

**PART - II : MATHEMATICS****SECTION - I (Single Correct Choice Type)**

This Section contains 8 multiple choice questions. Each question has four choices A), B), C) and D) out of which **ONLY ONE** is correct.

29. Let P, Q, R and S be the points on the plane with position vectors  $-2\hat{i} - \hat{j}$ ,  $4\hat{i}$ ,  $3\hat{i} + 3\hat{j}$  and  $-3\hat{i} + 2\hat{j}$  respectively. The quadrilateral PQRS must be a
- A) parallelogram, which is neither a rhombus nor a rectangle
- B) square
- C) rectangle, but not a square
- D) rhombus, but not a square

30. Let  $\omega$  be a complex cube root of unity with  $\omega \neq 1$ . A fair die is thrown three times. If  $r_1$ ,  $r_2$  and  $r_3$  are the numbers obtained on the die, then the probability that

$$\omega^{r_1} + \omega^{r_2} + \omega^{r_3} = 0 \text{ is}$$

- A)  $\frac{1}{18}$        B)  $\frac{1}{9}$        C)  $\frac{2}{9}$        D)  $\frac{1}{36}$
31. Equation of the plane containing the straight line  $\frac{x}{2} = \frac{y}{3} = \frac{z}{4}$  and perpendicular to the plane containing the straight lines  $\frac{x}{3} = \frac{y}{4} = \frac{z}{2}$  and  $\frac{x}{4} = \frac{y}{2} = \frac{z}{3}$  is
- A)  $x + 2y - 2z = 0$        B)  $3x + 2y - 2z = 0$        C)  $x - 2y + z = 0$        D)  $5x + 2y - 4z = 0$

32. If the angle A, B and C of a triangle are in an arithmetic progression and if a, b and c denote the lengths of the sides opposite to A, B and C respectively, then the value of the expression  $\frac{a}{c} \sin 2C$  and  $\frac{c}{a} \sin 2A$  is

- (a)  $\frac{1}{2}$       (b)  $\frac{\sqrt{3}}{2}$       (c) 1       (d)  $\sqrt{3}$

33. Let  $f$ ,  $g$  and  $h$  be real-valued functions defined on the interval  $[0, 1]$  by  $f(x) = e^{x^2} + e^{-x^2}$ ,  $g(x) = xe^{x^2} + e^{-x^2}$  and  $h(x) = x^2e^{x^2} + e^{-x^2}$ . If  $a$ ,  $b$  and  $c$  denote, respectively, the absolute maximum of  $f$ ,  $g$  and  $h$  on  $[0, 1]$ , then
- A)  $a = b$  and  $c \neq b$     B)  $a = c$  and  $a \neq b$   
 C)  $a \neq b$  and  $c \neq b$     **D)  $a = b = c$**
34. Let  $p$  and  $q$  be real numbers such that  $p \neq 0$ ,  $p^3 \neq q$  and  $p^3 \neq -q$ . If  $\alpha$  and  $\beta$  are nonzero complex numbers satisfying  $\alpha + \beta = -p$  and  $\alpha^3 + \beta^3 = q$ , then a quadratic equation having  $\frac{\alpha}{\beta}$  and  $\frac{\beta}{\alpha}$  as its roots is
- A)  $(p^3 + q)x^2 - (p^3 + 2q)x + (p^3 + q) = 0$     **B)  $(p^3 + q)x^2 - (p^3 - 2q)x + (p^3 + q) = 0$**   
 C)  $(p^3 - q)x^2 - (5p^3 - 2q)x + (p^3 - q) = 0$     D)  $(p^3 - q)x^2 - (5p^3 + 2q)x + (p^3 - q) = 0$
35. The value of  $\lim_{x \rightarrow 0} \frac{1}{x^3} \int_0^x \frac{t \ln(1+t)}{t^4 + 4} dt$  is
- A) 0    **B)  $\frac{1}{12}$**     C)  $\frac{1}{24}$     D)  $\frac{1}{64}$
36. The number of  $3 \times 3$  matrices  $A$  whose entries are either 0 or 1 and for which the system

$A \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$  has exactly two distinct solutions, is

**A) 0**    B)  $2^9 - 1$     C) 168    D) 2

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**SECTION - II | (Multiple Correct Choice Type)**

This Section contains **5 multiple choice questions**. Each question has four choices A), B), C) and D) out of which **ONE OR MORE** may be correct.

37. Let  $f$  be a real-valued function defined on the interval  $(0, \infty)$  by

$$f(x) = \ln x + \int_0^x \sqrt{1 + \sin t} \, dt. \text{ Then which of the following statement(s) is (are) true ?}$$

- (A)  $f''(x)$  exists for all  $x \in (0, \infty)$
- (B)  $f'(x)$  exists for all  $x \in (0, \infty)$  and  $f'$  is continuous on  $(0, \infty)$ , but not differentiable on  $(0, \infty)$
- (C) there exists  $\alpha > 1$  such that  $|f'(x)| < |f(x)|$  for all  $x \in (\alpha, \infty)$
- (D) there exists  $\beta > 0$  such that  $|f(x)| + |f'(x)| \leq \beta$  for all  $x \in (0, \infty)$

38. Let A and B be two distinct points on the parabola  $y^2 = 4x$ . If the axis of the parabola touches a circle of radius  $r$  having AB as its diameter, then the slope of the line joining A and B can be

- (A)  $-\frac{1}{r}$
- (B)  $\frac{1}{r}$
- (C)  $\frac{2}{r}$
- (D)  $-\frac{2}{r}$

39. The value(s) of  $\int_0^1 \frac{x^4(1-x)^4}{1+x^2} dx$  is (are)

- (A)  $\frac{22}{7} - \pi$
- (B)  $\frac{2}{105}$
- (C) 0
- (D)  $\frac{71}{15} - \frac{3\pi}{2}$

40. Let  $z_1$  and  $z_2$  be two distinct complex numbers and let  $z = (1-t)z_1 + tz_2$  for some real number  $t$  with  $0 < t < 1$ . If  $\text{Arg}(w)$  denotes the principal argument of a nonzero complex number  $w$ , then

- (A)  $|z - z_1| + |z - z_2| = |z_1 - z_2|$
- (B)  $\text{Arg}(z - z_1) = \text{Arg}(z - z_2)$
- (C)  $\begin{vmatrix} z - z_1 & \bar{z} - \bar{z}_1 \\ z_2 - z_1 & \bar{z}_2 - \bar{z}_1 \end{vmatrix} = 0$
- (D)  $\text{Arg}(z - z_1) = \text{Arg}(z_2 - z_1)$

41. Let ABC be a triangle such that  $\angle ACB = \frac{\pi}{6}$  and let  $a, b$  and  $c$  denote the lengths of the sides opposite to A, B and C respectively. The value(s) of  $x$  for which  $a = x^2 + x + 1, b = x^2 - 1$  and  $c = 2x + 1$  is (are)

- (A)  $(2 + \sqrt{3})$
- (B)  $1 + \sqrt{3}$
- (C)  $2 + \sqrt{3}$
- (D)  $4\sqrt{3}$

This Section contains **2 paragraphs**. Based upon the first paragraph **3 multiple choice questions** and based upon the second paragraph **2 multiple choice questions** have to be answered. Each of these questions has four choices A), B), C) and D) out of which **ONLY ONE** is correct.

**Paragraph for Questions 42 to 44**

Let  $p$  be an odd prime number and  $T_p$  be the following set of  $2 \times 2$  matrices :

$$T_p = \left\{ A = \begin{bmatrix} a & b \\ c & a \end{bmatrix} : a, b, c \in \{0, 1, 2, \dots, p-1\} \right\}$$

42. The number of  $A$  in  $T_p$  such that  $A$  is either symmetric or skew-symmetric or both, and  $\det(A)$  divisible by  $p$  is

- A)  $(p-1)^2$       B)  $2(p-1)$       C)  $(p-1)^2 + 1$       **D)  $2p-1$**

43. The number of  $A$  in  $T_p$  such that the trace of  $A$  is not divisible by  $p$  but  $\det(A)$  is divisible by  $p$  is

[Note : The trace of a matrix is the sum of its diagonal entries.]

- A)  $(p-1)(p^2-p+1)$       B)  $p^3-(p-1)^2$       **C)  $(p-1)^2$**       D)  $(p-1)(p^2-2)$

44. The number of  $A$  in  $T_p$  such that  $\det(A)$  is not divisible by  $p$  is

- A)  $2p^2$       B)  $p^3-5p$       C)  $p^3-3p$       **D)  $p^3-p^2$**

**Paragraph for Questions 45 to 46**

The circle  $x^2 + y^2 - 8x = 0$  and hyperbola  $\frac{x^2}{9} - \frac{y^2}{4} = 1$  intersect at the points A and B.

45. Equation of a common tangent with positive slope to the circle as well as to the hyperbola is

- A)  $2x - \sqrt{5}y - 20 = 0$       **B)  $2x - \sqrt{5}y + 4 = 0$**       C)  $3x - 4y + 8 = 0$       D)  $4x - 3y + 4 = 0$

46. Equation of the circle with AB as its diameter is

- A)  $x^2 + y^2 - 12x + 24 = 0$**       B)  $x^2 + y^2 + 12x + 24 = 0$   
 C)  $x^2 + y^2 + 24x - 12 = 0$       D)  $x^2 + y^2 - 24x - 12 = 0$

**SECTION - IV (Integer Type)**

This Section contains **TEN** questions. The answer to each question is a **single-digit integer**, ranging from 0 to 9. The correct digit below the question number in the ORS is to be bubbled.

47. If the distance between the plane  $Ax - 2y + z = d$  and the plane containing the lines

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} \text{ and } \frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5} \text{ is } \sqrt{6}, \text{ then } |d| \text{ is } \text{Ans. 6}$$

48. For any real number  $x$ , let  $[x]$  denote the largest integer less than or equal to  $x$ . Let  $f$  be a real valued function defined on the interval  $[-10, 10]$  by

$$f(x) = \begin{cases} x - [x] & \text{if } [x] \text{ is odd,} \\ 1 + [x] - x & \text{if } [x] \text{ is even} \end{cases}$$

Then the value of  $\frac{\pi^2}{10} \int_{-10}^{10} f(x) \cos \pi x \, dx$  is **Ans. 4**

49. Let  $\omega$  be the complex number  $\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3}$ . Then the number of distinct

complex numbers  $z$  satisfying 
$$\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix} = 0$$
 is equal to **Ans. 1**

50. Let  $S_k$ ,  $k = 1, 2, \dots, 100$ , denote the sum of the infinite geometric series whose first term

is  $\frac{k-1}{k!}$  and the common ratio is  $\frac{1}{k}$ . Then the value of  $\frac{100^2}{100!} + \sum_{k=1}^{100} \left| (k^2 - 3k + 1) S_k \right|$  is **Ans. 4**

51. The number of all possible values of  $\theta$ , where  $0 < \theta < \pi$ , for which the system of equations

$$(y + z) \cos 3\theta = (xyz) \sin 3\theta$$

$$x \sin 3\theta = \frac{2 \cos 3\theta}{y} + \frac{2 \sin 3\theta}{z}$$

$$(xyz) \sin 3\theta = (y + 2z) \cos 3\theta + y \sin 3\theta$$

have a solution  $(x_0, y_0, z_0)$  with  $y_0 z_0 \neq 0$ , is **Ans. 8**

52. Let  $f$  be a real-valued differentiable function on  $\mathbf{R}$  (the set of all real numbers) such that  $f(1) = 1$ . If the  $y$ -intercept of the tangent at any point  $P(x, y)$  on the curve  $y = f(x)$  is equal to the cube of the abscissa of  $P$ , then the value of  $f(-3)$  is equal to **Ans. 9**

53. The number of values of  $\theta$  in the interval  $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$  such that  $\theta \neq \frac{n\pi}{5}$  for

$n = 0, \pm 1, \pm 2$  and  $\tan \theta = \cot 5\theta$  as well as  $\sin 2\theta = \cos 4\theta$  is **Ans. 3**

54. The maximum value of the expression  $\sin^2 \theta + 3 \sin \theta \cos \theta + 5 \cos^2 \theta$  is **Ans. 2**

55. If  $\vec{a}$  and  $\vec{b}$  are vectors in space given by  $\vec{a} = \frac{1-2\vec{j}}{\sqrt{5}}$  and  $\vec{b} = \frac{2\vec{i} + \vec{j} + 3\vec{k}}{\sqrt{14}}$ , then the value of  $\frac{\vec{a} \times \vec{b}}{2\vec{a} + \vec{b}} \cdot \left[ \left( \frac{\vec{a} \times \vec{b}}{2\vec{a} + \vec{b}} \right) \times (\vec{a} - 2\vec{b}) \right]$  is **Ans. 5**

56. The line  $2x + y = 1$  is tangent to the hyperbola  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ . If this line passes through the point of intersection of the nearest directrix and the  $x$ -axis, then the eccentricity of the hyperbola is **Ans. 2**