t .



Solutions a. We choose the points $(2, 400)$ and $(4, 700)$, as shown on the graph. Then

$$m = \frac{\Delta a}{\Delta t} = \frac{300 \text{ feet}}{2 \text{ minutes}} = 150 \text{ feet/minute}$$

- b.** The slope gives the rate of change of altitude with respect to time. The skier rises at a rate of 150 feet per minute.
c. A linear model has the form

$$y = (\text{starting value}) + \text{rate} \times t$$

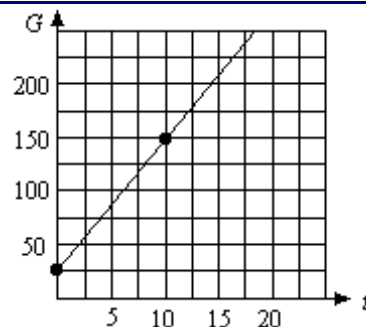
From the graph, we see that at $t = 0$ the skier's altitude was $a = 100$, and we calculated the rate of change of altitude as 150 feet per minute. Substituting these values into the formula, we find

$$a = 100 + 150t$$

Slope as a Rate of Change

- The slope of a line measures the **rate of change** of y with respect to x .
- The units of Δy and Δx can help us interpret the slope as a rate.

- Exercise 4** The graph shows the amount of garbage G (in tons) that has been deposited at a dump site t years after new regulations go into effect.
a. Choose two points on the graph and compute the slope, including units.
b. Explain what the slope measures in this problem.
c. Choose two different points and compute the slope again. Do you get the same value as before?

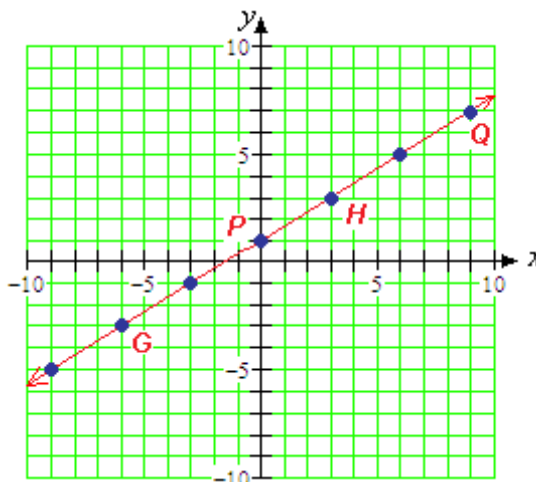


Lines Have Constant Slope

The slope of a line is constant: no matter which two points you pick to compute the slope, you will always get the same value.

For the line shown in the figure, try computing the slope using the points P and Q , and then using the points G and H . In each case, you should find that the slope is $\frac{2}{3}$.

Here is another way to look at slopes. If we start at any point on the line shown and



move 9 units to the right, what value of Δy will bring us back to the line? We can use the slope formula with $\Delta x = 9$.

$$m = \frac{\Delta y}{\Delta x} \quad \text{Substitute the known values.}$$

$$\frac{2}{3} = \frac{\Delta y}{9} \quad \text{Solve for } \Delta y.$$

$$\Delta y = 9 \left(\frac{2}{3} \right) = 6$$

You can use the graph to check this result for yourself; try starting at the point $(-6, -3)$.

The fact that lines have constant slope has two important consequences. First, because m is constant for a given line, we can use the formula $m = \frac{\Delta y}{\Delta x}$ to find Δy when we know Δx , or to find Δx when we know Δy .

RQ5 What formula can we use to find Δy when we know Δx , or to find Δx when we know Δy ?

- | | |
|-----------------------------|-------------------------------|
| a. The equation of the line | b. The general linear formula |
| c. The slope formula | d. $D = RT$ |

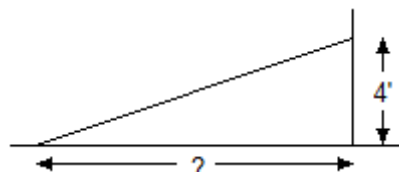
Example 5 A wheelchair ramp can have a slope of no more than 24%, or 0.24. What horizontal distance is needed if the ramp must climb an elevation of 4 feet?

Solution We first draw a sketch of the wheelchair ramp and label Δx and Δy . We are given that $\Delta y = 4$ feet, and we are looking for Δx . Substitute the known values into the slope formula, and solve for Δx .

$$0.24 = \frac{4}{\Delta x} \quad \text{Multiply both sides by } \Delta x.$$

$$0.24\Delta x = 4 \quad \text{Divide both sides by } 0.24.$$

$$\Delta x = \frac{4}{0.24} = 16.\bar{6}$$



The wheelchair ramp must have a horizontal length of $16\frac{2}{3}$ feet, or 16 feet 8 inches. ■

Exercise 5 A wheelchair ramp can have a slope of no more than 24%, or 0.24. What height can the wheelchair ramp climb over a horizontal distance of 10 feet?

Hint: Do we know the value of Δy or of Δx ?

Here is a second consequence of the fact that lines have constant slope: We can tell whether a collection of data points lies on a straight line by computing slopes. If the slope between any two data points is the same, the points lie on a straight line.

RQ6 How can we tell whether a collection of data points lies on a straight line?

- Plot them and look at the graph
- Calculate the slopes between points
- Find an equation for the line
- It depends on the scales on the axes

Example 6 Could this table represent a linear equation? Explain why or why not.

x	-6	-3	0	3	6
y	20	18	16	14	12

Solution We compute the slope between each consecutive pair of points. In each case

$$m = \frac{\Delta y}{\Delta x} = \frac{-2}{3}$$

Because the slope is the same for *all* pairs of points, the table could be linear. ■

Exercise 6 Could this table represent a linear equation? Explain why or why not.

t	5	10	15	20	25
P	0	3	6	12	24

Hint: Calculate the slopes between points.

Answers to Exercises

- 6.8 miles per hour
- 49 miles per hour
- a. -37.5 b. 37.5
- 12.5 tons per year is the rate at which garbage is accumulating.
- 2.4 ft
- No, the slope between points is not constant.

Now go to WeBWork and do the Reading Assignment for this Lesson.

Homework 4

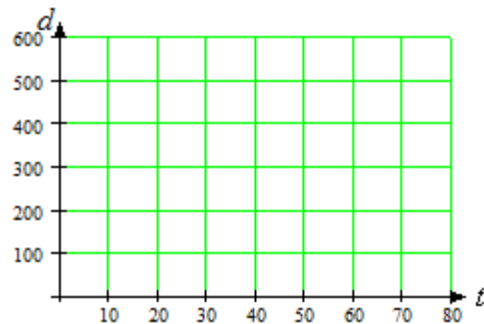
1. Ariel, Beryl, and Carol are training for a kayaking race. The girls practice on a quiet stretch of river starting 200 yards from the dock. There are markers placed at 100-yard intervals along the shore, and their teammates are stationed at these markers to time their progress. Yesterday they recorded the following times, in seconds, for each girl:

Marker (yards)	Time (seconds)		
	Ariel	Beryl	Carol
200	0	0	0
300	20	25	15
400	40	50	30
500	60	75	45

- a. For each girl, make a graph showing time on the horizontal axis and distance from the dock on the vertical axis. Fill in a table and plot the points.

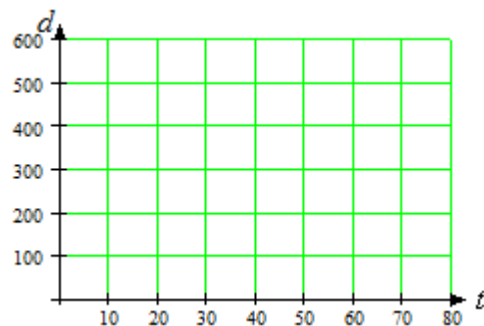
Ariel:

t	d
0	200
20	300
40	400
60	500



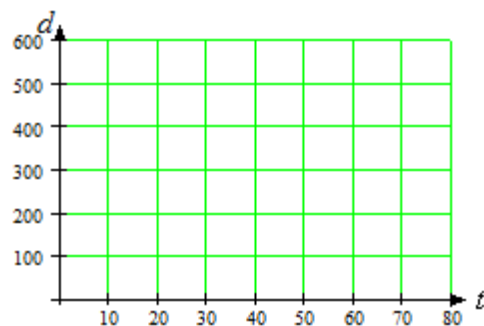
Beryl:

t	d



Carol:

t	d



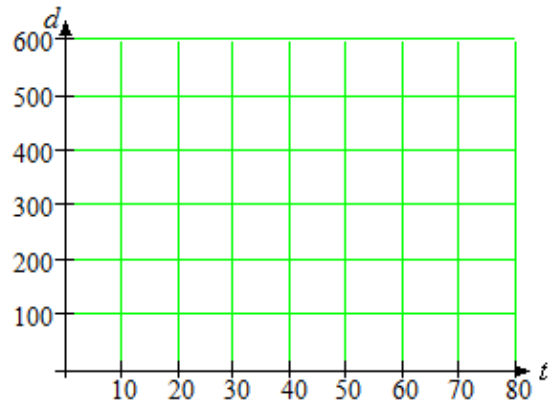
- b. For each girl, choose two data points and compute her average speed. Don't forget the units!

$$\begin{aligned} \text{Ariel's speed} &= \frac{\Delta d}{\Delta t} = \\ \text{Beryl's speed} &= \frac{\Delta d}{\Delta t} = \\ \text{Carol's speed} &= \frac{\Delta d}{\Delta t} = \end{aligned}$$

Illustrate each girl's speed on her distance graph.

- c. Which girl has the fastest average speed?

Plot all three graphs on the same grid, and compare. How can the different speeds be seen from the graphs?



- A** 2. The table gives the radius and circumference of various circles, rounded to three decimal places.
- a. If we plot the data, will the points lie on a straight line? Why or why not?

r	C
4	25.133
6	37.699
10	62.832
15	94.248

- b. What familiar number does the slope turn out to be? (**Hint:** Recall a formula from geometry.) What does the slope tell us about circles?

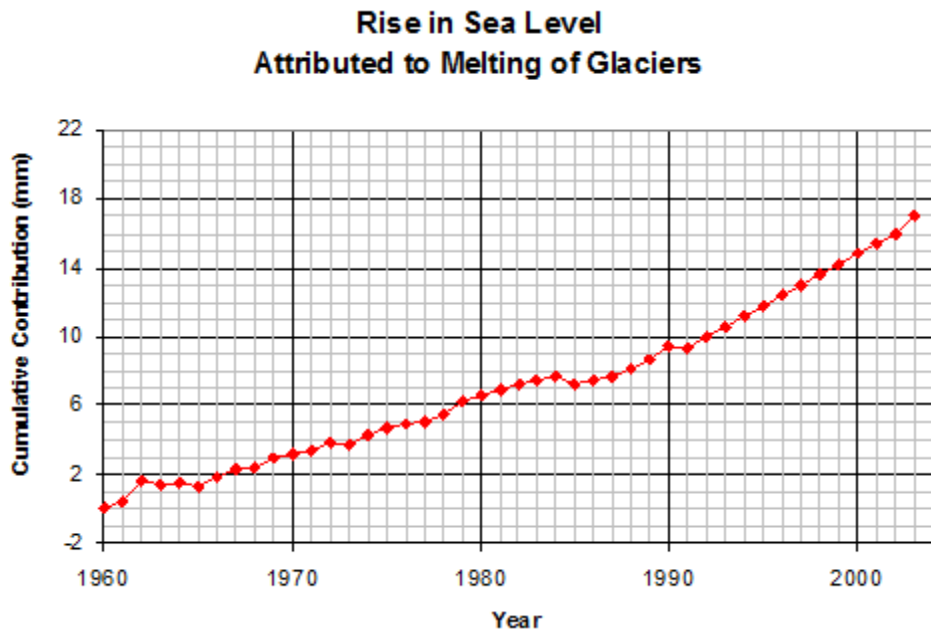
3. A spring is suspended from the ceiling. The table shows the length of the spring, in centimeters, as it is stretched by hanging various weights from it.

Weight, kg	3	4	8	10	12	15	22
Length, cm	25.76	25.88	26.36	26.6	26.84	27.2	28.04

- a. If you plot the data, will the points lie on a straight line? Why or why not?
- b. Interpret the slope as a rate of change. Include units in your answer.
4. Residential staircases are usually built with a slope of 70%. If the vertical distance between the first floor and the second floor is 10 feet, how much horizontal space does the staircase require?

10 Unit 1 Linear Models

5. The graph shows the rise in sea level attributed to the melting of land-based glaciers from 1960 to 2003.



- a. The graph appears to be almost linear from 1992 to 2002. Read the graph to complete the table, then compute the slope of the graph over that time interval, including units. What does the slope mean in this situation?

Year	Sea Level
1992	
2002	

- b. What was the total change in sea level from land-based glaciers over the time period from 1960 to 2003?
- c. Calculate the average rate of change of sea level from land-based glaciers from 1960 to 2003.
- d. From 1960 to 2003, the land-based glaciers decreased in thickness by about 8 meters (or 0.008 km). The total area of those glaciers is 785,000 square kilometers. Calculate the total volume of water released by melting. (**Hint:** $\text{Volume} = \text{area} \times \text{thickness}$)
- e. The surface area of the world's oceans is 361.6 million square kilometers. When the meltwater from the land-based glaciers (that's the volume you calculated in part (d)) enters the oceans, how much will the sea level rise, in kilometers? Convert your answer to millimeters, and check your answer against your answer to part (b).

**Now go to WeBWork and do the Skills
Practice for this Homework.**