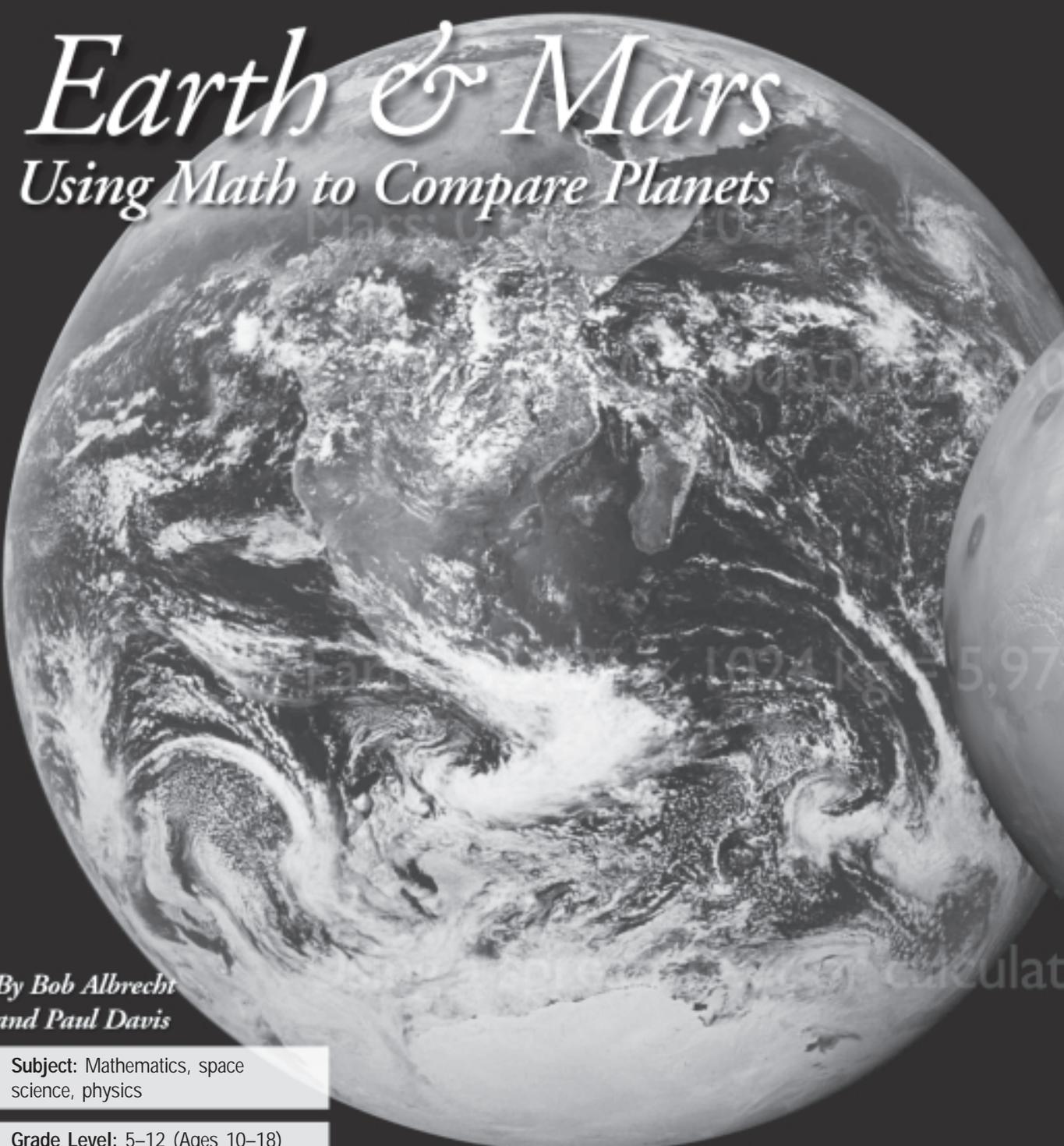


In the Curriculum

Starship Gaia

Earth & Mars

Using Math to Compare Planets



*By Bob Albrecht
and Paul Davis*

Subject: Mathematics, space science, physics

Grade Level: 5–12 (Ages 10–18)

Technology: Internet/Web, spreadsheet software, TI-83 graphing calculator (Texas Instruments)

Online Supplement: attached

Artwork modeled from *Earth & Mars: A Comparison*. From *Designing for Human Presence in Space* by Paul Wieland for NASA. Images courtesy of NASA.

Data Source

Our favorite online source of solar system data is the National Space Sciences Data Center (<http://nssdc.gsfc.nasa.gov>). It displays the latest data about the Sun, planets, and everything else that goes 'round and 'round the Sun.

The Mars Fact Sheet (<http://nssdc.gsfc.nasa.gov/planetary/factsheet/marsfact.html>) presents data about Mars and Earth side by side so you can compare the two planets. Table 1 lists planetary characteristics of Mars and Earth selected from the fact sheet. We'll use these data to pose questions, exercises, and investigations for you to ponder. Some are simple number-crunching exercises that you can do quickly with a calculator, spreadsheet, or *your* favorite power tool. Others are investigations that will require more time. Some may take a lot of time!

We'll post answers to the number-crunching exercises in the *Starship Gaia* e-mail newsletter. For the investigations, we'll share ideas, links to Internet sites, minitutorials, et cetera, et cetera. We'll also suggest more calculations and investigations. To subscribe to *Starship Gaia*, send an e-mail request to DragonFun@aol.com. Tell us in the message body, "Please send *Starship Gaia* e-mail newsletter."

Table 1. Mars and Earth data adapted from the Mars Fact Sheet of the National Space Sciences Data Center

Planetary Characteristic	Mars	Earth	Ratio (Mars/Earth)
Mass (10^{24} kg)	0.6419	5.9736	0.107
Volume (10^{10} km ³)	16.318	108.321	0.151
Equatorial radius (km)	3,397	6,378	0.533
Polar radius (km)	3,375	6,356	0.531
Volumetric mean radius (km)	3,390	6,371	0.532
Mean density (kg/m ³)	3,933	5,515	0.713
Surface gravity (m/s ²)	3.69	9.78	0.377
Sidereal rotation period (h)	24.6229	23.9345	1.029
Synodic rotation period (h)	24.6597	24.0000	1.029
Solar irradiance (W/m ²)	595	1,368	0.431

Mass

The mass of an object is a measure of how much “stuff” the object has—the quantity of matter in the object. If you look up *mass* and *matter* in a dictionary, you may find that they are defined in terms of each other, thus completing a circular definition. Mass in physics is like a postulate in geometry.

Scientists accept mass as a fundamental physical quantity and use it to explore and describe the universe. Mass is also a measure of an object’s *inertia* (its resistance to a force applied to the object in an effort to change its state of motion). Mass is not weight. Weight is the force of gravity on an object. An object’s mass is the same on Earth and Mars, but its weight is different on the two planets.

In Table 1, mass is given in the unit 10^{24} kg. Here 10^{24} means 10 to the 24th power, or 1 followed by 24 zeros, and “kg” is the abbreviation for kilogram. So 10^{24} kg equals 1,000,000,000,000,000,000,000,000 kilograms. The masses given in the table are:

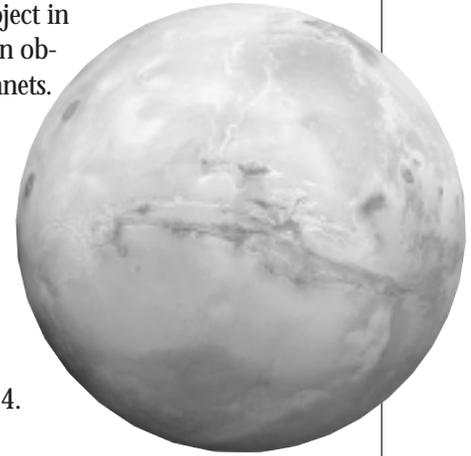
Mars: 0.6419×10^{24} kg =
641,900,000,000,000,000,000,000 kilograms.

Earth: 5.9736×10^{24} kg = 5,973,600,000,000,000,000,000,000 kilograms.

Using a spreadsheet or calculator, enter Mars’s mass as 0.6419E24 and Earth’s as 5.9736E24.

Your Turn:

1. Calculate the ratio of Mars’s mass to Earth’s and compare your answer to the ratio shown in Table 1.



Shape & Size

Earth and Mars are almost, but not quite, spheres. Both are a little fatter at the equator than at the poles, so the equatorial radius of each planet is greater than the polar radius. The equatorial and polar radii of Earth and Mars are shown in Table 1.

2. What is the equatorial diameter of Mars? of Earth?
3. What is the polar diameter of Mars? of Earth?
4. Investigation. Why are the equatorial diameters of Mars and Earth greater than their polar diameters?

Assume that Earth’s equator is a circle and you are a super athlete. You walk and swim around Earth along the equator. After performing this stupendous feat, you then start at the North Pole and snowshoe, walk, and swim along a meridian (great circle) around the South Pole and back to your starting point.

5. In circumnavigating Earth along the equator, how far do you walk and swim?
6. In circling Earth along a meridian, how far do you snowshoe, walk, and swim?

7. Investigation. In circumnavigating Earth, what countries do you pass through? What oceans, lakes, and rivers do you cross? What mountains do you climb? What towns and cities do you travel through?
8. You go to Mars and lope around its equator. How far do you travel?
9. Still fresh as a daisy, you circumnavigate Mars along a meridian. This takes you across both the north and south polar caps. How far do you walk and snowshoe?
10. Investigation. In circumnavigating Mars, what mountains do you climb? What craters do you cross? What valleys (sinuous valleys; singular: vallis) do you cross? Do you cross any oceans, lakes, or rivers?

There are 360 degrees in a circle. As you circumnavigate a planet along its equator or a meridian, you turn through 360 degrees relative to the planet’s center.

11. How far (in km) do you travel per degree along Earth’s equator? along an Earth meridian?

Continued on page 45.

Volume

Table 1's unit for volume is 10^{10} km³, which means 10 to the 10th cubic kilometers. The table also shows the *volumetric mean radius* for each planet. This is the radius of a sphere that has the same volume as the planet. The volume of a sphere of radius R is $(4/3)\pi R^3$. Earth's volumetric radius is 6,371 km. We'll use our TI-83 graphing calculator to calculate Earth's volume:

TI-83: [() 4 [+] 3 [)] [2nd][π] 6371 [^]
3 [ENTER]

Answer: 1.083206917E12 km³

Because 6,371 has only four significant digits, we can claim only four significant digits in our answer and round it to 1.083E12 km³, very close to NSSDC's value shown in Table 1.

16. The volumetric mean radius of Mars is 3,390 km. Calculate the volume of Mars and compare your answer with the value in Table 1.

Density

The density of a planet is its mass divided by its volume, shown in Table 1 as kg/m³. So we need to convert Earth's volume from km³ to m³. Abracadabra! $1.08321E12$ km³ = $1.08321E21$ m³. Please verify our magic while we calculate Earth's density.

TI-83: 5.9736E24 [+] 1.08321E21 [ENTER]

Answer: 5514.720137 kg/m³

Rounded to four digits, we get 5,515 kg/m³, the same as the value in the table.

17. Use the mass and volume of Mars to calculate its density. Compare your answer to the value given in Table 1.

Rotation

Earth rotates on its polar axis, an imaginary line passing through its north and south poles. Suppose you are standing on the equator. What is your speed because of Earth's rotation? To find out, divide Earth's equatorial circumference by the time for one rotation. Now here's a boggler: you'll divide by Earth's *sidereal rotation period*, which is 23.9345 hours (23h 56m 4s). This is the time it takes Earth to make one rotation *relative to the fixed stars*. Earth's *synodic rotation period*, its rotation period relative to the Sun, is 24 hours.

TI-83: 2 [2nd][π] 6378 [+] 23.9345
[ENTER]

Answer: 1674.32601 km/h

Zounds! Just standing there doing nothing you're traveling 1,674 kilometers per hour (1,040 miles per hour). And you're not even tired!

18. Convert 1,674 kilometers per hour to meters per second. (Answer: 465 m/s)
19. Suppose you are standing on Mars's equator. How fast are you traveling because of the rotation of Mars in kilometers per hour? in meters per second?
20. Circles of latitude get smaller as you go from the equator toward either pole, but the rotational angular velocity is the same at every latitude (well, it's zero at a pole). For Earth or Mars or both, make a table showing:
- latitude from 0° to 90° in 10° increments, and
 - the speed at which you are traveling at that latitude in kilometers per hour and meters per second.

Again, a spreadsheet is a great tool for this task.

Shape & Size continued from page 44.

12. How far (in km) do you travel per degree along Mars's equator? along a Mars meridian?
13. Extra for the trigonometrically unchallenged: In Shuhaw Hall at Santa Rosa Junior College in California, a benchmark is located at 122° 43' 10" west longitude, 38° 27' 20" north latitude. If you go around the meridian, how far do you travel? If you follow the latitude until you return to your starting point, how far do you travel?
14. Make a table showing:
- latitude from 0° to 90° in 10° increments,
 - radius of each circle of latitude in km,
 - circumference of each circle of latitude in km, and
 - distance in km/degree in traveling around each circle of latitude.

A spreadsheet is a great tool for this task.

15. Do "#14" for Mars.

Surface Gravity

To calculate the acceleration because of gravity on a planet's surface, use the equation $g = GM/R^2$, where G is the gravitational constant ($6.673E-11 \text{ N}\cdot\text{m}^2/\text{kg}^2$), M is the planet's mass in kilograms, and R is the planet's radius in meters. Let's do it for Earth at the equator:

```
TI-83: 6.673E-11 [×] 5.9736E24 [÷]
6378E3 [x²] [ENTER]
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Answer: 9.799146112 m/s²

Rounded to four significant digits, $g = 9.799 \text{ m/s}^2$, close to the value in Table 1. The value in the table allows for the effects of Earth's rotation. Earth's gravity varies from 9.78 m/s^2 at the equator to 9.83 m/s^2 at the poles.

21. Using the mass and polar radius given in Table 1, calculate Earth's gravity at its poles.
22. Calculate Mars's gravity at its equator and at its poles.

Solar Irradiance

Our benevolent Sun bestows radiant energy on Earth's surface at the rate of 1,368 watts per square meter (W/m^2). A solar cell array can convert 10%–15% of this energy to electricity—without polluting Earth's life support system. At Mars, the solar irradiance is only 595 W/m^2 . Mars is about 1.5 times as far from the Sun as Earth. Solar irradiance decreases as the inverse square of the distance from the source of radiation, so the value at Mars is about $1/1.5^2$ as much as at Earth.

23. Using Earth's solar irradiance, the Mars-to-Earth distance from the Sun ratio (1.5), and the inverse square relationship, calculate the solar irradiance on Mars and compare your answer with the value given in Table 1.
24. If a solar cell array can convert 15% of the solar irradiance to electricity, how many watts per square meter will it produce on Earth? on Mars?

Atmospheric Pressure & Composition

On Earth, the air you breathe is about 78% nitrogen, 21% oxygen, and small percentages of other gases at a sea-level pressure of 1,014 *millibars* (mbar). This is 1 *Earth atmosphere*. The atmosphere on Mars is about 95% carbon dioxide, 3% nitrogen, and small percentages of other gases at a pressure of about 6.1 millibars, less than 1% of Earth's. The average temperature is $-63 \text{ }^\circ\text{C}$ ($-81.4 \text{ }^\circ\text{F}$). To go for a walk on Mars, you must wear a walkabout suit much like a spacesuit. It provides air, water, and snacks, and maintains a comfortable temperature as you explore Mars on foot. Table 2 shows the surface pressures and compositions of the atmospheres of Earth and Mars.

The first people living on Mars will spend much of their time in habitats that provide the necessities and some of the amenities of life. They will obtain water from various sources, then split water into hydrogen and oxygen. Air miners will extract carbon dioxide and nitrogen from the atmosphere. These Martians can supply their habitats with Earth-like air at a pressure of 1,014 mbar. They can combine hydrogen obtained from water with carbon extracted from the atmosphere to make methane for use as rocket fuel, rover fuel, and other purposes.

Continued on page 47.

Table 2. Atmospheric pressure and composition of Mars and Earth

Atmospheric characteristic	Mars	Earth
Average pressure (mbar)		1,014
Composition (%)	6.1	1,014
Carbon dioxide, CO ₂	95.32	0.035
Nitrogen, N ₂	2.7	78.084
Argon, Ar	1.6	0.934
Oxygen, O ₂	0.13	20.946
Water vapor, H ₂ O	0.021	1

References

Zubrin, R., & Wagner, R. (1997). *The case for Mars: The plan to settle the red planet and why we must*. New York: Simon & Schuster.



Bob Albrecht (*dragonfun@aol.com*) is a writer and developer of science, math, and technology curricula. George Firedrake is his alter ego and takes the form of a dragon. Laran Stardrake, whose quotes sometimes lead off this column, is another of Bob's "accomplices." She's half-dragon and half-human. As Laran is fond of saying, "Reality expands to fill the available fantasies." The DragonFun image is from an original painting by Marcy Kier-Hawthorne.

Atmospheric Pressure & Composition continued from page 46.

25. Investigation. Where on Mars is water likely to be found and how can it be obtained?
26. Investigation. How can water (H_2O) be split into hydrogen (H) and oxygen (O)?
27. Investigation. How much oxygen does an average human adult consume per Martian solar day (24h 39m 35s)? How much water? How can this oxygen and water be recycled?
28. Investigation. In a habitat supplied with air that is 20% oxygen, 78% nitrogen, 1% argon, and 1% water vapor at a pressure of 1,014 millibars, what is the density of the air in kilograms per cubic meter? What is the total mass of the air in a habitat that has a volume of 1,000 cubic meters?
29. Investigation. Humans inhale oxygen and exhale carbon dioxide. How much carbon dioxide does one human produce per hour? In a closed environment, excess carbon dioxide must be removed from the air. How can this be done?
30. Investigation. Martians will grow plants for food. Plants consume carbon dioxide and give off oxygen. How much carbon dioxide does a bunch of plants consume per hour? How much oxygen does it produce per hour? Please quantify bunch (e.g., romaine lettuce with a total leafy surface area of one meter).
31. Investigation. How can atmospheric carbon dioxide (CO_2) be split into carbon (C) and oxygen (O)?
32. Investigation. How can methane (CH_4) be made from carbon (C) and hydrogen (H)? To burn methane as a fuel, what else is needed? When methane is burned, what are the exhaust products? Do they pollute the atmosphere of Mars?

You can find answers on the Internet and in books such as Bob Zubrin's *Case for Mars*. We'll work on answers and share our ideas in the *Starship Gaia* e-mail newsletter. May dragons of good fortune be with you as you investigate, investigate, investigate.



Paul Davis (ctec1@mchs.srsc.k12.ca.us) is a mathematics teacher at Maria Carrillo High School in Santa Rosa, California. He was a happy, normal teacher until he met Bob Albrecht and George Fire Drake in 1992 and became another one of their accomplices. Since then, Paul has been intertwining math, science, and technology in his classroom with the help of Bob and George.