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Multiplying Integers—Teachers’ Guide

3.6c

**The main point** of this lesson is for students to discover/assert ON THEIR OWN that when you multiply by a negative in our pictorial representation, you have to flip the picture to its opposite.

1). Whole-class activity

5

0

-5

How could you take this picture of 4 and use it to show 4 • 3 ?

5

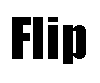
0

-5

Please draw a picture of 3 • 5

2). Small-group discussion

With your group, please decide how to show each problem, and give each answer:



5

0

-5

a). 5 • 2 = **10**

5

0

-5

b). 5 • -2 = **-10**

5

0

-5

c). -2 • 5 = **12**

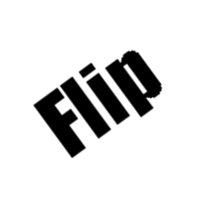
5

0

-5

d). -2 • ‑5 = **10**

**Note: Some students may switch the pictures for b) and c), or use the answer shown for c) as their response to both b) and c). This is justified by the commutative property. If a student does this, show their answer and elicit a discussion of why it works. However, it’s also important to bring out the idea of flipping when multiplying by a negative. The picture shown for b), including the flipping, is important.**



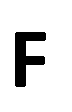
Whole-class discussion

* Points for reflection:
  + Do you agree with the way other groups drew their pictures?
  + In general, how do you show multiplying by a negative?
  + For example, in pictures, what’s the difference between (6)(4) and (6)( -4) ?

**At some point in/after the discussion, it’s worth it to introduce some notation for the when you’re flipping a picture. I use the letter “F” or the word “flip” rather than just letting students cross out the rectangle. That’s because we’ve used crossing-out to represent other concepts in the lessons on adding and subtracting. In addition, using an “F” reminds students of the meaning of what they’re doing.**

Individual Practice

3). Please show that you understand by doing these on your own:



5

0

-5

10

-10

(8)(-3) = **-24**

5

0

-5

10

-10

(-8)(-3) = **24**

5

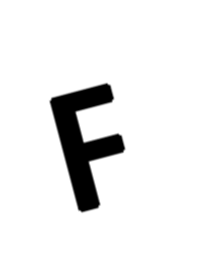
0

-5

10

-10

(-8)(3) = **-24**



“F” stands for “flip”

4). Try these without a picture:

a). (-2)(8) = **-16** b). (-2)(-8) = **16** c). -2 + -8 = **-10** d). (-2) + (8) = **6**

Small-group activity

5).

* Please choose two integers so that when you multiply them, the sign (+ or -) of the answer is different than what you would get if you added them. Create a visual diagram explaining why the signs are different.
* Then, using pictures, diagrams, or a rap or other performance, summarize the difference between the rules for adding and multiplying integers. (Performances must be less than 3 minutes).

Individual practice

6). Please practice on your own. Be careful—the questions mix adding, subtracting, and multiplying.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| (-2)(9) = **-18** | (-2)(-9) = **18** | (-2) + (-9) = **-11** | -2 – -9 = **7** | (2)(-9) = **-18** |
| -3 • -5 = **15** | -3 + 5 = **2** | 3 + -5 = **-2** | -3 • 5 = **-15** | 3 • -5 = **-15** |
| ‑3 - -5 = **2** | -3 – 5 = **-8** | (10)(-7) = **-70** | 10 + (-7) = **3** | ‑10 + -7 = **-17** |
| (-10)(-7) = **70** | 3\*-6 = **-18** | -3\*(-6) = **18** | -3 + -6 = **-9** | -3 + 6 = **9** |

7). Please use the order of operations to evaluate each expression. Check your work with the scrambled answers.

a). **-**3(4 + 5) – 1

**Example:**

**-**3( 9 ) + **-**1

**-**27 + **-**1

**-**28

b). **-**11 + (**-**4)(**-**6)

c). (**-**1 + **-**3)(4 + **-**2)

**Example:**

( **-**4 ) ( 2 )

**-**8

d). **-**4 + **-**2(**-**6+2)

e). 3 – 7 + 2(**-**2)

3 + **-**7 + **-**4

**-**3(4 + 5) + **-**1

3 + **-**7 + 2(**-**2)

**-**4 + **‑**4

**-**8

f). 4 – 3(5 + **-**9)

g). (2•**-**6 – 1) – 3(**-**2)

**Example:**

**Example:**

(2•**-**6 + **-**1) + **-**3(**-**2)

(**-**12 + **-**1) + 6

**-**13 + 6

**-**7

h). (2 – 8) – 5(5 + **-**8)

Scrambled Answers:

i). 2(-7) + **-**5(-3 – 1)

6

13

9

4

16

**4**

**13**

**16**

**9**

**6**

3.6c Extension

WHY Flip When Multiplying?

We can actually prove that a positive times a negative equals a negative. To understand, you have to know the Distributive Property. Please fill in the blanks below in the Distributive Property problems:

3(4 + 1) = 3(4) + 3(1)

5(2 + 6) = 5(2) +  **5** (6)

-8(5 + 4) = -8(5) + -8(4)

-3(4 + -2) =  **-3**  (4) +  **-3** ( **-2**)

Example:

Example:

You try:

You try:

1).

We’ve decided it makes sense to flip our pictures when multiplying by a negative, like this:

5

0

-5

3 • -2 = \_\_\_\_

Flip!

But can we prove that we’re supposed to flip?

The Hard Proof for Flipping when Multiplying

-3 + 3 = 0

We know this:

2(-3 + 3) = 2(0)

We can multiply both sides by 2:

2(-3) + 2(3) = 2(0)

Use the Distributive Property:

Multiply the parts we know:

2(-3) + 6 = 0

How does this prove that 2(-3) has to be negative? Please explain:

Answers will vary. The idea is that 2(-3) and 6 add to zero. Since 6 is positive, and it cancels out completely with 2(-3), that means 2(-3) is negative.

Challenge: Can you adapt this proof to prove that a negative times a negative equals a positive? Try it here:

-3 + 3 = 0

We know this:

-2(-3 + 3) = -2(0)

Multiply the parts we know:

-2(-3) + -6 = 0

We can multiply both sides by ‑2:

-2(-3) + -2(3) = -2(0)

Use the Distributive Property:

The sum is zero. Therefore, -2(-3) must be the opposite of -6. So ‑2(-3) must be positive.