REVERSE • ESREVER

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REVERSE was invented by Peter Lynn Sessions and published in *People's Computer Company*, May 1973. You can play REVERSE anywhere with paper and pencil. Write your own starting list or copy one of ours. You can play REVERSE on the blackboard or whiteboard in the classroom. You can play REVERSE alone or with friends.

- One person makes a list and the other players put the list in order. Different players may do it in different ways and require different numbers of reversings.
- Two-player cooperative game. Take turns reversing the numbers. a) No discussion is allowed. b) Discussion is allowed.
- Classroom: Teams of 3 or 4 students arrange the list in order.

Play REVERSE online at Reverse - A Game of Skill (http://www.snowcrest.net/donnelly/reverse.html).

How to Play

Begin with a list of numbers and try to put them in order with the smallest number on the left and the largest number on the right. The starting list might have three numbers or four numbers or five numbers or even more numbers. For example:

- Start with this list of five numbers: **2 5 1 4 3**
- Put the list into this order: **1 2 3 4 5**

Each turn you may reverse the first two numbers or the first three numbers or the first four numbers or all five numbers. For example, let's reverse the first three numbers, step by step.

Original list	Reverse the first 3 numbers			Final list
2 5 1 4 3	**2** **5 4 3** **1**	**2** **5 4 3** **1**	**2** **5 4 3** **1**	**1 5 2 4 3**

Here are the legal reversings for the original list shown above.

Original list: **2 5 1 4 3**
You may reverse the first 2 numbers: **<u>2 5</u> 1 4 3 → <u>5 2</u> 1 4 3**

Original list: **2 5 1 4 3**
You may reverse the first 3 numbers: **<u>2 5 1</u> 4 3 → <u>1 5 2</u> 4 3**

Original list: **2 5 1 4 3**
You may reverse the first 4 numbers: **<u>2 5 1 4</u> 3 → <u>4 1 5 2</u> 3**

Original list: **2 5 1 4 3**
You may reverse all 5 numbers: **<u>2 5 1 4 3</u> → <u>3 4 1 5 2</u>**

Practice

Start with an easy list that has only three numbers. Remember: You may reverse numbers **only from the left end of the list**. Given a list with three numbers, you may reverse the first two numbers or all three numbers. Here's an easy example:

- Start with this list of three numbers: **2 1 3**

You can put this list in order in one reversing. To do so

- Reverse the first two numbers and get: **1 2 3**

Your Turn

1. Start with this list of three numbers: **2 3 1**

You can put this list in order (1 2 3) in two reversings. How many numbers will you reverse in your first reversing? After you reverse those numbers, you'll have a modified list. How many numbers will you reverse in your second reversing?

2. Start with this list of three numbers: **1 3 2**

You can put this list in order (1 2 3) in three reversings. How many numbers will you reverse in your first reversing? How many numbers will you reverse in your second reversing? After your third reversing, is the list in order? If not, just keep reversing until the list is in order.

3. Make your own list of three numbers and put your list in order.

Try lists of four numbers. You can reverse the first two numbers, the first three numbers, or all four numbers.

1. Start with this list of four numbers: **2 3 4 1** You can do it in two reversings.
2. Start with this list of four numbers: **2 1 4 3** You can do it in three reversings.
3. Start with this list of four numbers: **1 3 4 2** You can do it in five reversings.
4. Write a note to a friend to tell her or him how to put a list of three numbers in order.
5. Write a note to a friend to tell him or her how to put a list of four numbers in order.

How Many Reversings?

You can put any scrambled list of three numbers in order in three or fewer reversings. You can put any list of four numbers in order in five or fewer reversings. You can put any list of five numbers in order in seven or fewer reversings. Try it with these lists of five numbers:

1. Original list: **4 3 2 5 1**
2. Original list: **1 5 2 4 3**
3. Original list: **2 1 4 3 5**
4. Original list: **3 5 1 2 4**
5. Original list: **3 2 5 4 1**

In general, you can put a list containing n numbers in order in $2n - 3$ or fewer moves. Please complete the following table showing the number of reversings to put the given list in order.

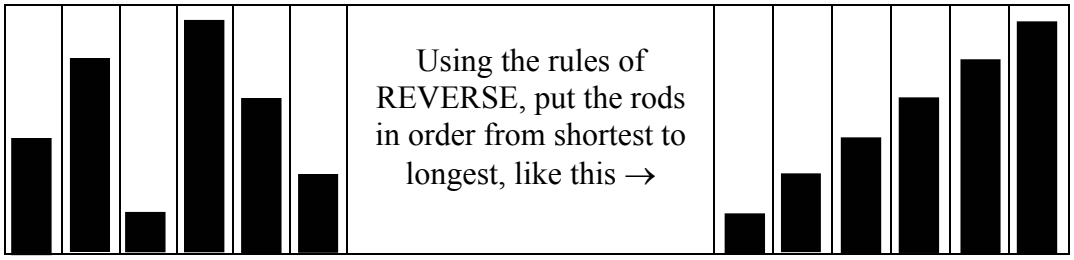
Number of numbers in the list:	**Order the list in this many or fewer reversings:**
2	1
3	3
4	5
5	7
6	
7	
8	
9	

Your Turn. Pick a scrambled list and put it in order.

1. Original list: **4 1 3 6 2 5**
2. Original list: **3 6 2 4 5 1**
3. Original list: **1 4 7 2 5 3 6**
4. Original list: **2 7 4 6 3 1 5**
5. Original list: **1 3 5 7 9 8 6 4 2**
6. Original list: **1 2 3 4 5 9 8 7 6**

Make your own lists of scrambled numbers. You and a friend can make lists for each other to put in order. You can make lists that are easy to put in order, requiring only a few moves, or lists that take many moves. You can arrange any list of n numbers in $2n - 3$ moves or fewer reversings. However, you can frequently do it in fewer moves by making partial orderings that pay off later.

- Play REVERSE on a blackboard or with paper and pencil.
- Play REVERSE with numbered tiles.
- Play REVERSE with letters instead of numbers: D F A C G B or P B M C Z Q E
- Play REVERSE with numbers from, say, -4 to 4: 2 -4 -2 3 0 4 -1 -3 1
- Play REVERSE with prime numbers: 23 11 7 79 37 2 53 61 47
- Play REVERSE with fractions: 2/3 1/4 1/2 3/5 4/7 3/8 2/5
- Play REVERSE with rods of different lengths. Arrange them from shortest to longest. If you don't have the rods, make rods with unifix cubes or multilink cubes or sticks cut to different lengths.



May dragons of good fortune be your playful companions.