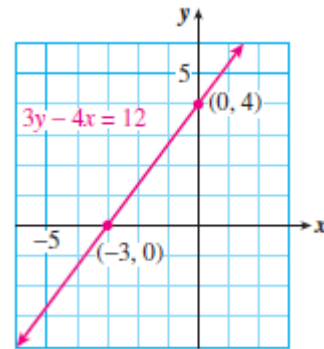


# Lesson 2 Intercepts

## Intercepts of a Graph

The points at which a graph crosses the axes are called the **intercepts** of the graph.

The  $x$ -intercept of the graph shown at right is  $(-3, 0)$ , and its  $y$ -intercept is  $(0, 4)$ .



**RQ1** What are the intercepts of a graph?

- The variables displayed on the axes.
- Points where the graphs intersect.
- The highest and lowest points.
- Points where the graph intersects the axes.

The coordinates of the intercepts are easy to find.

### Intercepts of a Graph

The points where a graph crosses the axes are called the **intercepts** of the graph.

- To find the  $x$ -intercept, we set  $y = 0$  and solve for  $x$ .
- To find the  $y$ -intercept, we set  $x = 0$  and solve for  $y$ .

**Example 1** In Lesson 1, we graphed an equation,  $g = 20 - \frac{1}{12}d$ , for the amount of gasoline,  $g$ , left in Leon's tank after he has driven for  $d$  miles. Find the intercepts of the graph.

**Solution** To find the  $d$ -intercept, we set  $g = 0$  and solve for  $d$ .

$$\begin{aligned} 0 &= 20 - \frac{1}{12}d && \text{Add } \frac{1}{12}d \text{ to both sides.} \\ \frac{1}{12}d &= 20 && \text{Multiply both sides by 12.} \\ d &= 240 \end{aligned}$$

The  $d$ -intercept is  $(240, 0)$ . To find the  $g$ -intercept, we set  $d = 0$  and solve for  $g$ .

$$g = 20 - \frac{1}{12}(0) = 20$$

The  $g$ -intercept is  $(0, 20)$ .

**Exercise 1** Find the intercepts of the graph of  $y = -9 - \frac{3}{2}x$ .

## Meaning of the Intercepts

The intercepts of a graph give us information about the situation it models.

**Example 2** What do the intercepts of the graph in Example 1 tell us about the problem situation?

**Solution** The  $d$ -intercept tells us that when  $d = 240$ ,  $g = 0$ , or that when Leon has traveled 240 miles, he has 0 gallons of gasoline left; the fuel tank is empty. The  $g$ -intercept tells us that when  $d = 0$ ,  $g = 20$ , or that when Leon has traveled 0 miles, he has 20 gallons of gasoline left. The fuel tank holds 20 gallons when full. ■

**Exercise 2** The gas tank in Rosa's Toyota Prius holds 11 gallons, and she gets 48 miles to the gallon.

a. Write an equation for the amount of gasoline,  $g$ , left in the tank after Rosa has driven for  $d$  miles.

b. Find the intercepts of the graph. What do they tell us about the problem situation?

**Hint:** Rewrite the sentence with mathematical symbols:

$$(\text{gasoline left}) = (\text{gallons in full tank}) - (\text{mileage rate}) \times (\text{miles driven})$$

## General Form for a Linear Equation

The graphs of the equations we have seen so far are all portions of straight lines. For this reason such equations are called **linear equations**.

The order of the terms in the equation does not matter. For example, the equation in Example 1,

$$C = 5 + 3t, \quad \text{can be written equivalently as} \quad -3t + C = 5$$

and the equation in Example 3,

$$g = 20 - \frac{1}{12}d, \quad \text{can be written as} \quad \frac{1}{12}d + g = 20$$

This form of a linear equation,  $Ax + By = C$ , is called the **general form**.

### General Form for a Linear Equation

The **general form** for a linear equation is

$$Ax + By = C \quad (A \text{ and } B \text{ cannot both be } 0)$$

**RQ2** What is the general form of a linear equation?

a.  $y = mx + b$

b.  $Ax + By = C$

c. Any equation whose graph is a straight line.

d. Set  $x = 0$  and solve for  $y$ .

Some linear models are easier to use when they are written in the general form.

**Example 3** The manager at Albert's Appliances has \$3000 to spend on advertising for the next fiscal quarter. A 30-second spot on television costs \$150 per broadcast, and a 30-second radio ad costs \$50.

a. The manager decides to buy  $x$  television ads and  $y$  radio ads. Write an equation relating  $x$  and  $y$ .

b. Make a table of values showing several choices for  $x$  and  $y$ .

c. Plot the points from your table, and graph the equation.

**Solutions** a. Each television ad costs \$150, so  $x$  ads will cost  $150x$ . Similarly,  $y$  radio ads will cost  $50y$ . The manager has \$3000 to spend, so the sum of the costs must be \$3000. Thus,

$$150x + 50y = 3000$$

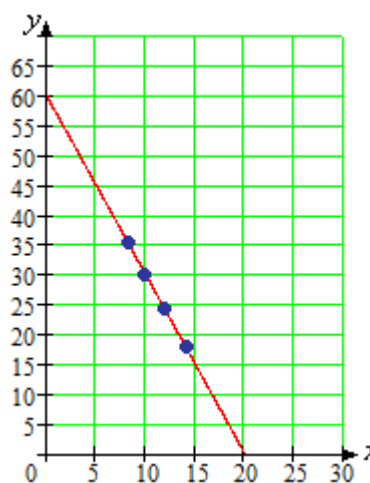
b. We choose some values of  $x$ , and solve the equation for the corresponding value of  $y$ . For example, if  $x = 10$ , then

$$\begin{aligned} 150(10) + 50y &= 3000 \\ 1500 + 50y &= 3000 \\ 50y &= 1500 \\ y &= 30 \end{aligned}$$

If the manager buys 10 television ads, she can also buy 30 radio ads. You can verify the other entries in the table.

$x$	8	10	12	14
$y$	36	30	24	18

c. We plot the points from the table. All the solutions lie on a straight line.



**Exercise 3** The manager at Breadbasket Bakery has \$120 to spend on advertising. An ad in the local flyer costs \$15, and posters cost \$4 each. She decides to buy  $x$  ads and  $y$  posters. Write an equation relating  $x$  and  $y$ .

**Hint:** Use the general form for a linear equation. What is the total amount of money the manager will spend?

## Intercept Method of Graphing

Because we really only need two points to graph a linear equation, we might as well find the intercepts and use them to draw the graph.

### Intercept Method of Graphing

To graph a linear equation by the **intercept method**:

1. Find the horizontal and vertical intercepts.
2. Plot the intercepts, and draw the line through the two points.

**RQ3** Describe the intercept method of graphing.

- a. Make a table of values and plot the points.
- b. Extend the line until it crosses both axes.
- c. Solve for  $y$  in terms of  $x$ .
- d. Plot the points  $(x, 0)$  and  $(0, y)$  and draw the line through them.

**Example 4a.** Find the  $x$ - and  $y$ -intercepts of the equation

$$150x + 50y = 3000$$

**b.** Use the intercepts to graph the equation.

**Solutions a.** To find the  $x$ -intercept, we set  $y = 0$ .

$$\begin{aligned} 150x + 50(0) &= 3000 && \text{Simplify.} \\ 150x &= 3000 && \text{Divide both sides by 150.} \\ x &= 20 \end{aligned}$$

The  $x$ -intercept is the point  $(20, 0)$ . To find the  $y$ -intercept, we set  $x = 0$ .

$$\begin{aligned} 150(0) + 50y &= 3000 && \text{Simplify.} \\ 50y &= 3000 && \text{Divide both sides by 50.} \\ y &= 60 \end{aligned}$$

The  $y$ -intercept is the point  $(0, 60)$ .

**b.** We scale both axes in intervals of 10, then plot the two intercepts,  $(20, 0)$  and  $(0, 60)$ . We draw the line through them, as shown in Example 3. ■

**Exercise 4** Find the  $x$ - and  $y$ -intercepts of the equation in Exercise 3, and use the intercepts to graph the equation.

**Hint:** Choose convenient scales for the  $x$ - and  $y$ -axes.

## Solving for $y$ in Terms of $x$

We have now seen two forms for linear equations: the general linear form,

$$Ax + By = C$$

and the form for a linear model,

$$y = (\text{starting value}) + (\text{rate}) \times t$$

It is sometimes useful to convert an equation from one form to the other.

**Example 5a.** Solve the equation  $4x - 3y = 6$  for  $y$  in terms of  $x$ .

**b.** Write the equation  $y = -9 - \frac{3}{2}x$  in general linear form.

**Solutions a.** We first isolate the  $y$ -term on one side of the equation.

$$\begin{aligned} 4x - 3y &= 6 && \text{Subtract } 4x \text{ from both sides.} \\ -3y &= 6 - 4x && \text{Divide both sides by } -3. \\ \frac{-3y}{-3} &= \frac{6 - 4x}{-3} && \text{Simplify: divide each term by } -3. \\ y &= -2 + \frac{4}{3}x \end{aligned}$$

**b.** We write the equation in the form  $Ax + By = C$ .

$$\begin{aligned} y &= -9 - \frac{3}{2}x && \text{Add } \frac{3}{2}x \text{ to both sides.} \\ \frac{3}{2}x + y &= -9 \end{aligned}$$

We can write the equation with integer coefficients by clearing the fractions. We multiply both sides of the equation by **2**.

$$2\left(\frac{3}{2}x + y\right) = (9)2$$

$$3x + 2y = 18$$



**Caution!** Do not confuse solving for  $y$  in terms of  $x$  with finding the  $y$ -intercept.

Compare:

**a.** In Example 5a, we solved  $4x - 3y = 6$  for  $y$  in terms of  $x$  to get  $y = -2 + \frac{4}{3}x$ . This is still **an equation** in two variables; it is just another (equivalent) form of the original equation.

**b.** To find the  $y$ -intercept of the same equation, we first set  $x = 0$ , then solve for  $y$ , as follows:

$$4(0) - 3y = 6$$

$$y = -2$$

This gives us **a particular point** on the graph, namely,  $(0, -2)$ . the point whose  $x$ -coordinate is 0.



**Exercise 5a.** Solve the equation  $150x + 50y = 3000$  for  $y$  in terms of  $x$ .

**b.** Write the equation  $y = 0.15x - 3.8$  in general linear form with integer coefficients.

### Answers to Exercises

**1.**  $(0, -9), (-6, 0)$

**2a.**  $g = 11 - \frac{1}{48}m$

**b.**  $(0, 11)$  The tank has 11 gallons before Rosa begins driving.  
 $(528, 0)$  After Rosa drives 528 miles, the tank will be empty.

**3.**  $15x + 4y = 120$

**4.**  $(0, 30), (8, 0)$



**5a.**  $y = -3x + 60$

**b.**  $-15x + 100y = 380$

**Now go to WeBWork and do the Reading Assignment for this Lesson.**

## Homework 2

1. A deep-sea diver is taking some readings at a depth of 400 feet. He begins rising at a rate of 20 feet per minute.

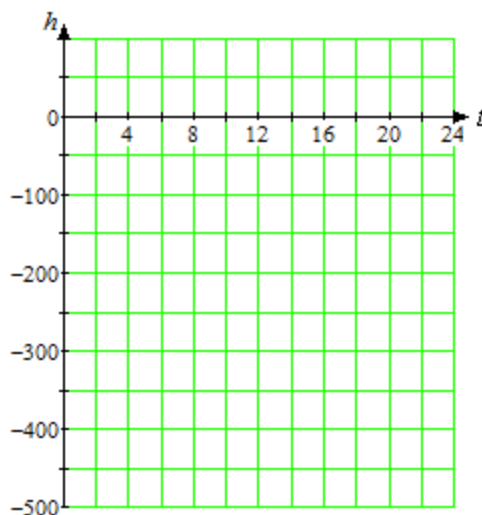
a. Complete the table of values for the diver's altitude  $h$  after  $t$  minutes. (A depth of 400 feet is the same as an altitude of  $-400$  feet.)

$t$	0	5	10	15	20
$h$					

- b. Write an equation for the diver's altitude,  $h$ , in terms of the number of minutes,  $t$ , elapsed.
- c. Find the intercepts and sketch the graph.

$t$	$h$
0	
	0

- d. Explain what each intercept tells us about this problem.



2. In central Nebraska, each acre of corn requires 25 acre-inches of water per year, and each acre of winter wheat requires 18 acre-inches of water. (An acre-inch is the amount of water needed to cover one acre of land to a depth of one inch.) A farmer can count on 9000 acre-inches of water for the coming year. (Source: Institute of Agriculture and Natural Resources, University of Nebraska)

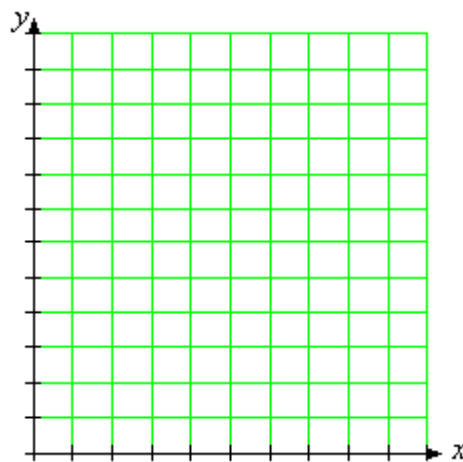
- a. Write an equation relating the number of acres of corn,  $x$ , and the number of acres of wheat,  $y$ , that the farmer can plant.
- b. Complete the table.

$x$	50	100	150	200
$y$				

- c. Find the intercepts of the graph.

$x$	$y$
0	
	0

- d. Use the intercepts to help you choose appropriate scales for the axes, and graph the equation.
- e. What do the intercepts tell us about the problem?
- f. What does the point  $(288, 100)$  mean in this context?



Now go to WeBWork and do the Skills Practice for this Homework.