

# **Algebra 1**

## **Predicting Patterns & Examining Experiments**

**Unit 7: You Should Probably Change**  
**Section 4: Making Choices**

# Student Question #5...



**How many ways could the class choose a committee of four students?**

In choosing a committee (combinations), order is irrelevant (as the next slide shows).

# Student Question #5...

**These are all the same committee:**

**ABCD ACBD CABD BACD BCAD CBAD  
ABDC ACDB CADB BADC BCDA CBDA  
ADBC ADCB CDAB BDAC BDCA CDBA  
DABC DACB DCAB DBAC DBCA DCBA**

**How many ways could the class choose a committee of four students?**

**If the class had four students, there is only one way to choose four for a committee. But our class has eleven....**

In choosing a committee (combinations), order is irrelevant (as the next slide shows).

# Student Question #5...



**How many ways could the class choose a committee of four students?**

**Out of eleven students, we can arrange four students 7920 ways.**

$$\frac{11!}{(11-4)!} = \frac{11!}{7!} = \frac{11 \cdot 10 \cdot 9 \cdot 8 \cdot 7!}{7!} = 11 \cdot 10 \cdot 9 \cdot 8 = 7920 \quad \text{But....}$$

In choosing a committee (combinations), order is irrelevant (as the next slide shows).

# Student Question #5...



**How many ways could the class choose a committee of four students?**

**Out of eleven students, we can arrange four students 7920 ways, but we are not arranging them (just choosing them). So, the 24 arrangements for each group needs to be removed.**

In choosing a committee (combinations), order is irrelevant (as the next slide shows).

# Student Question #5...



How many ways could the class choose a committee of four students?

Out of eleven students, we can **CHOOSE** four students 330 different ways.

$$\frac{7920}{24} = 330$$

In choosing a committee (combinations), order is irrelevant (as the next slide shows).

# Combination Notation



Combinations defined

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A combination is a choosing of objects **NOT** in a specific order.

${}_n C_r = r$  objects chosen out of  $n$  total objects

$${}_n C_r = \frac{n!}{(n-r)! \cdot r!}$$

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# Student Question #5...



How many ways could the class choose a committee of four students?

Out of eleven students, we can **CHOOSE** four students 330 different ways:

$${}_n C_r = \frac{n!}{(n-r)! \cdot r!}$$

$${}_{11} C_4 = \frac{11!}{(11-4)! \cdot 4!} = \frac{11 \cdot 10 \cdot 9 \cdot \cancel{8} \cdot \cancel{7}!}{\cancel{7}! \cdot \cancel{4} \cdot 3 \cdot \cancel{2} \cdot 1} = \frac{990}{3} = 330$$

In choosing a committee (combinations), order is irrelevant (as the next slide shows).

# How many hands?

How many different 5-card hands can be dealt out of a 52-card deck?



Notice that order is irrelevant, so we will just be using a combination.

# How many hands?

How many different 5-card hands can be dealt out of a 52-card deck?

$$\begin{aligned} {}_{52}C_5 &= \frac{52!}{(52-5)! \cdot 5!} = \frac{52!}{47! \cdot 5!} \\ &= \frac{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48 \cdot \color{red}{47!}}{\color{red}{47!} \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} \\ &= \frac{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} \\ &= \frac{311,875,200}{120} \\ &= 2,598,960 \end{aligned}$$



There are 52 cards and we are choosing 5. Notice the expansion of the numerator so that we can divide out the common factors (47!).

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**There are almost 2.6 million different hands.**



# How many smoothies?

At a local juice store, you can mix and match ingredients to create your own smoothie.

From nine juices you can choose three, from five sherbets you can choose two, and from eight vitamin powders you can choose one.

How many different smoothies can be made?

Combination with multiple choices to make. Notice that when we are mixing a fruit drink, order is irrelevant (combination, not a permutation).

Solution slides follow.

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**Choices**

**Juices:**

Separately, we can find the choices available for each ingredient.

solution process

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**Choices**

**Juices:**  ${}_9C_3 =$

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## Choices

Juices:  ${}_9C_3 = \frac{9!}{(9-3)! \cdot 3!} = \frac{9 \cdot 8 \cdot 7 \cdot 6!}{6! \cdot 3!} = 84$

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$$\text{Choices} = 84 \cdot 10 \cdot 8$$

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$$\text{Choices} = 84 \cdot 10 \cdot 8 = 6720$$

There are 6720 different smoothies that can be made at the juice store.

# Probability of a Limon

If you randomly grab five skittles, what is the probability that you will pick exactly 3 green and 2 yellow candies?

What do we need to know?

# Probability of a Limon

If you randomly grab five skittles, what is the probability that you will pick exactly 3 green and 2 yellow candies?

## Facts:

- There are 40 skittles in the bowl.
- There are equal numbers of each of the five colors.

# Probability of a Limon

If you randomly grab five skittles, what is the probability that you will pick exactly 3 green and 2 yellow candies?

There are 8 of each color, so:

the number of ways to choose 3 greens:  ${}_8C_3 = \frac{8!}{5! \cdot 3!} = 56$

# Probability of a Limon

If you randomly grab five skittles, what is the probability that you will pick exactly 3 green and 2 yellow candies?

There are 8 of each color, so:

the number of ways to choose 2 yellows:  ${}_8C_2 = \frac{8!}{6!2!} = 28$

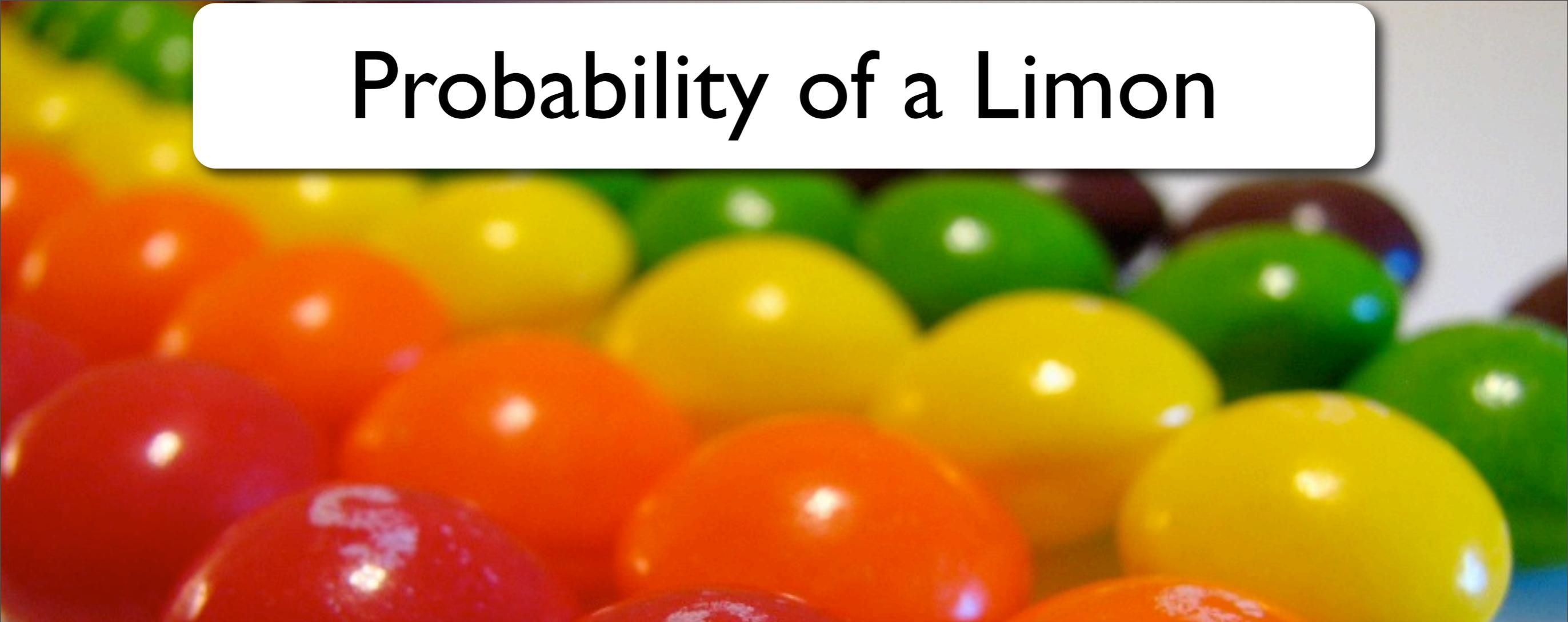
# Probability of a Limon

If you randomly grab five skittles, what is the probability that you will pick exactly 3 green and 2 yellow candies?

There are 40 skittles, so:

the number of ways to choose 5 candies:  ${}_{40}C_5 = \frac{40!}{35! \cdot 5!} = 658,008$

# Probability of a Limon



If you randomly grab five skittles, what is the probability that you will pick exactly 3 green and 2 yellow candies?

$$\text{Probability} = \frac{\text{event}}{\text{total outcomes}}$$

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$$\text{Probability} = \frac{\text{event}}{\text{total outcomes}} = \frac{{}_8C_3 \cdot {}_8C_2}{{}_{40}C_5} = \frac{56 \cdot 28}{658,008}$$

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There is a .24% chance of picking exactly 3 green and 2 yellow candies.

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## **6 degrees of cards**

<http://www.flickr.com/photos/insightimaging/3899220362/>

## **Fresh fruit juice**

<http://www.flickr.com/photos/blandm/3582158037/>

## **Skittles**

<http://www.flickr.com/photos/ambiebambie39507/2377420546/>