

Facilitator's Guide

Building Algebraic Thinking Through Pattern, Number and Function

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Introduction to the Course

Welcome to facilitation of the on-line course *Building Algebraic Thinking through Pattern, Function and Number*. This guide provides the information and support needed to run a successful course. We hope that you enjoy the facilitation experience and that your participants have a fruitful professional development experience.

Course Overview

Building Algebraic Thinking through Pattern, Function and Number examines the questions: What is algebraic thinking? Why is algebraic thinking important to learning math? How can teachers foster the development of algebraic thinking in their students?

Content

There are many aspects of algebra that regularly present challenges for students. While this course cannot address all such issues, it does highlight several —identifying, extending and generalizing linear patterns, interpreting graphs, translating between words and algebraic notation, solving linear equations, and proportional reasoning. These topics for focus were identified on the basis of recommendations of math teachers and research on student learning in algebra.

Pedagogy

This course highlights three pedagogical strategies that promote deeper understanding of the math content. Each of these strategies has, at its center, an emphasis on the student's reasoning about mathematics.

- *Strategy 1: Focus on Problems and Activities That Promote Conceptual Understanding*
Effective student learning in mathematics is made up of three types of knowledge: conceptual understanding, procedural understanding, and factual knowledge. Many students are lacking in at least one of these, most often in conceptual understanding. This course highlights various problems and follow-up activities that encourage the development of conceptual understanding in students.
- *Strategy 2: Utilize Effective Questioning Techniques*
Teachers and students pose questions for a variety of purposes. This course examines the nature of questions intended to elicit students' algebraic thinking. The course also focuses on helping teachers learn how to pose questions based on the purpose and use questioning to optimize students' learning.
- *Strategy 3: Provide Opportunities for Reflection on Ideas and Learning*
Having time to synthesize and reflect upon ideas can promote learning and retention. The course features multiple and regular opportunities for written reflection and feedback from the facilitator. Course participants are asked to write in a learning log and to reflect in writing upon questions for further thought. These activities are intended to help participants make connections across activities, research and their own experiences and discoveries throughout the course.

Philosophy

There are a number of underlying philosophical assumptions that shape this course. It is helpful to be aware of these assumptions as they affect the expectations that are placed upon course participants.

They are:

- Effective student learning in mathematics is made up of a foundation of three types of knowledge: conceptual understanding, procedural understanding, and factual knowledge. Many students are lacking in at least one of these, most often in conceptual understanding.
- Some problems can encourage the development of conceptual understanding. For their own development, it is important for teachers to do the math in these problems as mathematical thinkers and learners themselves, not just as teachers of students. This level of engagement lets them examine and reflect on the mathematical content and processes inherent in a particular problem while they become aware of their own solution strategies.
- It is vital to examine students' thinking to understand how the students come to understand different topics, not just for the purpose of evaluation. It is also important that they become aware of the ways in which students think about the mathematics that are different than those of the teacher.
- Conducting discussions in math class that encourage students to make their thinking apparent will deepen their learning by helping them see why and how things work, and how different approaches can lead to a solution. This can be done through posing particular kinds of questions that draw out students' thinking and encourage analysis, synthesis, and generalization.
- All students can attain this level of mathematical understanding, given the appropriate mathematical tools, approaches, and instruction.

Course Structure

This course focuses on algebraic thinking. It opens by asking participants to reflect upon their definition of algebraic thinking, provides many opportunities to explore how to develop the algebraic thinking of their students and closes by asking participants to reconsider their idea of what algebraic thinking. It examines algebra problems, explores instructional strategies, including questioning technique, and promotes reflection to help teachers support students in building their algebraic thinking

Each session asks participants to engage in some combination of the following tasks:

- **Readings**

The readings in this course draws heavily on Mark Driscoll's book *Fostering Algebraic Thinking* as well as brief articles drawn from various sources, most often from NCTM publications and other education journals. These readings provide important background information for each session.

- **Activities**

There are numerous math problems throughout the course that participants are expected to complete. Many of them model approaches that encourage the development of conceptual understanding about a topic. Additionally, there are other activities for participants to complete such as responding to questions for further thought, analyzing some examples of written student work, and writing a final project.

In some sessions, there are online “applets” or animations—small, focused pieces of computer software designed for a very specific purpose—that accompany some of the math problems and provide a means for thinking about an aspect of mathematics in a more visual and dynamic way.

Participants will also watch video clips that are intended to capture genuine classroom interactions and/or students’ thinking in their own words.

- **Final Project**

While the final project is due at the end of the course, it has been structured so participants can complete the parts of the project throughout. This allows them to immediately apply what they’ve learned in a session to something they can use in their classroom, easing the burden of completing the whole project in one session.

Each session can comfortably be completed in one week’s time, for a total course length of 8 weeks. Participants have flexibility in when and how they complete the tasks during the session. They will submit their assignments to a course facilitator by email and will receive feedback, announcements and reminders from the facilitator.

COURSE FACILITATION

Your Role As Facilitator

Elements of Each Session of the Course

Below are the kinds of tasks that can appear in each session and some notes about your responsibilities as facilitator for each:

Readings

In almost every session, participants have one or two brief readings that lay some pedagogical or mathematical foundation for the session. Encourage participants to complete the readings early in the session, as the readings provide valuable background for the content of the session and the discussions. Also, make sure that you are familiar with the readings, so that you can relate it to students' responses in learning logs or other assignments for which you will provide feedback.

Math Problems

Many of the sessions include math problems for participants to solve. It is important to make clear to participants that these math problems are intended for them to complete as learners and doers of mathematics themselves, and not simply as resources to use with their students (though many—not all!—of the problems can also be used with their students.) You may want to set up a separate discussion forum if you have participants who would like to discuss various solutions methods to the problems. Each solution is provided through a solutions tab at the end of the session. Providing the solution to participants in this manner may be helpful if someone gets stuck. You should also invite participants to email you with any questions about the math problems or the solutions that are provided.

Activities

In addition to the math problems, participants are asked to do a variety of other kinds of activities, such as analyzing pieces of student work, writing in a learning log and watching classroom video. Some of these activities are assessment activities that participants will submit to you. You will read and provide feedback to participants on these items that they submit for your review. More details are provided for you later in this guide in the section entitled "Providing Feedback to Participants."

Student Work

Participants will analyze examples of student work. "Student work" in this course refers both to written student work as well as video clips of students talking about their thinking on a particular problem. Encourage participants to consider the student work for its educational merits and not to dwell on how closely the work mimics something their students would do.

Videos

There are several video clips used in the course. The video clips focus either on interactions with individual students, with pairs or small groups of students, or in some cases, a whole class interaction with the teacher. In every case, the videos are intended to capture students' thinking in

the students' own words, or real student-teacher interactions in real classroom settings. **None of the videos were scripted in any way.**

When viewing videos of teacher/student interactions, it is sometimes tempting for viewers to focus on critiquing the instructional choices of the teacher in the video. Encourage participants to focus instead on the student and the students' thinking and dialogue. These videos are not intended as exemplars of instructional technique but rather as catalysts for participant thought and application of ideas.

Questions for Further Thought

Each session includes a question or two for the participants to consider and write about. The purpose of these questions is to provide a structure for reflection and synthesis of the content of the session. Their notes might also be helpful to them in designing and writing their final projects. These questions for further thought could also be used in a discussion forum, should you choose to create a format for this. This is a great way to increase the dialogue around this topic, since a discussion board will enable participants to share ideas with one another, rather than just back and forth with the facilitator. In this event that you establish a discussion forum, your role would be to keep the discussion moving forward, to encourage as much participation from all course participants as possible, and to provide alternative viewpoints for consideration if they do not come up naturally from other participants. You might also pose follow up questions and bring in comments from the session readings.

General Facilitation Goals and Suggestions

Depending upon your situation, your responsibilities as a facilitator of the course may include some or all of the following:

1. To inspire interest in the course and the topic and encourage completion of the activities contained in the course.
2. To answer questions, provide support and guide participants in their completion of the course.
3. To provide accountability to course participants by collecting assignments, providing feedback and tracking progress toward completion of the course requirements.
4. To pursue ways to extend and connect the learning and experiences of the course to other professional development opportunities.

Strong communication is an important component of each of these responsibilities. Especially since there is not a required discussion component of this course, a participant's contact with the facilitator is critical. Your ability to convey expectations and information in an enthusiastic and approachable manner will enhance your participants' experiences in the course. Helping them to feel a connection with you will encourage their participation and investment in the course.

If you do not have face-to-face opportunities to build a relationship with course participants, you will need to rely upon email communication to do this. Providing frequent, consistent and upbeat email correspondence will help you to clarify expectations and keep your participants feeling connected and supported. Your email communication should aim to:

1. **Make a positive first impression in your initial email contact.** Make sure participants know that you are there to support them in their coursework. Let them know how you will do that. Some sample emails are provided in Facilitation Resources section of this facilitation guide.
2. **Clearly communicate the expectations of the course.** Make sure that participants understand what is expected of them and what they can expect from you. Refer them to the sections of the course that help to outline and organize the expectations.
3. **Foster and maintain a connection with participants throughout the course.** A few ways to foster this connection include:
 - Sending feedback on assignments promptly and personalizing that feedback as much as possible;
 - Providing reminders of upcoming due dates;
 - Summarizing sessions by sharing their ideas, ideas offered by other participants and your own ideas.

Before You Begin

6 weeks or longer in advance of the Session One:

1. Identify Participants for the Course. The course is intended for teachers of mathematics at grades 6-10. The focus of the course is algebraic thinking. The course will be completed largely at a computer, and assignments are to be emailed to a facilitator. The amount of time required to read and provide feedback to participants will increase as the number of participants increase. Participants will submit 3 learning logs and a final project. You might also opt to have participants email you their questions for further thought in each session. In addition, time spent on individual follow-up with participants and record-keeping will increase with the size of the group.
2. Familiarize yourself with the course. Ensure that you can access all the features of the course on your computer. Also consider what is required to ensure computer access for participants. Will they do the course on machines at school or in their home or will access be provided in both locations? Read the Course Introduction carefully. Browse the Sessions and Resources. Sample the activities of the course. Check that you can view animations and video and that links function properly.
3. Gather the necessary course readings. The Resources section provides details on how to purchase books and articles. Plan to have these available to participants at the very start of the course, and allow enough lead time to get each of the readings.
4. Create a Course Calendar. Be sure to consider school holidays, vacations and other events if participants will not be expected to work on the course during these days. An example is provided below and a blank calendar is included in the Facilitation Resources section of this guide.

Sample Calendar:

Session	Start Date	Completion Date	Assignments to Submit by Email	Due Date
1	Friday 9/19/08	Thurs. 9/25/08	Opening Reflection Question for Further Thought (optional)	9/25/08
2	Friday 9/26/08	Thurs. 10/2/08	Question for Further Thought (optional)	10/02/08
3	Friday 10/3/08	Thurs. 10/9/08	Question for Further Thought (optional)	10/09/08
4	Friday 10/10/08	Thurs. 10/16/08	Learning Log 1 Question for Further Thought (optional)	10/16/08
5	Friday 10/17/08	Thurs. 10/23/08	Question for Further Thought (required)	10/23/08
6	Friday 10/24/08	Thurs. 10/30/08	What Questions Did You Ask? Learning Log 2 Question for Further Thought (optional)	11/3/08
7	Friday 10/31/08	Thurs. 11/6/08	Question for Further Thought (optional)	11/06/08
8	Friday 11/7/08	Friday 11/14/08	Learning Log 3 Final Project	11/14/08 11/14/08

4-6 weeks before Session One

1. Send an invitation/announcement to participants providing details for their participation in the course. If possible provide this information by email and include a link to on-line course, access information or passwords, course calendar, facilitator contact information. See the sample emails provided in the Facilitation Resources section of this guide.

1 – 4 weeks before Session One

1. Make sure that participants know how to access the course and all of its features and have no technology problems.

2. Make sure that participants have access to the course book (*Building Algebraic Thinking* by Mark Driscoll) and all the other course readings. See the Resources section of the course for details.
3. Study the sessions in more depth, making sure that you have completed the readings and activities prior to the start of each session. Highlight readings and make notes for your reference in providing feedback to participants.

Getting the Course Underway

For Each Session

1. Prior to the start of each session, complete all of the readings and activities yourself. Record your ideas and your solutions for reference when providing feedback to participants.
2. Send weekly email announcement late the day before or early in the first day of the session. See the sample emails provided in *Email Announcements* in the Facilitation Resources section of this guide.
3. Provide written feedback by email for any assignments that have been submitted during the previous session. Feedback is most relevant when it is descriptive and offered reasonably promptly. Set a date by which you will send feedback and let participants know when they can expect it. Try to provide feedback within three days of the date of your receipt or due date of the assignment. See *Suggestions for Providing Feedback* in this guide.
4. Send email reminders on day 6 of Sessions 4, 5, 6 and 8 that an assignment is due the following day. If you are collecting the Questions for Further Thought, send reminders each week.

At the End of Sessions 4, 5, 6 and 8

1. Send an email to each participant either confirming receipt of assignment or reminding him/her to submit an assignment that is past due.

Session By Session Checklist

Session One

Facilitation Goals:

- To support participants in accessing all of the features of the course and troubleshoot technical difficulties
- To establish email communication and get to know the participants through this exchange

To Do:

- Send email announcement
- Send email reminder of assignment due

- Reply to emails about a-ha moments

Session Two

Facilitation Goals:

- To support participant completion of the reading and activities of the session
- To respond to questions from participants

To Do:

- Send email announcement
- Respond to any inquiries from participants
- Make contact with any participants who did not submit Session One's assignment

Session Three

Facilitation Goals:

- To support participant completion of the reading and activities of the session
- To respond to questions from participants

To Do:

- Send email announcement
- Respond to any inquiries from participants
- Make email/phone contact with participants as needed

Session Four

Facilitation Goals:

- To support participant completion of the reading and activities of the session
- To respond to questions from participants and support their completion of course requirements
- To monitor participants' progress toward completion of course requirements and intervene as necessary to help participants get on track.

To Do:

- Send email announcement
- Send email reminder of assignment due
- Read and provide written feedback to Learning Log 1
- Email/phone participants as needed to inquire about progress and offer support

Session Five

Facilitation Goals:

- To support participants as they prepare and conduct a student interview
- To provide feedback to support the learning of participants

To Do:

- Send email announcement
- Send email reminder of assignment due
- Follow up with participants who have not submitted Learning Log 1
- Read and reply to Question for Further Thought

Session Six

Facilitation Goals:

- To support participants in their examination of math tasks and student video
- To facilitate their initial planning of their Final Project

To Do:

- Send email announcement
- Send email reminder of assignment due
- Follow up with participants who have not submitted Question for Further Thought
- Read and reply to Learning Log 2

Session Seven

Facilitation Goals:

- To support participant completion of the reading and activities of the session
- To respond to questions from participants

To Do:

- Send email announcement
- Follow up with participants who have not submitted Learning Log 2 or other past due assignments
- Invite questions about the Final Project

Session Eight**Facilitation Goals:**

- To support participants in the completion of their Final Project
- To monitor progress toward completion of course requirements

To Do:

- Send email announcement
- Send email reminder of Learning Log 3 and Final Project due
- Follow up with participants who have not submitted any course requirements
- Read and reply to assignments
- Conclude the course

Providing Feedback to Participants

Providing feedback to participants on their assignments is an important role of the facilitator. This section of the guide provides some suggestions and examples to support you in assessing participants' work. This opportunity to provide feedback allows you to learn about what your participants are thinking and experiencing as participants in the course. It also enables you to see how they may be applying their learning to their own practice.

The following table outlines the items that participants will be expected to email to you. You should provide feedback within 3-7 days of having received their work. (NOTE: If you have established a discussion board for the course, you will need to monitor the discussion and respond to their entries in each of the sessions. In this case, the questions for further thought will become the discussion question. In this scenario, you will not need them to email you the Question for Further Thought in Session 5)

ASSIGNMENT	DUE DATE
Opening Reflection	End of Session 1
Learning Log 1	End of Session 4
Question for Further Thought	End of Session 5
What Questions Did You Ask	End of Session 6
Learning Log 2	End of Session 6
Learning Log 3	End of Session 8
Final Project	End of Session 8

To support you in tracking completion of assignments and feedback, see the Assignments Record in the Facilitation Resources section of this guide.

Suggestions for Feedback By Assignment:

Opening Reflection

The primary goal of this assignment is to establish email communication between the participant and the facilitator. A secondary goal is to get the participant thinking about his/her own experiences in teaching or learning algebra. In responding to a participant's reflection, make an effort to connect with some of their ideas. Also, it is helpful if you reflect upon this question yourself, so that you can share your own a-ha moment in your response to them. Keep a record of completion of this task. If you do not receive an email from a participant by the end of the session, follow up by email or phone to remind them to send it and to inquire about any problems.

Learning Logs

These assignments are a key feature of the course. Through the completion of these learning logs, participants process and share their ideas from the course. Sometimes a learning log entry will draw on content from more than one session. Completion of these logs is an important expectation, and failure to do so may have an impact on the overall assessment of a participant.

Below you will find some sample learning logs with some facilitator feedback. These may be helpful in structuring your feedback to participants. They are not provided as exemplars, but rather as examples of participant work with some facilitator comments.



Learning Log #1 SAMPLE PARTICIPANT RESPONSE

<p>In Session 2, you were asked to choose one of four lessons to do with your students. Briefly describe the lesson you did with your students for the assignment with your students.</p>	
<p>I had two of my students work on the Letter Perfect and the Toothpick houses problems. I gave my students the option of completing that nights homework in class or to do these extra activities for 5 bonus points on the up coming test. I sat with the two students who chose the bonus activity and explained to them that I was there to watch. They could ask each other questions, but I was only there to watch. I explained to them that they were to work individually on the toothpick houses problem and then they could work together on the letter perfect and go over together the toothpick houses problem. My two students are so different in their mathematical ways that this lesson was very interesting. One of the students dislikes showing his work and does his best to do it all in his head where my female student shows every single step. I knew that the results of the two working together would be interesting.</p>	
<p>Take a look at the work you collected from your own students based on that lesson. Choose a few samples that show different approaches to solving the assigned problem, or that seem especially interesting to you, and answer these questions:</p> <p>What algebraic thinking is evident in your students' work? Refer to the Guiding Questions for Algebraic Thinking (Table 1-1, page 4, Fostering Algebraic Thinking)</p>	
<p>I found the toothpick problem more challenging and interesting to analyze so I will discuss my student's work from that problem. Both of my students had a clear understanding that they were looking for a pattern or a rule, but only one checked the rule. One of my students believed that because one house had 6 toothpicks that they could just make an equation $6x=y$. This student without even completely checking the given houses above completed the entire table and didn't see an error in this method until discussing the problem with the other student. My other student really went into depth with his work (surprisingly because he never shows his work) . He drew multiple houses to check his equation of $5h+1=t$. He completed the whole graph after looking at the first four (checking each one with an illustration) and finding that his equation did work for all cases. He also looked at the equation backwards and could easily produce an equation that would work for knowing a certain number or toothpicks</p>	<p><i>Facilitator Comments:</i> <i>Interesting dichotomy in approach, algebraic thinking present, and outcomes. And, it's interesting in that there were surprises for you in the outcome.</i></p>

<p>and solving how many houses they would create. He definitely though algebraically about the problem in terms of does my equation work? Will it work for all cases? And if I have to work backwards does the problem still make sense?</p>	
<p>What are 3 to 5 questions you could pose to your students the next time you do this activity to encourage their algebraic thinking? Explain how your questions would help further this goal.</p>	
<p>Is there an equation that you can create to complete this table and if there is does that equation check out to be correct if you create the toothpick house? This makes the students check their work, which I strongly believe in. It also allows the student to see errors and go back and rethink the problem.</p> <p>Without using an equation can you guess what the next number of toothpicks might be? Will it be higher or lower? This just gets the students on the right track of knowing that the houses double in size, but not quite in the number of toothpicks. I like making students think about problems first before trying to solve the problem that way they don't come up with an answer so fast and then just stop thinking about the problem and just plug in values.</p> <p>Thinking about all of your operations (addition, subtraction, multiplication and division) which operations seem to be useful in this equation? Do you need more than one? I like students to learn to combine and use operations together, for example multiplication is associated with addition more than subtraction and division is associated with subtraction. This small mathematical knowledge will help them algebraically figure out which operations will help them be successful when thinking about how the number of toothpicks changes and the number of houses increases.</p>	<p><i>Great way to slow them down from jumping to their first idea!!</i></p> <p><i>Nice strategy to help develop the ability to generalize!</i></p>
<p>Considering what you now know about algebraic thinking and patterns, explain what you would do differently next time, and why.</p>	
<p>I would probably give the students a little more time to work individually as I felt that once paired up one student took over the conversation. I felt that the weaker of the two math students let the other student lead and didn't do their own thinking and analyzing of the problem. I might try to let the students both complete the problems before discussing so there would be varying answers and then have each student test out the other student's equation to see if it worked. If it didn't work the students could make suggestions to each other then had the papers back and re think their</p>	<p><i>The experience of being paired with someone who "gets it" can be discouraging for the other student. So, I think your efforts to avoid this are great. Sometimes grouping by similar ability can provide a good opportunity</i></p>

steps to come up with a new solution.

for them to discuss their thinking and build on the thoughts of another without their being given the answer. Having students work together can provide many challenges, can't it? Nice job on your learning log. I enjoyed reading it!



Learning Log

Learning Log #1 SAMPLE PARTICIPANT RESPONSE

<p>In Session 2, you were asked to choose one of four lessons to do with your students.</p> <p>Briefly describe the lesson you did with your students for the assignment with your students.</p>	
<p>I started the lesson for the painting faces problem by explaining to my students what a face was what the problem meant by different size rods and what a rod actually was using cubs. We discussed the problem together to make sure everyone understood what the problem was asking. I then gave the students the opportunity to complete the problem online using the applet. Each student worked on their own for awhile and then I let them work with a partner to discuss each others answers. We then came back together and shared as a group. As all of this was going on I was only a facilitator making sure students stayed on task and fostering some discussions.</p>	<p><i>Facilitator Comments</i></p> <p><i>Nice! I like how you structured the task and the role you chose for yourself.</i></p>
<p>Take a look at the work you collected from your own students based on that lesson. Choose a few samples that show different approaches to solving the assigned problem, or that seem especially interesting to you, and answer these questions:</p> <p>What algebraic thinking is evident in your students' work? Refer to the Guiding Questions for Algebraic Thinking (Table 1-1, page 4, Fostering Algebraic Thinking)</p>	
<p>My students were able to build rules to represent the changes in the number of faces being painted from each length rod to the next. They could see that the number of faces increased by four each time a block was added to the rod. Some of my students were also able to find not only the vertical change in the number of painted faces but also the horizontal change between the length of rod and the number of faces. They were able to see a pattern from the length of rod to the number of faces. They were able to show this pattern as a rule in both word sentence form and also algebraic form. Granted not all the students were able to do this on their own, but most could understand it when discussing with a partner.</p>	<p><i>Sounds like a very</i></p>

	<i>successful lesson since most could generalize the rule, even if not as an algebraic expression.</i>
What are 3 to 5 questions you could pose to your students the next time you do this activity to encourage their algebraic thinking? Explain how your questions would help further this goal.	
<p>How is the number of faces column related to the one that came before it? This would encourage my students to think about how the numbers are changing.</p> <p>Can you find a more helpful way to write the rule? This would help my students see that coming up with a rule that can show how the length of rod effects the number of faces will make finding any length of rod from any number of faces a simple calculation.</p> <p>Which process reverses the one you are using? This will help students see the importance of doing and undoing a problem to find solutions.</p>	<i>Nice questioning. You've identified a variety of lines of questioning to support the specific learning goals.</i>
Considering what you now know about algebraic thinking and patterns, explain what you would do differently next time, and why.	
I would like to set up centers where students can work this problem using the best method for them. I would set up manipulatives such as blocks and algebra tiles, a technology center using the applet, and possibly paper and pencil for those that like to do everything in their head. Then just asking more questions to make students think algebraically.	<i>Would you suggest to students which center(s) to consider based on their learning style? This approach sends a clear message that there are many different ways to successfully complete this (and most) math problems. I like it! Thanks for sharing your experiences and reflections. I enjoyed reading this.</i>



Learning Log #2 SAMPLE PARTICIPANT RESPONSE

<p>In the video for this session, you explored the Comparing Functions applet and watched video of a student interview in which students explored the applet.</p> <p>What did you learn about the students' understanding of these functions from the interview?</p>	
<p>First, I found these four students to have a pretty good understanding of mathematics and how it works. They seemed, through their comments and the thought process leading to those comments, to have a good grasp of how numbers behave. They were perplexed by the challenge of predicting, but they weren't overwhelmed. (I have lots of students who would say "I don't know" and not really think about it, while I have other students who like to think about math and challenge themselves. The students in the video all seemed to fall into the latter category.)</p> <p>I tried to think back to when I was in my early teens – what would I think when confronted by situations I hadn't yet encountered. I find even today I have to stop and think about a situation I've formerly encountered. What must it have been like the first time I encountered something new?</p> <p>The students understood exponents (although one student did say "two times" when talking about the second power, I think he did understand how they work). Although they had a clear understanding that the product of the exponential function would get less and less as the exponent decreased, they seemed to be in agreement that it would dip into the negative values when the exponent went into the negatives. The boy in the white t-shirt understood that a number squared would be positive regardless of whether the base number was positive or negative, which he justified by stating a negative number multiplied by itself would yield a positive result because a negative number multiplied by a negative number gives a positive product.</p>	<p><i>Facilitator Comments:</i></p> <p><i>So nice that you've thought about what you learned both from the standpoint of what content they know but also their disposition toward math, especially when they are stuck. I also appreciate your efforts to put yourself in their place. This is a great reason for lifelong learning experiences, isn't it?</i></p> <p><i>Nice description of you observations of their understandings.</i></p>
<p>As their teacher, what would you do in class to follow up on what you learned here?</p>	

I do plan to do something with this activity. All our students have laptops. I could give them the web address for the applet, or I could try to have it saved on the server, which might be an easier way for them to fish it up. I would plan to do a preliminary introduction with the LCD projector, do a class prompt similar to the prompt done in the video, then set the students loose with the guiding questions that were provided by this course.

Something else I would have them do is work on some value tables on paper, as I think processing this with paper and pencil would help them see patterns in the table that are represented in the graph. Then we could see and discuss how those patterns that they see in the tables appear in the graphs.

Last thing I would try to do is a “so what?” as the question of “what does this have to do with real life?” seems to have surfaced more this year than in the past years. One thing that comes to mind is to look at a couple having two kids, these kids having two kids, and so on to represent the 2 to the x power. I’m not sure how I would represent situations with negative exponents, as nothing is popping to mind where this would apply in “the real world” in a way eighth grade students could relate to it.

I like this effort to connect the ideas across different representations. The graphs and table are great ways for them to examine the patterns that emerge in these functions.

I view kids wanting to make these connections as a positive step (even though their delivery is not always so positive!) I think this relates to the importance of understanding why. There does not always need to be a concrete and directly applicable benefit to everything they do in the math class but if the work is mostly relevant they can better accept the more theoretical parts of class! Thanks for sharing your ideas and for your effort to make the mathematics meaningful.

Learning Log #2 SAMPLE PARTICIPANT RESPONSE

<p>In the video for this session, you explored the Comparing Functions applet and watched video of a student interview in which students explored the applet.</p> <p>What did you learn about the students' understanding of these functions from the interview?</p>	<p><i>Facilitator Comments:</i></p> <p><i>Thanks for sharing your reflections in this learning log. I enjoyed reading them and have shared some comments in response.</i></p>
<p>I saw that the students had a limited understanding of negative exponents. They weren't really sure what would happen to the number if it were raised to a negative number. They did know that if a negative number is squared that you get a positive so the function of $y=x^a$ was a little easier for them to see what would happen to the Y values as the X values increased or decreased. They came up with some interesting predictions about each function. As they worked through the applet some of their predictions were right and some were wrong. But there was no doubt that some learning took place during this exercise.</p>	<p><i>Nice description of your observations and solid conclusions about what you think is going on for these students. In particular, you write clearly about what they seem to know and not know about each type of function. This kind of assessment is really valuable in making instructional decisions. Aren't applets a great tool for student exploration and discovery. I like this one a lot as it offers an entry point and a lot of opportunity to think deeply about nonlinear functions.</i></p>
<p>As their teacher, what would you do in class to follow up on what you learned here?</p>	
<p>I would definitely have a quick lesson over negative and zero exponents, if just for a refresher. I would do some more online practice with predictions as well as graphing the functions. Conducting student interviews is always a productive way of finding out what the students know and what they don't know.</p>	<p><i>What/how would you do this?</i></p> <p><i>I like this idea. For some learners, actually creating the graphs themselves can help them grasp what is changing and the important elements of the function.</i></p> <p><i>Yes, this follows well. In the first step you identified quite astutely some predictions about what they know and</i></p>

	<p><i>don't know and it's great to confirm this assessment before going too far down a certain path of instruction! Nice idea.</i></p>
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Learning Log

Learning Log #3 SAMPLE PARTICIPANT RESPONSE

Write your current definition of algebraic thinking.	
Algebraic thinking is the ability to see rules or generalizations and be able to state them without specific numerical data. It is the ability to work backwards and manipulative equations for different purposes. It is the consistent real-life approach that always bears in mind what is really being asked and what is really happening; always with the question, “Could this be a real workable response or solution?”.	
Now, look back at the discussion from Session 1 and compare your initial definition of algebraic thinking with your current one. In what ways has your conception of algebraic thinking changed?	
It hasn’t changed that much, but I am more aware of it as I work with 7th grade students. I know more about questions to ask and pitfalls to avoid.	<i>Having a greater awareness and being more in touch with questions to ask or how students think is HUGE.</i>
In light of what you learned in this course, what, if anything, would you do differently in your classroom? Describe those differences generally, or if you prefer, in the context of how you would conduct specific lessons or activities.	
I am more highly aware of the need to anticipate and/or not be surprised by student misconceptions. I feel better about not getting through the whole curriculum; I want students to really understand each skill and concept presented. I will do much more class discussion, computer-based activities, and real-life problem-solving. I also intend to do more writing (oral discussion with learning disabled students) that focuses on their thinking. There are many more activities that have been presented in this course that I have yet to try. I look forward to this. Much of this corresponds nicely with my required curriculum. Thank you.	<i>Thank you! It sounds like you have a lot of new goals and plans. I wish you the best of luck in implementing them and thank you for your participation in the course. It was nice getting to know you online.</i>



Learning Log

Learning Log #3 SAMPLE PARTICIPANT RESPONSE

Write your current definition of algebraic thinking.	<i>Facilitator Comments</i>
<p>Algebraic thinking is the ability to approach abstract situations and represent these situations mathematically or symbolically in ways that gives the situation meaning allows for the prediction of patterns and trends which cannot be directly observed.</p>	<p><i>Nicely articulated definition.</i></p>
<p>Now, look back at the discussion from Session 1 and compare your initial definition of algebraic thinking with your current one.</p> <p>In what ways has your conception of algebraic thinking changed?</p>	
<p>My earlier definition: Algebraic thinking: I think of this term as relating to some sort of change. For example, what first comes to mind is math with variables, the solution changing relative to the value of the variables. Given the variety of solutions (based on variety of variables), this type of thinking opens the door to a huge landscape of mathematical exploration.</p> <p>My understanding has changed in three fundamental ways:</p> <p>I mentioned abstract, and I think this has been a result of this class and of my teaching of algebra over the past month to my “non-Algebra” students. Many students struggle with algebra because it is so abstract. I am finding that even the use of manipulatives and applets only helps with short-term understanding. The jump from the concrete to the abstract can be a large, dangerous move for some kids, and therefore necessary precautions must be in place.</p>	

A situation should have meaning. Algebra should be taught in context to make it easier for kids to understand. They need to be invested in the study of algebra to make it worthwhile. I have never been asked as frequently “Why do we need to know this?” as I have been this year. Algebra is a tool to represent real situations in an abstract way which allows the information to be manipulated. The connection to real life situations is a key element to algebra and if it is not stressed it has little meaning to most children.

My third key advance in thinking relates a bit to my second. It is the application of algebra, especially when applied to future events. Predicting the future is something we all, as humans, like to do. Algebra is essential in the making of these predictions, and in the representation of our findings..

Excellent observations and development of your thinking about this. I'm impressed!

<p>In light of what you learned in this course, what, if anything, would you do differently in your classroom? Describe those differences generally, or if you prefer, in the context of how you would conduct specific lessons or activities.</p>	
<p>I would like to connect what goes on in class to what is happening in students' lives more directly. The CMP books we use gives real life situations, but they are someone else's lives and often seem far from real and far from lively. So, I would like to start to integrate algebra more into experiences like the huge service learning project our team is undertaking this spring. This will be a good goal for me – I know it can be done, but it will take some forethought and some careful planning to make it work.</p> <p>Also, I will worry less about mastery. I think exposure should be my goal in areas where I find the students not quite developmentally ready for a particularly abstract concept.</p>	<p><i>Yes!! These activities and the CMP lessons are written to meet the needs of a range of students. It's important to determine what it is that you want ALL children to come away with and I would be surprised if it was mastery of each component!</i></p> <p><i>Excellent job on this entry!</i></p>

Question for Further Thought

At the end of Session 5, participants will email you their response to the following questions:

Share one surprise (if there were any) and one significant aspect of the interview that you think will inform your teaching and other people's teaching. In particular, tell us:

1. What about your students' thinking really struck you as significant?
2. To what extent did you see your students able to make generalizations and justify them?
3. What might you do in the classroom as a result of what you've learned?

Review and respond to participant's ideas. Keep in mind that the purpose of the student interview is for the teacher to gain information about how the student is thinking. Unlike other teacher/student interactions, its purpose is NOT to guide students to a solution. This is often a shift for teachers, depending upon their philosophy and style. In instances where responses seem to stress moving them to a solution, you might pose some questions such as "What algebraic thinking did you notice in the students work or explanation?"

If participants include other questions for further thought, you can provide feedback to those if you wish. This feedback response will likely be brief, but will enable you to gauge how participants are doing and provide a check-in opportunity.

What Questions Did You Ask?

Responses will vary, depending upon a participant's approach to the problems. The chart that follows outlines some of the questions from Driscoll's book. Be sure that you have done this task yourself first before responding to participants. Your experience will inform your feedback. You can draw similarities to their experiences and contrast differences in approach.

Sessions	1	2	6	6	6
	Crossing the River	Painting Faces	The Bus Stop Queue	The Bicycle Journey	Lins Go On An Outing
How is this number in the sequence related to the one that came before?					
What if I start at the end?					
Which process reverses the one I'm using?					
Can I decompose this number or expression into helpful components?					
Is there a rule or relationship here?					
How does the rule work, and how is it helpful?					
Why does the rule work the way it does?					
How are things changing?					
Is there information here that lets me predict what's going to happen?					
Does my rule work for all cases?					
What steps am I doing over and over?					
Can I write down a mechanical rule that will do this job once and for all?					
How can I describe the steps without using specific inputs?					
When I do the same thing with different numbers, what still holds true? What changes?					

Now that I have an equation, how do the numbers (parameters) in the equation relate to the problem context?					
How is this calculating situation like/unlike that one?					
How can I predict what's going to happen without doing all the calculations?					
What are my operation shortcut options for getting from here to there?					
What are other ways to write that expression that will bring out hidden meaning?					
How can I write the expression in terms of things I care about?					
How does this expression behave like that one?					

Final Project

The Final Project provides an opportunity for participants to connect the work of their class to their own practice and to share some of their thoughts and discoveries. The Final Project Template provides a framework for completion. In addition, the course offers suggestions about which pieces of the project to do in which session, in order to structure the completion of the project. In your email correspondence beginning in Session 6, be sure to offer reminders to participants to get started on the project, so that they do not have to do all of it in Session 8. Sometimes, participants who leave too much to the end, are not successful in thoroughly and promptly completing the Final Project.

As examples, some final projects with facilitator feedback are provided below. These are not provided as exemplars, but instead, as examples of how participants may complete the project and how a facilitator might respond.

Final Project – SAMPLE PROJECT #1

FACILITATOR COMMENTS: Thanks for sending your project. I enjoyed reading it and have included some comments in the body of the project in bolded and italicized font!

Step 1

Tiling Garden Beds

Grade : 8

If this activity is not taken from the course or the **Fostering Algebraic Thinking** book, please briefly describe the activity. If it is taken from the course, list the session or page number from the book. Please write your descriptions clearly enough that someone else could make use of it.

This is an activity that was taken from session 5 of this course.

Step 2

Describe the potential misconceptions or areas of difficulty that your students may have with the activity you selected.

Before getting to the specific areas of misconception and possible difficulties, I need to give you an overview of my classes, students and their abilities. I teach four classes of math. Three of these are heterogeneously grouped classes that are taught 8th grade math. The fourth class is the 2nd half of an Algebra 1 class, of which the first half took place during the 7th grade. These students were identified by teacher recommendation and standardized test scores, along with some parental input. The school is very diverse, both ethnically and socioeconomically. The students come from families that speak over 31 different languages. Our English language learners (ELL) are mainstreamed as early as possible. A few ELL students in my class read as low as 2nd grade level. At the same time, I have some students who are working well above the 8th grade level. ***You have your work cut out for you, don't you?*** I feel that this information is important, because I have seen that the algebraic reasoning of these students do not necessarily coincide with their placement. Several students who were not selected to take Algebra 1 have strong algebraic thinking skills. They see the patterns and make connections to tables, graphs, and equations. They developed, in a mathematical sense, over the 7th grade year, but do not have a chance to switch into the Algebra 1 class, because they have missed the first year. These kids will benefit in the long run from the type of problems that this course discussed. They will be able to make connections that some of the Algebra 1 kids

may not make. Some of my Algebra 1 students are not as confident with their algebraical reasoning. I feel that they could have been placed based less on their math skills and more because they have a strong work ethic. Basically, they were good students.

By the time I gave this activity to the Algebra 1 students, we had already done a unit on linear equations. because of this, I anticipated their work to be more advanced than the other students'.

Knowing my students, I think that the first hurdle will be to be sure that everyone understand the problem. With many low ability readers and ELL students, it is so important that they are given time to understand the problem. ***Right, setting the context of the garden and the tiles is critical.***

Secondly, I anticipate that the kids will try to take short cuts, such as doubling the number of tiles when you double the garden length. This took place on the video that we watched and I am sure that it is a common misconception.

I think that the most common difficulty that I will have with this activity is that many students will see that increase of 2 tiles for every garden length, but will be unable to develop or express a rule. All students have difficulty discussing the nth term. That concept will need to be reviewed and clarified.

At the end of the activity, the students are asked to work backwards to develop the garden length from the number of tiles. I anticipate several, "How do I do that?" type of questions.

There is also the possibility that some students will quickly understand and complete this work. In order to accommodate all learners, I will need to develop extensions for these kids.

Great anticipatory thinking! You've compiled on a strong list of potential difficulties.

Step 3

Describe the goal of your discussion:

- a) What do you hope students will gain from the discussion?
- b) How will you decide when the discussion is "done?"

The goals of my discussion are twofold. I want to see where my students are with their algebraical reasoning skills. I want to take that information and bring them to the next step. Secondly, I want to help students come up with their own "questions" to ask themselves when they are given problems such as this one. In other words, I want to give them strategies to reason their way through a similar problem if I am not standing over them. ***Great idea to model questioning in order to help them internalize this technique as a problem-solving tool. Love it!***

List 4-6 possible discussion questions that you might use during the discussion. Your discussion questions should try to target some of the areas of misconception that you described in Step 3. Refer to the framework for Categories of Teacher Questions on p. 6 of Fostering Algebraic Thinking.

*What's the problem asking of you? (Clarifying)

*What is going on in the three frames? (Clarifying)

*How did you get that number? (Wrong answer) (Orienting)

*How can you check your answer? Does the tile length double when you double a garden length of 2 to 4? ***Why? Why Not?*** (Orienting) ***nice***

*Why did you reach different conclusions? (When two students come up with correct , but different equations?) (Prompting Mathematical Reflection)

*Can you explain in words how the garden is growing? What if you had a garden of 10, 15, 100, 200? What is changing from one picture to the next? What is staying the same from one picture to the next? (Eliciting Algebraic Thinking) **Excellent!**

* Can you find any connections between your equation and the table? What would this look like graphed? What would this pattern be if the garden grew as a square? Can you come up with an equation for this situation? (Extending)

Beautiful progression of questions.

Step 4

Conducting Your Discussion

Think/Pair/Share and Whole Class

I chose to do a Think /Pair/Share and a whole class discussion. I felt that this gave my students the time that they needed to think about the problem, to be able to all discuss and share their results and finally to discuss any connections or extensions that I saw to be appropriate.

I am reporting out on all of the classes to whom I gave this activity. This includes both the Algebra 1 class and the general grade 8 classes. Although they meet at different times and have different curriculum, I wanted to see all of the students' algebraic thinking. As I discuss the results though, I am going to refer to different classes, because the discussions were very different.

The suspense is building.....I'm so eager to read on!!

Step 5

Write a reflection on your discussion:

- To what extent do you think you reached your goal for the discussion? How satisfied were you with the discussion?
- What happened during your discussion? What questions did you ask? What new questions came up?
- What evidence of algebraic thinking did you notice in your students' responses?
- What will you do differently next time?

Reflection

I feel pleased with this lesson, and the discussion that resulted from the lesson. I was able to see where different people were algebraically. Because of the think/pair/share setup, I was able to listen to the partner discussions, enhance those discussions by questioning, and finally to lead a class discussion that resulted in deeper understanding of the problem. I felt that I met the goals that I set for myself before the lesson.

As expected a few of the students did not understand the question. I visited tables where I knew this could be a problem, so that those students would get a chance to solve the problem. Upon clarification, I encouraged those students to think independently for a few minutes so that they could formulate a response. I gave the students 10 - 15 minutes to work independently. I walked about the class at this time to be sure that everyone understood the problem, yet I tried to let them think for that time period. My pencil was at my desk! After the thinking session, I visited tables and listened to their discussions. Each person reported his/her thinking, then the other person was allowed to ask questions for clarification. ***Nice job!***

I slowly went through the groups to look for misconceptions. In general, the tile numbers for the three gardens shown was correct. Some students had included the garden, but I clarified the problem

them and they went forward. Many students had not noticed that the garden length went from 3 to 6 then 8. They had just continued counting by twos, filling out 14 and 16 respectively for 6 and 8. I took out some graph paper for them and asked them to draw out the "6" garden. They immediately saw their problem and, usually by counting by two's, corrected their mistake. To get to the "15" garden, many of the kids continued to count by 2's to get to 36 tiles. By far the most common mistake was the doubling of the "15" garden to get to the "30" garden. I asked one girl to explain how she got 72 for a "30" garden. She explained that she had doubled the result of the "15" garden. I asked her why that works. She was not clear. I asked her if the "6" garden was twice the "3" garden. She went back to the table and saw that it clearly wasn't true. She hesitated. She then started to count by 2's again. I asked her if she would do that for a garden of 100. She thought it might take a long time. At this point I asked her what was changing from frame to frame. I then asked her what was staying the same. She thought for a minute and began to see the way to formulate a rule. "Oh, these ends are staying the same. The middle part is getting bigger." So I asked how she would find the "30" garden. She immediately said $30 + 30 +$ the ends. She eventually wrote $L \square 2 + 6$. ***This is great that she could relate the model to her rule. This is one of the things that makes these tasks so powerful. What is changing/staying the same as the garden changes???***

From that class another girl wrote $L + 2 \times 2 + 2$. I asked her to explain this. As she did, I told her that if I started to perform those operations, I would begin with the 2×2 , at which point she quickly added parentheses to make an expression of $((L + 2) \square 2) + 2$. The discussions among each group was interesting. Some of the kids, who were not able to get past the change of two for every length of garden, were able to discuss different patterns that they saw. I tried to get to those students who had not formulated a rule. I asked them to draw the garden on graph paper of various lengths. I let them block out the areas that did not change. As we made several different lengths, they began to see a rule and were able to verbalize it. All of this helped me to formulate the next days lesson, which was Octagonal trains, in the hope that each student would get to that next step. It also helped me to know who I should zero in on first for help.

Our discussion started with all of the expressions on the board. The students came to a point where they understood how to get each of them. I had to coax and question them to see that they were all equivalent expressions. That was not as important to me, however, as the ability to use symbols and formulate their rule.

The next time that I do this activity, I will have introduced the 8th grade class to constant change over time, or slope. I don't think that you need to do this. The students will certainly benefit from the activity alone. However, I think for those students who are able to look deeper, it is a huge bridge into the traditional algebra curriculum, where most of these students are heading. I also would have had an extension for the problem, such as adding a third dimension or changing the dimensions of the garden, in order to keep the algebraic reasoning developing for those kids who saw the pattern immediately.

The Algebra class very quickly tried to come up with an expression. A boy who completely struggles with the concepts in Algebra in this same class, was done in less than a minute. He could not believe how easy it was. His expression of $2x + 6$ worked and he could manipulate it forwards and backwards. This particular boy has one of the best mathematical minds that I have taught, yet he puts up blocks against solving equations in an algebraic way. It is interesting that he, who struggles so much, completely understood the activity.

Isn't this amazing! It goes so far to support the idea of different learning styles, doesn't it? So often, I've experienced kids who struggle in math really get this. Conversely, kids who have done well in math sometimes struggle when they have to think on their own.

A second person, a girl who is also at the top of the class, started by only looking at the table.

She just could not come up with an expression using that table. I tried to redirect her to the pictures, hoping that it would help. I asked her to think about what was changing and what was staying the same. All of a sudden it became so easy for her and she said the ends stay the same - there are 6 - and the length changes. It can be so much more difficult for some to go from the table to an expression, rather than a picture to an expression.

My final discussion was particularly interesting with this class. We put all of the expressions on the board and analyzed them. I asked each person to explain his/her thinking to the class. All of the expressions in the Tiling Garden Beds Animation were there with the exception of $2(n+3)$, yet they added one more. This came from a student who is identified as gifted and talented. I would have expected that he would have quickly seen the $2x + 6$ expression. Instead he muddled for a while in order to come up with this: In the garden, each end square touched 5 tiles. Each of the interior garden squares touched only 2- one above and one below. Therefore, his expression was $2(n-2) + 2(5)$.

I love this task for talking about equivalent expressions. Look at all that came out of this discussion! Wow!

At one point someone said that each of the expressions meant the same thing. So we looked at each one and saw that the simplified version was $2x + 6$. Because they had been studied $y = mx + b$, I wanted to try to connect to this activity to it. I first wrote the table up on the board. I asked them what it would look like if we graphed the data. As a group we put it on the coordinate plane to discuss. I asked them what they saw, and they said "Points that make up a line." I drew the line, connecting the points only. I asked what we knew about straight lines. All of a sudden it was as if a flood gate had been unleashed. They started making connections! They saw the slope in the graph. We went back to the table and found that it was the change over a unit. For every additional garden length, the tiles went up by 2. Of course that would be the slope. Someone asked to extend the line to the y-axis to find the intercept. Again, an a-ha moment. Of course the y-intercept is where x equals 0 - in this case when there is no garden length! It was a great class. All students were focused and trying to make connections. It showed me the importance of algebraic thinking for ALL students. It also showed me the importance of going back to reasoning as well as the traditional $y = mx + b$ work. They need to see the patterns that correspond to the equations, tables and graphs. Unless they are given time to investigate, they can't develop these reasoning skills.

The next time that I do this activity with this level class, I think that I would follow it up with a non-linear problem for comparison. It is important for the students to see this difference. I would also take a problem that perhaps changes 3 for every 2 units. That could also enhance the discussion, particularly looking at the table.

Nice extension idea to help them notice the characteristic difference between these two.

I really enjoyed this type of activity and I cannot emphasize enough the benefit of the think/pair/share. It gives the student the opportunity to think independently, to express and communicate his/her thinking, and to be exposed to other's thinking. It gives the teacher the opportunity to question, to correct misconceptions, and to help the students move along the continuum towards a deeper understanding of algebraic thinking.

Thanks very much for your thorough and thoughtful sharing of your experiences. This was beautifully done and I really enjoyed reading it.

Final Project –Sample Project #2

FACILITATOR COMMENTS: Thanks for sending your project. I enjoyed reading it and have included some comments in the body of the project in bold italicized font.

“Painting Faces” (Activity from Lesson 2) Grades: Broad Range but good for 6-8

Potential Misconceptions or Difficulties

The potential misconceptions that I anticipate my students may have with this activity are: 1) trying to base their predictions of the number of stamps needed for a longer length upon the numbers already charted (i.e. if a rod with a length of 7 requires 30 paint squares, 14 would require 2×7 or 14 squares); 2) when working backwards from the number of paint squares to solving for the length, they will try to divide before subtracting. I foresee difficulties in coming up with an actual equation, even in word form. I then see possible difficulties in writing the equation correctly when working backwards; even those who have little difficulty coming up with the rule, may forget parentheses. For some of my students, I will need to check to be sure that their charts are completely correct before they work on a rule.

Great anticipatory thinking here! I’m curious to read how it turned out.

Goal of My Discussion

I hope my students will gain a better understanding of discovering a rule based on the careful observation of steps that are repeated and changes that have something in common. I also hope to have students hear responses from others that will help them grasp the concepts; perhaps hearing the same thing in various ways will help. I hope to see the process of working backwards reinforced along with the awareness of order of operations.

I will decide that discussion is done, unfortunately, when the period ends. I will continue discussion from my notes the next day, but fear that it may not be quite as effective. Ideally, I hope to continue until I feel like every student can express understanding of at least parts of the problem solving.

The time is always tricky, isn’t it. I agree it’s a challenge to pick up the next day and maintain the flow of the discussion. I love to see that you hope the discussion continues to build their understanding. This is not just an opportunity to assess what they brought out of the investigation and it is great to be listening for changes in their thinking that are inspired by the conversation!

Potential Questions for Discussion

1. Does your rule work for all the data in the table?
2. In looking at several rod lengths, what do you see that is the same for all of them? ***Great question for getting to variable/constant distinction!***
3. How many of the faces on each cube are always painted? ***I like this always kind of question. This is great to reinforcing mathematical statements, too. Another related question is “How many***

faces on each cube are painted. This kind of “it depends” question also helps them to define the pattern.

4. Describe your rule to us using the manipulatives as visual aids. *Nice way to have them connecting rule to the model --- essential, don't you think?*
5. Write several of their equations on the board and have them guide me in working through them to see if they work. What must I do next, and next, etc.?
6. How can I work backwards or “undo” this process?

Awesome collection of questions,! I wish I could be a student in your class!

Class Discussion

I usually combine my whole group discussion with some small group discussion, both with a “Think-Pair-Share” model and a “Numbered-Heads Together” model. This time, since they were already working in pairs, I used Think-Pair-Share” for some of my discussion questions and a whole group approach (with several students answering the same questions) for the rest.

Reflection

As I had anticipated, time was an issue for completing the discussion, although we were able to get through a complete piece with some closure. I feel that my goal was partially reached; somehow I need to facilitate discussion to be more efficient and able to include every student individually at some level. I was satisfied with much of the discussion, but do not feel that I have an excellent handle on every student's thinking. *So hard to do with limited time --- an even with unlimited time is tough to do this and keep everyone reasonably engaged. Maybe setting a goal of over x discussions (on different lessons), I hope to hear from everyone and understand their thinking is a more attainable goal.*

The discussion went surprisingly according to my expectations. Students were excited to share and even “wrong” thinking became teaching moments, many times through the responses of other students. I am not aware of any questions I asked other than those already listed, but I feel like I asked them many times and several different ways. We did end up examining the rods and breaking down the numbers in the table to better see what approximately half of the students had already been able to express as a rule, and many as an equation.

I noticed the algebraic thinking most in the ability of several students in each of my classes being able to state quite clearly the backwards thinking needed to solve the problems in which the given data was the number of painted squares. With a couple of well-placed (I think) questions, they were able to even write the equation correctly so that anyone could use it (use of order of operations). Although there were still many who could not write the rule in an algebraic equation, they were able to articulate it to the point where we could examine it together to write the rule with algebraic symbols. A lot of quickly raised hands with “Oh, I know!” responses made me feel good about that piece.

This, to me, is suggestive of a great lesson because it allowed you to individualize instruction in a way that allowed all to have success but for some to take the task to higher level of sophistication. Is it important that they all got all of the assignment, or rather that they all got part of the assignment and some could take an additional challenge? I would expect these expectations to evolve as the class has more experience with these tasks, Great work!

I don't think I would do very much differently next time except to maybe do a little less talking myself. During the original activity time, I think I would like to sit with each group and have "mini" discussions with them. This would require having something constructive for the others to do, perhaps a computer activity or a writing assignments asking them to respond to some of the questions that would be discussed later. For my schedule, this whole group discussion would have to wait until the next day to complete it well. The more I think about it, the more I think that having all students write responses to the questions for discussion would work out well for our schedule, even if that task is a homework assignment.

Sounds great! I loved reading about your experience. Excellent job on this project. Thanks for sharing it and for all of your outstanding contributions to this course!

Facilitation Resources

- Course Calendar Template
- Sample Email Announcements
- Assignment Record

Course Calendar

Building Algebraic Thinking Through Number, Pattern and Function

Session	Session Start Date	Session End Date	Assignments to Submit by Email	Due Date
1			Opening Reflection Question for Further Thought (optional)	
2			Question for Further Thought (optional)	
3			Question for Further Thought (optional)	
4			Learning Log 1 Question for Further Thought (optional)	
5			Question for Further Thought (required)	
6			What Questions Did You Ask? Learning Log 2 Question for Further Thought (optional)	
7			Question for Further Thought (optional)	
8			Learning Log Final Project	

Sample Email Announcements

This section provides sample text for corresponding with participants. Feel free to cut and paste this sample text and edit it to meet your needs. In many instances, you will need to tailor the email for your particular situation. Remember that your email communication is your best opportunity for connecting with participants and building their interest in the course.

Course Invitation Email (To be sent out to recruit course participants)

Dear Teacher/Colleague,

I am writing to announce an exciting professional development opportunity. In the coming weeks/months, I will be facilitating an on-line course examining the topic of algebraic thinking. It is an eight-week course for teachers of 6-10th grade mathematics and addresses the following questions: What is algebraic thinking? How is algebraic thinking important in learning mathematics? How can teachers foster their students' algebraic thinking?

The course, entitled Building Algebraic Thinking Through Pattern Number and Function, will enable us to explore together a collection of math activities and instructional strategies to promote algebraic thinking. The course will require about 5 hours of work each week, and since it is an on-line course, you have flexibility in scheduling the week's coursework around your schedule. Upon successful completion of the course, you will receive _____.

I would like to begin the course in ____ [insert date] ____ to run 8 weeks and end on ____ [insert date].

If you would like to learn more about the course, please visit [insert link]

I am excited about this opportunity and hope you will consider participation in this course. Please email me if you are interested in enrolling or if you have any questions.

Sincerely,

[your name]

Course Enrollment Acknowledgment Email

Dear __ [insert name] __,

Thank you for enrolling in Building Algebraic Thinking Through Pattern, Number and Function. This promises to be a great experience and I look forward to working with you as I facilitate the course. I want to inform you of some logistical issues. The course will begin one _____ and end on _____. You will have some assignments to turn in to me for review and feedback along the way. Course requirement details are provided in the link to the course and I will send reminders.

My primary vehicle for communication will be email and I hope you will feel free to write to me with any questions or problems. I can also be reached by telephone at _____.

To access the course, [Insert Instructions for accessing the course]. I will be in touch again in advance of the start of Session One.

Sincerely,

[your name]

Session One Announcement Email

Welcome to Session One of Building Algebraic Thinking through Pattern and Function! You have three primary tasks to complete in this session: 1) orient yourself to the online learning space and course

expectations, 2) introduce yourself to me, your facilitator and 3) start to explore the concept of algebraic thinking.

Familiarize yourself with the technical requirements of the course, explore the course website, and read the course goals and expectations. As part of the orientation, you will email an introduction to your facilitator(s). Please include a little bit of background about yourself and describe an "aha!" moment either you or one of your students had around algebra and how you got there.

During this session, we will also dive into the course material. You will do two short readings, a math activity, and a short analysis of the types of thinking you used to complete the math activity. The question for further thought focuses on creating your own definition of algebraic thinking.

Feel free to email me with questions or concerns.

Email About Math Tasks (to be mailed 2-3 days after the start of Session One)

Hello everyone,

In most of the sessions, there will be at least one math problem for you to complete; please complete it as mathematics learner yourself, rather than as your students would complete it, since we want to first focus on how you think mathematically. As you are getting into Session 1, you'll see that the math problem for this session is called "Crossing the River."

If you scroll down the page of the session, you will see a solutions button. This is provided so that you can check your answers after you've tried the problem, or so you can see one way to solve it.

I do encourage you to try the problem on your own first before checking the solutions. As we explore the nature of algebraic thinking, it will be both helpful and important for you to be able to reflect on the way you thought about the problem yourself.

If you are feeling stuck at any point with a math problem, please feel absolutely free to email me and I'll be happy to help.

Thanks

[your name]

Session Two Introduction Email

Welcome to Session 2! Thank you all for your participation in Session 1. I appreciate the energy, enthusiasm and thoughtfulness you put into your introductions..

Session Two has begun and runs from [insert date]. This session asks you to focus on your own thinking when doing patterns problems. (Session Three will shift to focusing on your students' thinking about patterns). You'll read an article about patterns, engage in solving a few patterns problems, and consider

what strategies you used to solve them. In the question for further thought you'll reflect on your approaches solving the patterns problems presented in this session.

Additionally, sometime between now and the start of Session Four, please plan to have your students complete one of the patterns problems presented in this session. You may choose to use it with your whole class or a smaller group of students. Additional details on this assignment can be found in the Session Two folder within the "Sessions" area of our course.

Let me know if you have any questions or concerns, and have fun with Session 2!

[your name]

Session Three Introduction Email

Session 3... here we come!

Today we will begin Session 3, which runs from [insert date] . In Session 2, we asked you to do the patterns activities and focus on your own thinking, and now we will shift over to looking at students' thinking about patterns. In this session you will read two short articles and then examine some sample student work. You will also watch a video of those same students explaining their thinking process. Please note - the videos are found on the CD that was mailed to you along with Fostering Algebraic Thinking. If you have any trouble seeing the video, let your facilitator know as soon as possible. The question for further thought this week centers around comparing your initial reaction to the student work to the interviews and what kind of algebraic thinking you saw in both.

I am attaching the class list under "Orientation". There you will find an attachment for participants' names, schools, and the grades they work with.

If you haven't already, plan to have your students complete one of the patterns problems presented in this session before the start of Session 4 (insert date). You may choose to use it with your whole class or a smaller group of students. Additional details on this assignment can be found in the Session 2 assignments area.

Let me know if you have any questions or concerns, and have fun with Session 3!

[your name]

Session Four Introduction Email

Welcome to Session 4 and nearing the halfway point for the course!

Session 4 runs from [insert dates] and focuses on developing good questioning techniques that help students develop algebraic thinking. You'll read an article on how a teacher developed his current questioning techniques and discuss your reactions. You'll do an activity on a Letter V pattern, watch a video of a teacher conducting a classroom discussion about a similar activity, and analyze the teacher's questions. Last, you'll write about the experience of giving your students one of the math activities from Session 2. You will write about this in your Learning Log - the first of three such assignments. The Learning Log is a Word document you will download and then email to me at this address before the end of the Session. Since this is the first time we are doing this, please don't hesitate to contact me with any issues.

Thanks again for your dedication to this course. I hope you enjoy Session 4, and as always, let us know if you have issues or questions.

[your name]

Session Five Introduction Email

Here we are at Session 5, delving into the second half of the course! We hope you are enjoying it so far and are gaining insights and knowledge that will benefit your teaching and your students' learning.

This session will run from [insert dates].

Thanks to those of you who sent in your Learning Log already from Session 4! If you haven't yet sent in your Log, please do so as soon as possible. Don't hesitate to contact me for help with it. Also, keep up your work on the session math problems and the questions for further thought. This time devoted to thought and reflection should make the course more beneficial.

*In Session 5, we ask you to conduct your student interviews. You may have already completed several of the preparation steps for the interview in previous sessions, and this week we ask you to interview the two students you chose on the Tiling Garden Beds problem. Before you do the interview, we also ask that you read chapter five of *Fostering Algebraic Thinking*, do the Tiling Garden Beds activity yourself, and watch an animation of the Tiling Garden Beds problem and a video of a sample interview. Our question for further thought this week will center around your experience doing the student interviews, so the earlier you can do them, the sooner you can work on this reflection.*

Please don't hesitate to contact me with any issues, questions, or concerns. Good luck with your interviews!

[your name]

Session Six Introduction Email

We are now into Session 6, which means that there are only 3 more sessions remaining in the course! We are more than halfway through. You should take the time to congratulate yourselves in getting thus far. We really appreciate all your participation and feedback, and hope that the course is also making a positive impact on your teaching practices as well. Keep up the good work

Moving on to Session 6 - we move away from our study of patterns and into our study of functions. This first session on functions asks you to do a short reading on student misconceptions when it comes to graphing. You will then do a series of math problems (three on paper and one using a computer applet), once again paying attention to your thinking as you complete the work. You will watch about ten minutes of video of another student interview and complete another Learning Log about the students' understanding of function from the interview. Our question for further thought this week asks you to share ideas from the session you might try with your class about student understanding of graphs. Session 6 ends on [insert date]

Please be in touch with any questions

[your name]

Session Seven Introduction Email

Session 7 will run from [insert day and date] through [insert day and date]. This session continues our work with algebraic thinking related to functions, focusing on the use of multiple representations. You will complete a series of activities - some using technology - that provide some examples of activities that help students understand the relationship between different representations of functions. You will also watch several video segments of a class discussing some of the problems and then you'll reflect upon the students' understanding. Finally, you will complete the next several steps of your final project. At the conclusion of Session 8, you will be asked to turn in a final project that involves planning, conducting and reflecting on a classroom discussion focused on algebraic thinking. Plan to conduct your discussion so that you will have time to write up your reflection by the end of Session 8.

Keep up the good work!

[your name]

Session Eight Introduction Email

Our last session, Session 8, and wrap-up will go from [insert dates]. You will complete your final project during this session, as well as fill out the final project template asking you to reflect on your experience with the final project. Additionally, there is a short learning log about algebraic thinking that asks you to refer back to our very first question for further thought and consider how that definition might have changed as a result of the experiences of the course.

Thank you again for all your hard work. I look forward to reading the final projects and assignments. I hope the completion of this work and course is rewarding. Please let me know if you have questions.

[your name]