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Note: to my knowledge, the representation used here is new. It combines the best of the 2 most common pictures to integers: number lines and algebra tiles. Below is a quick example of how the picture works. The new representation pays off later by making it much easier for students to see the connections between the rules for adding, subtracting, multiplying, and dividing integers. However, in this lesson, we focus only on adding. (For more examples, see the Answer Key, which shows worked-out solutions to all questions. The Key is available on Curriki).

5

0

-5

5

0

-5

Equation: **-2 + 4 = 2**

Equation: **-4 + -2 = -6**

**2**

**Add up**

**-6**

Pairs of opposites cancel.

These do not cancel, so we just count them up.

Attack & Defense: The Game of Integers

3.6a

What Are Integers?

Before we talk about integers, let’s review whole numbers. What are some examples of whole numbers?

 **Whole number examples:** **NON-examples of whole numbers:**

Integers are just like whole numbers, but they also include negatives. What are some examples?

**Integer examples:** **NON-examples of integers:**

Whole-Class Discussion

We’ll use integers to play a game called Attack & Defense. What will make your score go **up**, positive chips or negative chips? \_\_\_\_\_\_\_\_\_\_. What will make your score go down? \_\_\_\_\_\_\_\_\_\_\_\_.

Here’s how we’re going to draw pictures of positive and negative chips:

5

0

-5

**Example:**

3 negative chips

5

0

-5

**Practice:** Show 4 positive chips

5

0

-5

**Practice:** Show ‑4 chips plus -1 chip

If the number of Positives you have is the same as the number of Negatives, then your total chips is 0.

1). What are some combinations of chips would give you a total of 0? Draw 2 examples:

5

0

-5

5

0

-5

Equation:

Equation:

 For each Pos. that you add, your total goes up by 1.

 For each Pos. you **take away**, what do you think happens? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

 When you add a Neg. to your chips, does your total go up or down?\_\_\_\_\_\_\_\_\_\_\_\_\_

 But if you take away a Neg., what do you think happens? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

What are some ways you could lower your total by 2 chips?

2).

What are some ways you could raise your total by 3 chips?

3).

a).

b).

c).

d).

a).

b).

c).

d).

4). Let’s say you had 2 Pos and added 5 Neg. Then your new total would be -3, since the 2 Pos chips would cancel out 2 Neg chips, leaving 3 Neg chips at the end. We can draw a picture of it like this:

5

0

-5

Equation:

5

0

-5

5

0

-5

Eq’n:

Eq'n:

5).

What are some combinations of chips that make a total of +2? Please give two examples.

6).

What are some combinations of chips that make a total of -6? Please give two examples:

.

5

0

-5

5

0

-5

Eq’n:

Eq’n:

10

10

-10

-10

7). For each situation below, draw a picture and write the equation.

5

0

-5

5

0

-5

Eq’n:

Eq’n:

10

10

-10

-10

a).

You add 3 Pos chips and 5 Pos chips:

5

0

-5

5

0

-5

Eq’n:

Eq’n:

10

10

-10

-10

b).

You add 4 Pos chips and 9 Neg chips:

c).

You add 9 Pos chips and 4 Neg chips:

d).

You add 4 Neg chips and 8 Neg chips:

Practice on Your Own.

8). Try these questions on your own. Each answer will be shown after you have a chance to work.

5

0

-5

Eq’n: 10 + 6 = \_\_\_\_

10

-10

5

0

-5

5

0

-5

Eq’n: -3 + -6 = \_\_\_\_

Eq’n: 8 + -5 = \_\_\_\_

10

10

-10

-10

Small-Group Inquiry

9). In the pictures in #7-8, **sometimes some chips** **CANCEL** out, and **sometimes they all ADD UP**.

 a). Describe all the situations in which some chips *cancel out*. Use a complete sentence in your answer.

 b).Describe all the situations in which the amounts of chips *add up*. Use a complete sentence in your answer.

10). a). In the first box below, make up an example where the amounts add up. Label the example “Amounts Add Up.”

b). In the next box, make up an example where they cancel part of each other out. Label it “Amounts Partially Cancel.”

**2 Kinds of Integer Addition Problems**

Label:

 Amounts \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

5

0

-5

Equation: \_\_\_\_\_\_\_\_\_\_\_\_

Label:

 Amounts \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

5

0

-5

Equation: \_\_\_\_\_\_\_\_\_\_\_\_

11). a). If the amounts are going to cancel out, how can you calculate how much will be left? For example, if you had 437 chips of one kind, and 922 chips of the other kind, how many would be left after they canceled part of each other out? Show work and explain your reasoning.

b). In part a), how do you know whether what’s left will be Pos chips or Neg chips? Use a complete sentence in your answer.

c). Of the expressions below, circle the one in which the amounts will *cancel part of each other out*. In a complete sentence, explain your reasoning.

 77 + 245 -932 + -58 -68 + 250

 d). In the problem you circled, after the amounts cancel part of each other out, how much will be left, and will it be positive or negative? Use a complete sentence in your answer.

12).a). Write your own addition problem in which the amounts *add up* and give a *negative answer*.

(You don’t have to solve it yet.)

 b). Now solve your addition problem.

13).a). Write your own addition problem in which the amounts *add up* and give a *positive answer*.

(You don’t have to solve it yet.)

 b). Now solve your addition problem.

14).For each addition problem, please fill in the table.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Addition Problem** | **Do the chips add up, or cancel out?** | **How many chips are left at the end?** | **Is what’s left Positive or Negative?** | **What’s the sum?** |
| -8 + -21 |  |  |  |  |
| -8 + 21 |  |  |  |  |
| +8 + -21 |  |  |  |  |
| +19 + -13 |  |  |  |  |
| -19 + 13 |  |  |  |  |
| -19 + -13 |  |  |  |  |
| 19 + +13 |  |  |  |  |

15).Now practice with larger amounts.

 -88 + 213

 a). Do the chips add up, or cancel out? \_\_\_\_\_\_\_\_\_\_\_\_\_\_.

 b). How many chips will there be at the end? Show your scratch work below:

 c). When you wrote the scratch work in part b), did it matter whether you put 88 or 213 on top? Why or why not?

 d). Will the sum be positive or negative? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

 e). What’s the sum? \_\_\_\_\_\_\_\_\_.

16). When you have to do scratch-work to subtract two positive numbers, the larger number has to be on top, like this:

Example:

4 5 1

2 8

 –

Non-Example:

4 5 1

2 8

 –

Question: How do these examples relate to #15? Please answer in a complete sentence.

17). Please calculate each answer. SHOW SCRATCH WORK for each question. Please circle your answers, and don’t forget to put a negative sign on them if they’re negative.

 a). -88 + -213 b). 54 + -400 c). 355 + -198

18).Try these problems.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Addition Problem | Do the chips add up, or cancel out? | How many chips are left at the end? **Show scratch work!** | Is what’s left Positive or Negative? | What’s the sum? |
| -80 + 210 |  |  |  |  |
| -42 + -214 |  |  |  |  |
| -6,532 + 412 |  |  |  |  |
| 1,258 + -1,875 |  |  |  |  |
| +195 + -137 |  |  |  |  |
| -169 + -474 |  |  |  |  |
| 857 + -1,388 |  |  |  |  |