

Name _____

Direct Variations

There are many relationships that two variables can have. One of these relationships is called a direct variation. Use the description and example of direct variation to help you complete this activity.

Description of direct variation:

Two variables, x and y , vary directly if you can find some number ' c ' so that $y = cx$ for every value of x .

Example

Answer this question: What is the c in this direct variation? (Hint: What number can you use to multiply each x number by to get the corresponding y number?)

Fill in the equation with the c you found: $y = \underline{\quad}x$

Directions— Here are three pairs of variations. In each pair, one is a direct variation and one is not. Examine each graph and follow the directions.

1.

- Circle the graph that shows direct variation.
- Find the constant c that relates x to y .
- Use it to write the equation that represents the graph.

2.

- Circle the graph that shows direct variation.
- Find the constant c that relates x to y .
- Use it to write the equation that represents the graph.

3.

- Circle the graph that shows direct variation.
- Find the constant c that relates x to y .
- Use it to write the equation that represents the graph.

4. Write sets of directions for how to check for direct variation.

How to check for direct variation by analyzing the graph :

How to check for direct variation by analyzing the table:

Directions— Make a table of values and graph that are an example of direct variation.

5. What is the constant of variation in your example?

6. Write the equation that represents your direct variation.

Teacher Notes for

Direct Variations

The intent of this activity is for students to use the description of a direct variation to recognize examples of direct variations and write their corresponding equations. They also come up with directions for how to check for direct variations based on tables and graphs. At the end of the activity, they come up with their own example of a direct variation.

Possible answers:

1. Students should circle the second graph and chart.
c is equal to 4
The equation is $y = 4x$.
2. Students should circle the second graph and chart.
c is equal to 1
The equation is $y = x$. There is no c in this case.
3. Students should circle the first graph and chart.
c is equal to .5 or $1/2$
The equation is $y = .5x$ or $y = x$.

Important:

Notice that x and y are used to represent variables that take on an infinite number of values, with c being used as a constant. This subtle difference in how variables are used should be brought to the students attention. This is part of algebra that rarely gets talked about, but should be in order to help students become powerful algebra users.

4. Answers will vary for this one.

Points to look for in analyzing the graph for direct variation are:

- check to see that the graph goes through the origin
- all the examples are lines

Points to look for in analyzing the table for direct variation are:

- you have to be able to find one number that you can multiply all x's by to get their corresponding y's.
- the y column must have equal intervals between the values since the x values are all going up by one.

5. Answers will vary since students will be making up their own.

6. Same as above.

Supplemental Activity:

In order for students to see the wide range of possible linear direct variations, you could have students put their examples on bigger graph paper to display around the room.

Name _____

Inverse Variations

Inverse variation is another type of relationship that two variables can have. Refer to the description and example of an inverse variation to help you complete this activity.

Description of inverse variation:

Two variables, x and y , vary inversely if you can find some number c so that each x multiplied by its corresponding y is c . Written as an equation, this translates to $xy = c$. However inverse variations are usually written in this form: $y =$

Answer this question: What's the number c for this inverse variation? (Remember that x times y must constantly equal the same c no matter what pair of x and y you choose)

Fill in the ' c ' in the equation here:

Directions– Here are three pairs of variations. In each pair, one is an inverse variation and one is not. Examine each graph and table and answer the questions.

1.

- Circle the inverse variation.
- What is the c that any pair of x times y equals?
- Write the equation that represents the graph of the inverse variation.

2.

- Circle the inverse variation.
- What is the c that any pair of x times y equals?
- Write the equation that represents the graph of the inverse variation.

3.

- Circle the inverse variation.
- What is the c that any pair of x times y equals?
- Write the equation that represents the graph of the inverse variation.

4. Come up with two sets of directions on how to check for inverse variation.

How to check for inverse variation by analyzing the graph :

How to check for inverse variation by analyzing the table:

Make up your own!!

Make up your own inverse variation by completing the table of values and the graph.

5. What is the constant of variation in your example?

6. Write the equation that represents your inverse variation.

Teacher Notes for

Inverse Variations

The intent for this activity is the same as for Direct Variations except for that this activity deals with inverse variations. The format of the whole activity is also very similar.

Possible answers:

Underneath the example, students are to complete the equation. For this one, they should notice that every pair of x and y values when multiplied together gives the result of 10.

1. Students should circle the first graph and chart.

The 'c' is 12.

The equation is $y =$

2. Students should circle the second graph and chart.

The 'c' is 20.

The equation is $y =$

3. Students should circle the first graph and chart.

The 'c' is 15.

The equation is $y =$

Note: The second graph and chart is the same as the first one except that 5 is added to every y value. This difference makes this function not an inverse variation.

4. Answers will vary but here are some points to look for:

When analyzing the graph for inverse variation:

- it is a curve
- it starts out high and then decreases, fast at first, and then more slowly
- the graph stays very close to the x and y axis

When analyzing the chart for inverse variation:

- all of the x values multiplied by their corresponding y values give the same answer
- you can tell what the constant is by looking at what y equals at x equal to one
- none of the x or y values can ever be equal to zero

5. Answers will vary as students are making up their own inverse variations.

6. Same as above.

Just as in the first activity, you could have students put their own examples of inverse variations on bigger graph paper to display in the room.

Name _____

Equation Writing

Now that you have experience recognizing and creating charts and graphs of direct and inverse variations, it's time to get more practice writing the equations that create the graphs and charts.

Remember:

Direct variation describes x and y when you can find a number c so that $y = cx$.

An inverse variation results from an equation $y =$

where c is some number.

Directions: Complete each table of values and write an equation that represents the relationship between x and y .

I	
X	Y
1	4
2	8
3	
4	16
5	
6	24
Equation:	

II	
X	Y
1	
2	
3	21
4	28
5	35
6	42
Equation:	

III	
X	Y
1	.5
2	1
3	1.5
4	
5	2.5
6	
Equation:	

IV	
X	Y
1	6
2	3
3	
4	1.67
5	1.2
6	
Equation:	

V	
X	Y
1	
2	1
3	.67
4	.5
5	.4
6	
Equation:	

VI	
X	Y
1	18
2	
3	6
4	
5	3.6
6	3
Equation:	

Directions – Make up your own examples of direct variation and inverse variation by writing an equation and completing the corresponding table of values.

Direct variation equation:

Inverse variation equation:

Table of values:

Table of values:

Teacher Notes for

Equation Writing

This activity provides students with more practice writing the equations that represent direct variations and inverse variations. Students complete tables of values and then write the corresponding equation. At the end of the activity, students write their own direct variation equation and inverse variation equation.

Completed tables of values:

Equation:

$$y = 4x$$

Equation:

$$y = 7x$$

Equation:

$$y = .5x$$

Equation:

$$y =$$

Equation:

$$y =$$

Equation:

$$y =$$

Answers will vary for the equations that students create on their own.

Note: The table of values for student to fill in is left blank on purpose. You can let students choose for themselves their 'x' values. Or you could tell the students specific values for 'x'. For example, you could require that they have the values 1, 2, 5, 8, 15, and 50 in the 'x' column. This way they would have to calculate for values other than 1 through 6 and they would be getting practice using the equation to calculate their answers.

Name _____

Variations in Antarctica

Direct and inverse variation are useful ways to describe relationships we find in the real world. Variables in the real world don't always correspond so nicely as the pure math examples we've used so far. But it can still be useful to call a relationship "direct" or "inverse", even if it doesn't fit the definition exactly.

Directions: Use ArchiTech to complete the following charts. Two of the charts most closely represent sets of variables that vary directly. The other one most closely represents variables that vary inversely. As you fill in each chart, see if you can tell which is which.

Directions: Graph the data from the three charts below.

I

II

Directions: Answer these questions about the charts and graphs you just completed.

1. Which sets of variables do you think most closely represent a direct variation? Give reasons for your choices based on the numbers in the charts and the shapes of the graphs.

2. Which set of variables most closely represents an inverse variation? Give reasons for your choice.

3. List some reasons why real world examples don't always resemble pure math examples of direct and inverse variations?

Teacher Notes for

Variations in Antarctica

This activity shows students one place where direct variations and inverse variations are seen in the real world. Students will also see that the real world doesn't always match exactly the mathematical world. They will have to use their knowledge about how direct and inverse variations look like in the mathematical world to match as best they can the real world examples.

Here are completed tables and graphs.

Answers will vary for questions. Here are some possible responses:

1. The first graph varies directly because it is a straight line which starts from the origin. There is a difference of 371 in the numbers.
The second graph is a straight line but it doesn't go through the origin. There is a constant difference between the numbers.
2. The last graph seems to be an inverse variation. The graph is a curve. The numbers don't have the relationship that the x's multiplied by the y's is a constant result.
3. It could be that the ones that don't come out looking exactly right are a combination of two or more different functions. For example, in the examples that most closely resemble direct variation, you might have $y = 4x + 10$. Adding on the 10 would cause the graph to come out above the y-axis. Instead of crossing the y-axis at 0, it would cross at $y=10$. You would have similar results with inverse variations. For example, if a constant was added on, the graph would be shifted up.

Name _____

Further Investigation

on Direct Variation

Here is a graph of a direct variation and its table of values. Use it to help you complete this activity.

1. Draw in the remaining two rectangles on the graph following the same pattern that has been started. Use each point as the upper right-hand corner of each new rectangle.
2. What do you notice about the shapes of the rectangles that have been drawn?

3. The rectangles that you've drawn are proportional. Show that this statement is true by using the dimensions of the rectangles that you've drawn. (Hint: Try writing a proportion.)

4. Draw in five rectangles just like you did on the first graph. Use each point as the upper right-hand corner of each rectangle.
5. Are these rectangles proportional?

If yes, use the dimensions to show this.

If no, what is different about this graph compared to the first one?

Teacher Notes for
Further Investigation
on Direct Variations

This activity allows students the opportunity to explore the properties of shapes drawn from the graphs of direct variations. This activity focuses on rectangles drawn using points from the graph as the upper right-hand corner of the rectangles. You could further explore other shapes drawn from direct variations.

2. The response intended for is that the rectangles are growing larger proportionally. If your students are saying things like, "They are getting bigger". You may want to ask them to expand on that response. "How are they getting bigger." "Can you describe the growth of the rectangles more specifically?"

3. Depending on the experience your students have had, they may or may not be able to write true proportions from the table. You may have to go through one with them first. The proportions can be set up in different ways.

Another method is to set up a chart like the one below:

rectangle 1
rectangle 2

height

1
2

base

2
4

This would then translate to the proportion

.

5. The rectangles are not proportional. You can not set up any true proportions. This graph is different from the first one in that it doesn't go through the origin.

Name _____

Further Investigation on Inverse Variation

Here is a graph of an inverse variation along with its table of values. Use it to help you complete this activity.

1. Continue the pattern of rectangles using the points on the graph as the upper right-hand corner of the rectangles. Notice that the rectangles overlap each other.
2. Record your observations about the shapes of the rectangles in relationship to each other.
3. Inverse variations are symmetric. In the above example, when $x = 2$, $y = 5$ and when $x = 5$, $y = 2$.

Come up with at least two other examples of this symmetric property. Explain why the symmetric property works for inverse variations.

4. These rectangles are not proportional. Use the dimensions of the rectangles to prove this statement is correct.
5. Complete the following chart of the areas and perimeters of each rectangle. The rectangle on the far left is #1.

Rectangle
Area
Perimeter

2

3

4

5

6

7

8

9

10

6. Record the patterns you notice between the numbers in this chart.

7. Record the relationship you notice between the numbers in the chart and the shapes of the actual rectangles.

Teacher Notes for
Further Investigation on

Inverse Variations

This activity investigates properties about rectangles that are drawn from the graph of an inverse variation. The focus is on the area and perimeter of the rectangles and how the numbers relate to the shapes of the rectangles.

Possible answers:

2. The shapes of the rectangles are getting shorter and wider as they move along from left to right. First they are tall and thin. Then they progress to very short and wide.

3. One example would be that when $x = 1$, $y = 10$ and symmetrically when $x = 10$, $y = 1$. You could use any other example from the table of values. When $x = 4$, $y = 2.5$. That means that when $x = 2.5$, $y = 4$.

4. You can not set up any true proportions using the values from the table.

5. Here is a completed chart:

Rectangle

Area

Perimeter

1

10

22

2

10

14

3

10

12.66

4

10

13

5

10

14

6

10

15.34

7

10
16.86

8
10
18.5

9
10
20.22

10
10
22

6. The areas always stay the same. The first and last perimeters are the same. Then the perimeter goes down fast and starts to slowly increase to the ending value which is the same as the first one.

7. Even though the shapes of the rectangles change there is always the same amount of space inside of them. The lower values for perimeter correspond to the rectangles that are close to being squares. The larger perimeter values correspond to the long skinny ones.

Optional in class activity:

In addition to, or instead of, you could take your class through the following whole class activity in order to investigate the rectangles made from inverse variations.

1. Split your class up into groups of 3-4 students. Put an equation of an inverse variation on the board. Let's say you chose $y =$

.

2. Have each group come up with at least two different rectangles that have an area of 16 units.

3. Draw the graph of the chosen equation on the board. Have each group come up to the board and tape their rectangle underneath the graph so that the upper-right corner touches the graph of the inverse variation.

Some questions to ask students:

Can you fit two rectangles that are the same shape underneath the graph in two different places? Why or why not.

How many rectangles of the correct area will fit underneath the curve?

How are the perimeters of the rectangles related? Where are the rectangles with the larger perimeters? How about the rectangles with smaller perimeters?

How can you use the graph to find the square root of the constant of variation?
(If you draw the line of symmetry $y = x$, the place where it intersects the graph of the inverse variation will give you the square root.)

Name _____

Directly Related

You've learned that direct variations result in graphs that are lines. But are all lines direct variations? This is the question you will investigate in this activity.

Directions: Here are two graphs of lines with their table of values. The first one is a direct variation. The second one is not. Use them to help you complete the first part of the activity.

$$y=3x$$

$$y=3x+5$$

1. Direct variations are directly proportional. This means that you can write proportions from their table of values. Write at least three true proportions from the first table of values.
2. What happens when you try to write true proportions for the second table of values?
3. Compare the two equations. What is different about them? What is the same?
4. How do the similarities and differences relate to what the graphs look like?
5. What could you do to the second equation to make it into a direct variation?
6. Going back to the original question:
Do all lines represent direct variation?

Give a thorough explanation of your answer.

Use at least two examples to help further describe your thinking.
Teacher Notes for
Directly Related

This activity has students compare properties of direct variations with other linear graphs that are not direct variations.

Possible answers:

1. Students should see that they can write many more than three true proportions from the first table. Here are three possibilities:
2. You get proportions that are not true.
3. The second equation has something added to the $3x$. They both have a 3 in them. X is being multiplied by some number in both of them.
Important: You should probably go over this question together with the class to make sure that they noticed these similarities and differences. It's also important for them to realize that the two examples being used are both only one of many more of that particular category. You could have students come up with other examples of direct variations. You could also have them come up with other examples of the second type of linear function.
4. The fact that there is a ' $3x$ ' in both equation accounts for the steepness or slope of the line. The 5 that is added on to the end of the second equation causes the graph to be raised 5 points in every place. It has been 'shifted' up.
Note: This question could be further investigated by coming up with other examples that are similar to the two sample equations. Students could make conjectures based on the given examples, and then test them out on other related examples.
5. If the 5 is taken off the original equation, it then becomes a direct variation.
6. Here is an example of a very thorough response to the question. You may or may not expect your students to display the degree of understanding shown here. After your students have responded to the question, you could have a class discussion bringing up some of the points not brought up by your class.

Sample response:

No, all lines do not represent direct variation. We learned earlier that direct variations maintain the relationship $y = cx$, where c is a constant. It is true that all graphs based on the relationship $y = cx$ will form a straight line going through the origin. (We know that the line will pass through the origin because when x equals 0, y will also equal 0 no matter what the constant is since zero times any number is zero. Therefore, the point $(0, 0)$ is definitely on the graph $y = cx$.) The value of the constant c actually turns out to be the slope of the line. ($c = y / x$, which is the rise over the run between the point (x, y) and the origin.)

However, this doesn't prove that all lines will fit the requirements of a direct variation. The general equation of a line (in slope-intercept format) is $y = mx + b$, where m is the slope of the line and b is the y -intercept (where the line crosses the y axis). Let's look at those two equations side by side:

$$\begin{array}{ll} y = cx & \text{(Equation for direct variation)} \\ y = mx + b & \text{(Equation for a line)} \end{array}$$

The equation of a line is almost the same as the equation for a direct variation, except that the equation of a line also says "+ b ". This makes sense, because we've already determined that direct variations are lines which must pass through the origin. That's the same thing as saying that direct variations are lines which have a y -intercept of 0. Instead of writing $y = mx + 0$, we can just write $y = mx$ (or $y = cx$ if we use c to represent the constant instead of m .) Lines that do not pass through the origin must cross the y -axis at some other place, either above or below the origin. The value of b tells exactly where on the y -axis that point is.

The second sample graph, $y = 3x + 5$, is an example of a line which does not pass through the origin (it has a y -intercept of 5), so although it is a line, it is not a direct variation.

Name _____

Please Advise

Hello:

I'm writing in to ask for some clarification. I'm trying to make some cookies from this old recipe I have. The problem is that some of it is missing. My grandfather happened to be around as I ran into this dilemma so I asked him for some advice. That was a mistake! He started going on about how the measurements are all directly proportional and all I have to do is to figure out what the constant of variation is and then I'm home free. Then he took out some paper and starting graphing some equation. "It's all there!", he said. I nodded my head, but all I kept thinking was "What??? Where???"

That's when I decided to write in to you. I'd really appreciate the answer to my problem. I'd also like to be able to understand what he meant. The more thorough you are, the better. Please include examples and diagrams of what you're talking about.

Here's the problem:

I was looking through my mom's old recipes and I noticed a great chocolate chip cookie recipe. The only problem was that there seemed to be two versions, and neither one of them was complete. For example, one said 2 cups of flour while the other one said 6 cups. The first one had one cup of chocolate chips and the second one didn't say anything about the chocolate chips. I want to make a lot of cookies, so I need to know how many chocolate chips go with the 5 cups of flour! How do I figure this out? And what if I want

to make even more? Is this where that constant of variation comes in? And how could a graph help me see anything?

Thanks for your help. I look forward to going back to my grandfather and impressing him with my new understanding of all this.

Teacher Notes for
Please Advise

This activity can be used as an end of the unit assessment for direct variation.

Here are some things that students may do to help solve the problem.

Make a table and fill in the known values:

of cups of flour
chocolate chips

1

2

1

3

4

5

6

Using the fact that the measurements are directly proportional, set up a proportion to help find the missing amount requested. In this case, the writer wants to know how many chocolate chips are needed for 5 cups of flour.

Solving this proportion

gives us 2.5 cups of chocolate chips for 5 cups of flour.

In order to solve this, you can notice that 1 is half of 2 and so half of 5 is 2.5. You can then fill in the rest of the chart and come up with an equation that shows this relationship. This will also give you the constant of variation.

of cups of flour

chocolate chips

1

.5

2

1

3

1.5

4

2

5

2.5

6

3

The equation is $y =$
 x and the constant of variation is

.

You can use this constant to find out how many cups of chocolate chips you would need for any number of cups of flour. You would just multiply the number of cups of flour by

.

Graphing these functions will allow you to get a quick overall picture of the relationship of flour to chocolate chips.

Direct and Inverse Variations