

Algebra I

Predicting Patterns & Examining Experiments

Unit 2: Maintaining Balance

Section 3: Balancing tough situations

Again, If the skill of multi-step equations is difficult then perhaps multiple days worth of class are merited in practicing this all-important skill in Algebra 1.

Discuss

What four consecutive numbers
have a sum of 2010?

(Small Group Discussion) Students may solve this numerically by random guessing or dividing 2010 by 4 and working with that result. Students may need to learn the definition of 'consecutive'.

Discuss

What four consecutive numbers
have a sum of 2010?

Guess & Check (Numeric) Solution:

$$100+101+102+103 = 406$$

$$300+301+302+303 = 1206$$

$$500+501+502+503 = 2006$$

$$501+502+503+504 = 2010$$

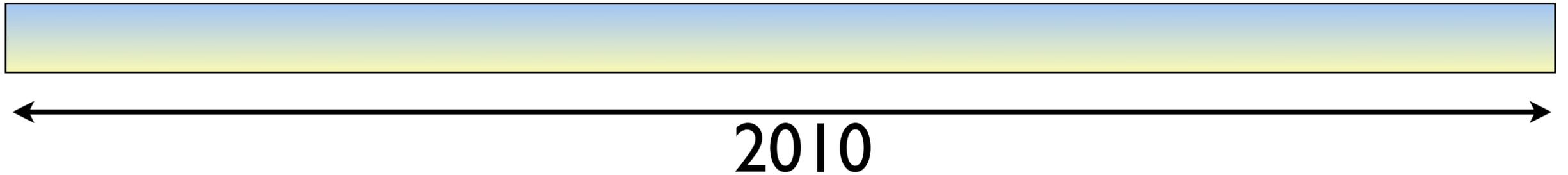
**The four consecutive numbers are
501, 502, 503, and 504.**

(Small Group Discussion) Students may solve this numerically by random guessing or dividing 2012 by 4 and working with that result (the next slide). After we look at these two methods, we will develop an algebraic method as well.

Discuss

What four consecutive numbers
have a sum of 2010?

Graphical Solution:

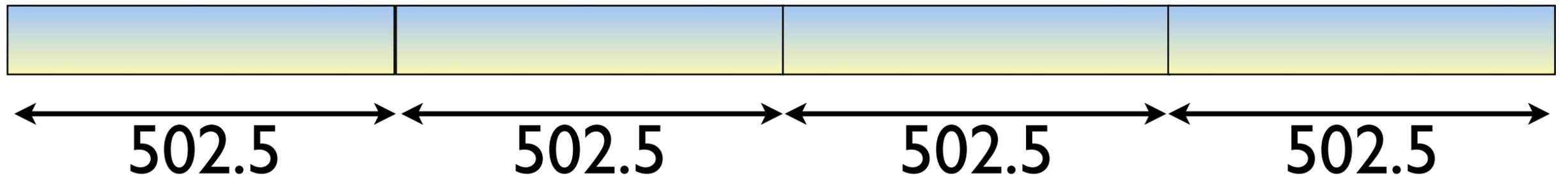


(Small Group Discussion) Students may solve this numerically by random guessing or dividing 2010 by 4 and working with that result.

Discuss

What four consecutive numbers
have a sum of 2010?

Graphical Solution:



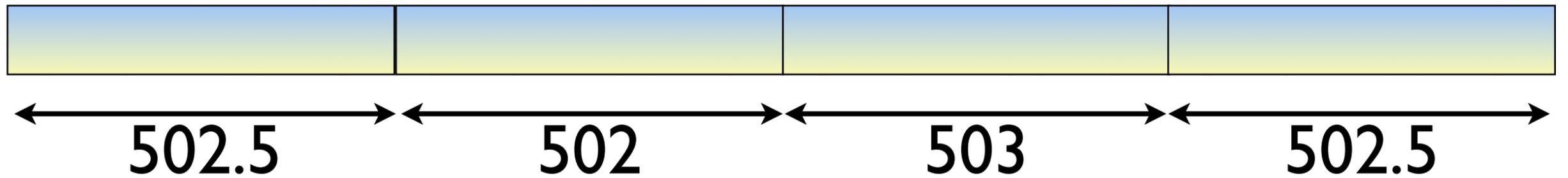
4 equal pieces of 502.5 each

Graphically we can see a progression that leads to our answer.

Discuss

What four consecutive numbers
have a sum of 2010?

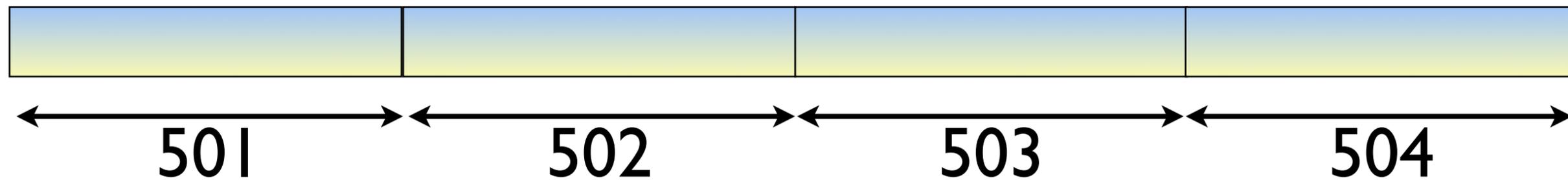
Graphical Solution:



Discuss

What four consecutive numbers
have a sum of 2010?

Graphical Solution:



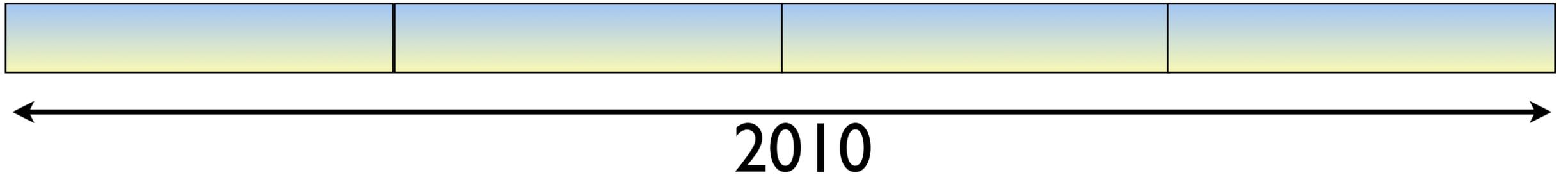
4 almost equal pieces around 502.5 each

A graphical solution that works its way to the answer.

Discuss

What four consecutive numbers
have a sum of 2010?

Analytic Solution (with help from the graphic):

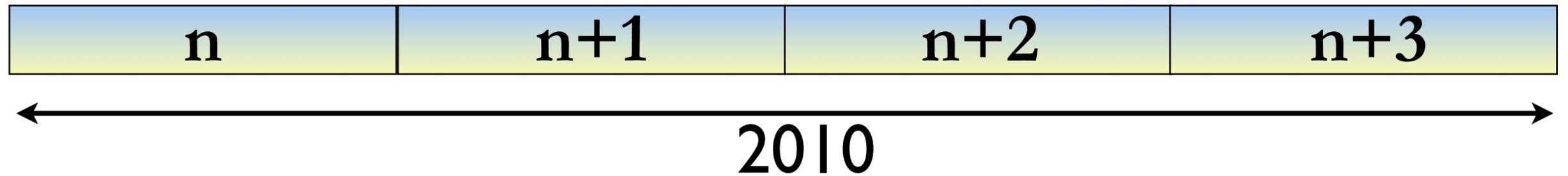


Analytic solution will use the graphic representation for some help.

Discuss

What four consecutive numbers
have a sum of 2010?

Analytic Solution (with help from the graphic):



$$n + n+1 + n+2 + n+3 = 2010$$

$$4n + 6 = 2010$$

How do we solve this?

multiplication
•4

addition
+6

Multi-step Equations



Alright, we've been looking at simple (one-step) equations, now we need to look at how to solve multi-step equations.

How do we solve...

$$4n + 6 = 2010$$

Again, If the skill of multi-step equations is difficult then perhaps multiple days worth of class are merited in practicing this all-important skill in Algebra 1.

How do we solve...

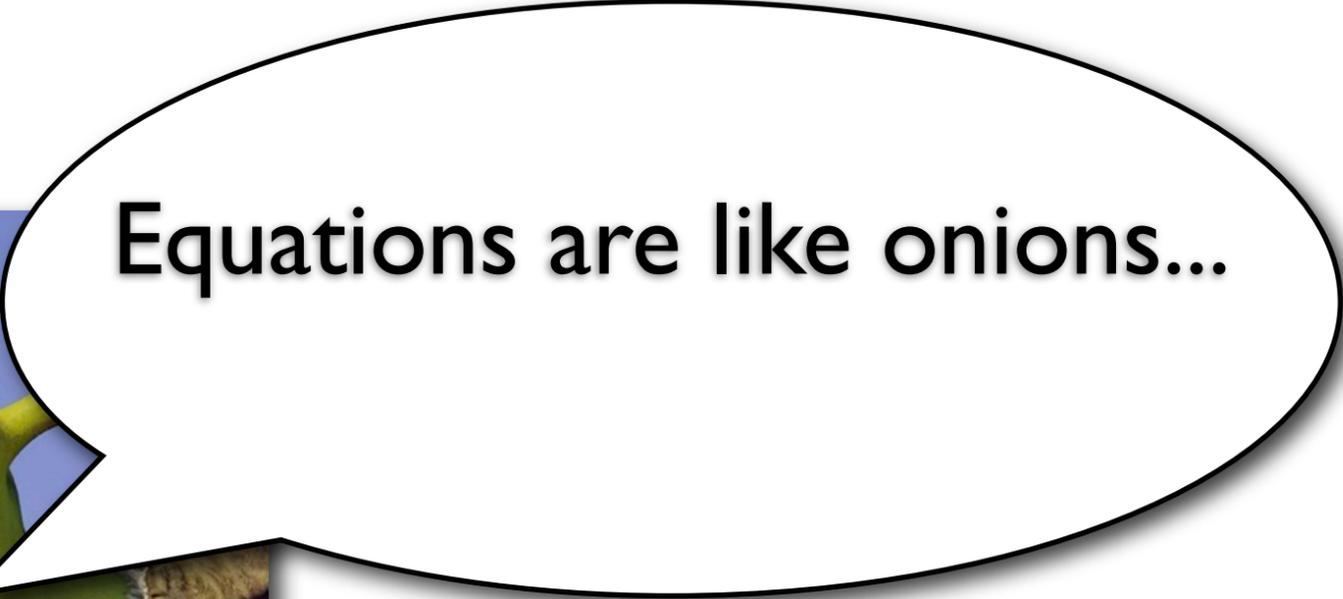
$$4n + 6 = 2010$$



A word from Shrek on multi-step equations.

How do we solve...

$$4n + 6 = 2010$$



Equations are like onions...



A word from Shrek on multi-step equations (see if students can complete the sentence...)

How do we solve...

$$4n + 6 = 2010$$

A close-up image of the character Shrek, a green ogre, with his mouth open as if speaking. He is wearing a brown, textured tunic. The background shows a blue sky and some grass.

Equations are like onions...
they have layers

A word from Shrek on multi-step equations. Next, we will see the layers of this multi-step equation.

How do we solve...



layers

$$4n + 6 = 2010$$

Let's break down the equation into the layers and see what process we need to follow in order to get our answer.

How do we solve...



layers

$$4n + 6 = 2010$$



Let's break down the equation into the layers and see what process we need to follow in order to get our answer.

How do we solve...



layers

$$4n + 6 = 2010$$



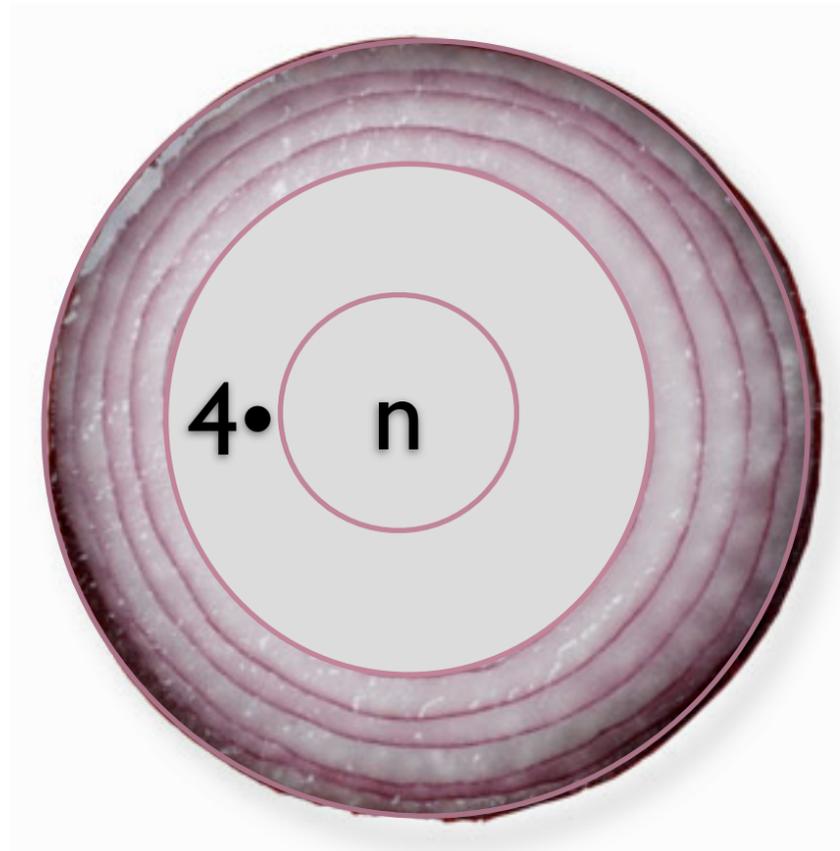
Innermost layer (core) : the variable we are trying to solve for: n

How do we solve...



layers

$$4n + 6 = 2010$$



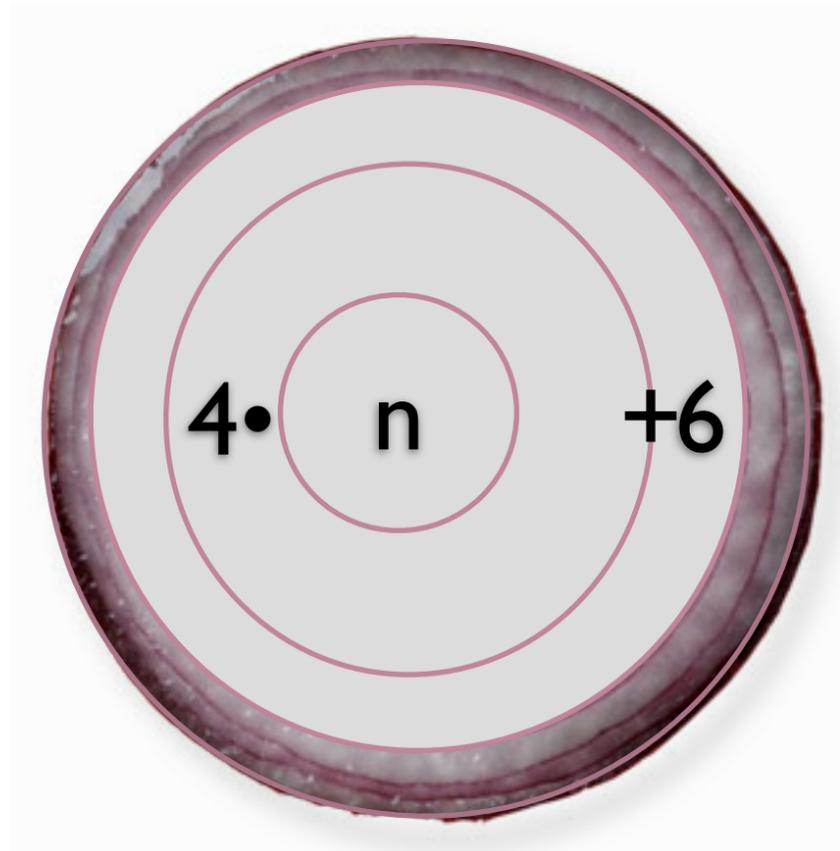
Layer 1 : we multiply by four.

How do we solve...



layers

$$4n + 6 = 2010$$



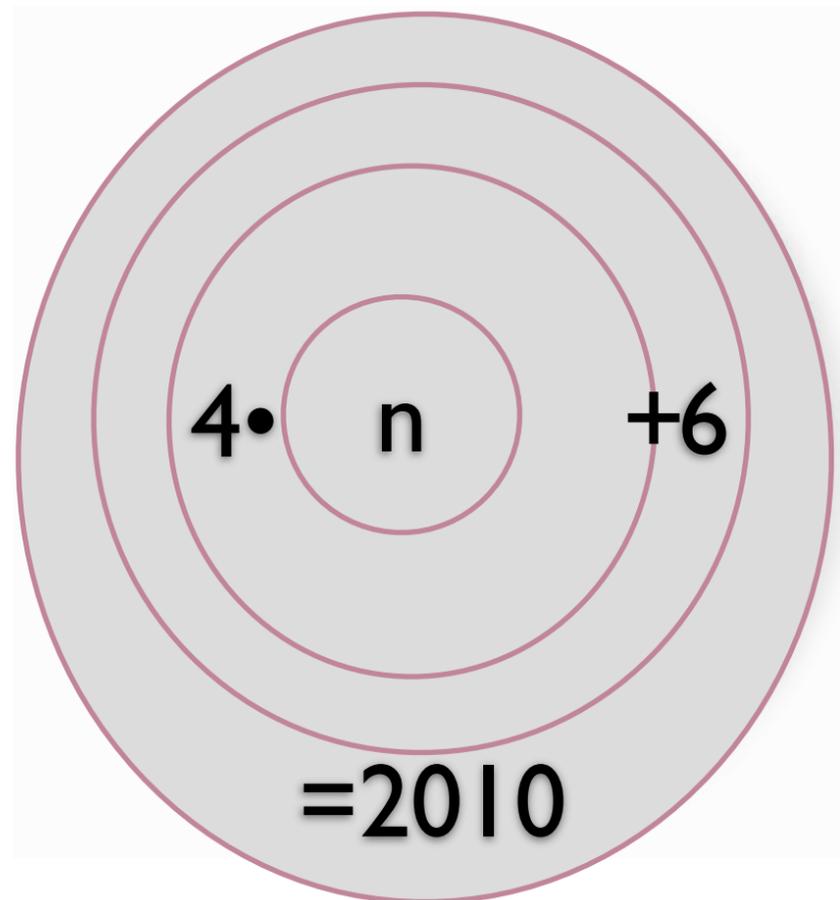
Layer 2 : we add six

How do we solve...



layers

$$4n + 6 = 2010$$



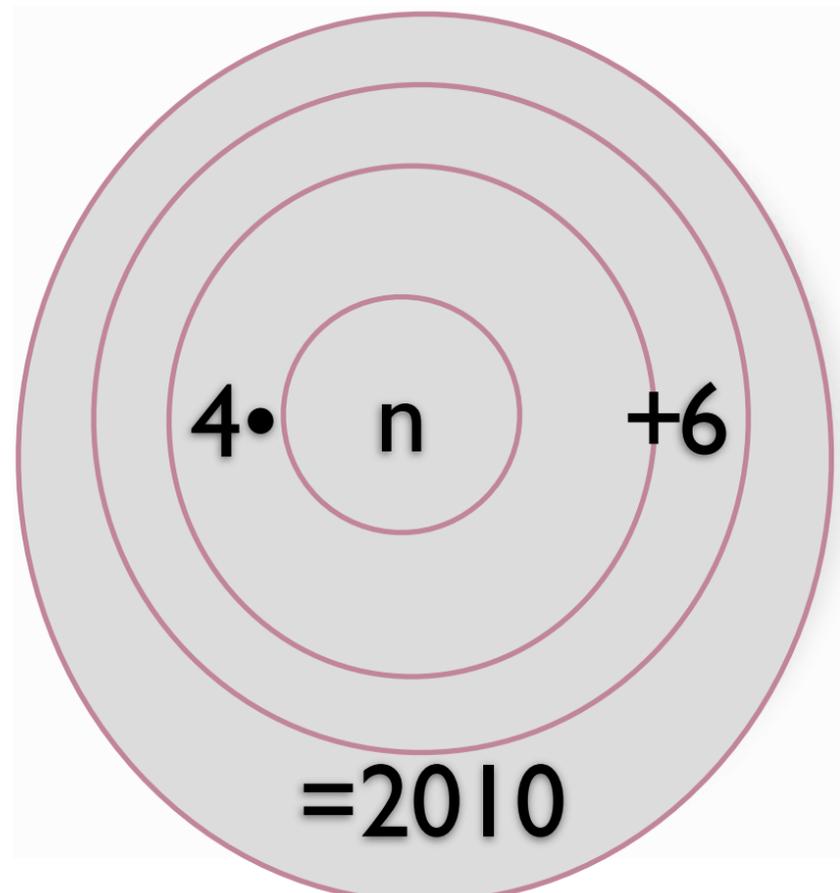
Outside Layer : we get an answer of 2010.

How do we solve...



layers

$$4n + 6 = 2010$$



Now we need to start on the outside layer and work back to our variable.

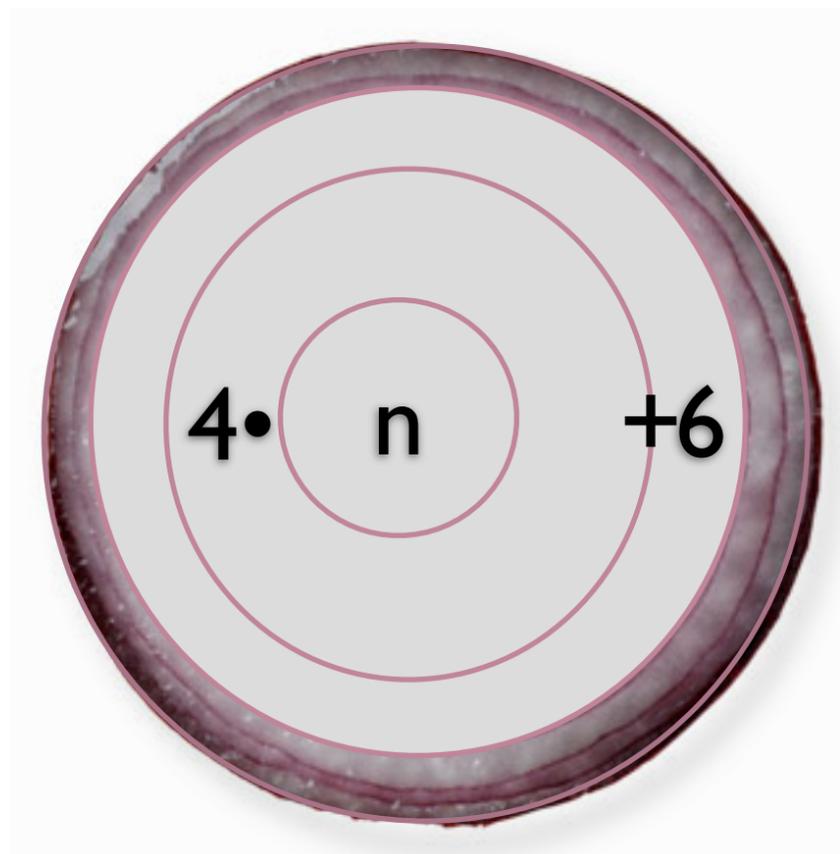
Outside Layer : we get an answer of 2010.

How do we solve...



layers

$$4n + 6 = 2010$$



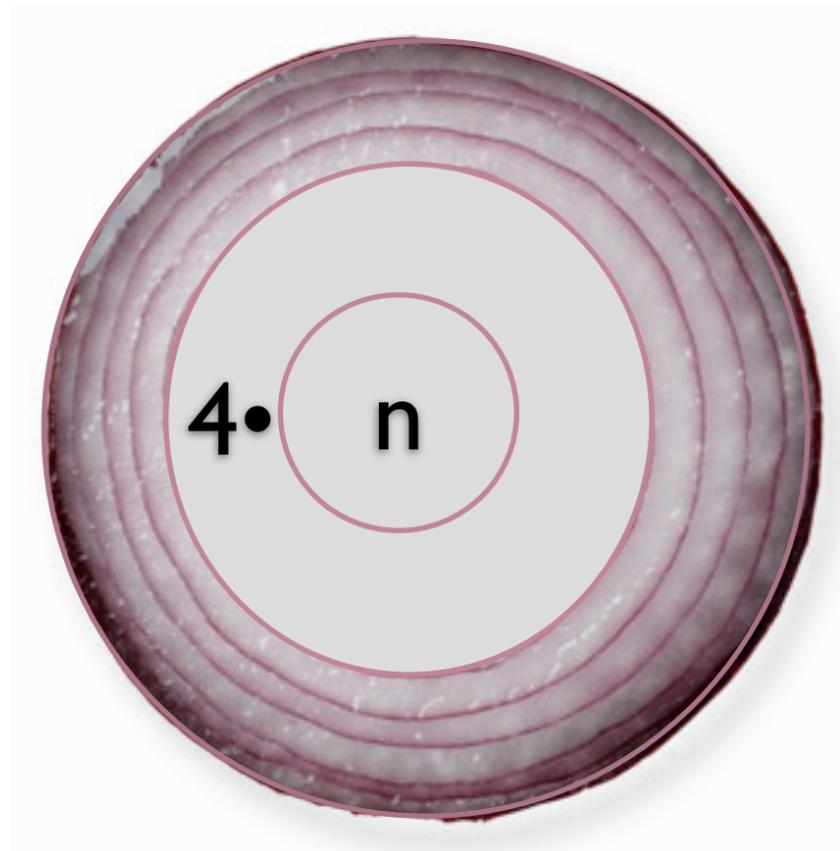
How do we take off the layer of “ +6 “

Alternately, “What is the opposite of adding six?”

How do we solve...



layers



$$4n + 6 = 2010$$

$$\begin{array}{r} -6 \qquad -6 \\ \hline \end{array}$$

$$4n = 2004$$

How do we take off the layer of “ +6 “ : subtract 6

Alternately, “What is the opposite of adding six?”

How do we solve...



layers



$$4n + 6 = 2010$$

$$\quad -6 \quad \quad -6$$

$$4n = 2004$$

How do we take off the layer of “•4”

Alternately, “What is the opposite of multiplying by four?”

How do we solve...



layers



$$4n + 6 = 2010$$

$$\quad -6 \quad \quad -6$$

$$\frac{4n}{4} = \frac{2004}{4}$$

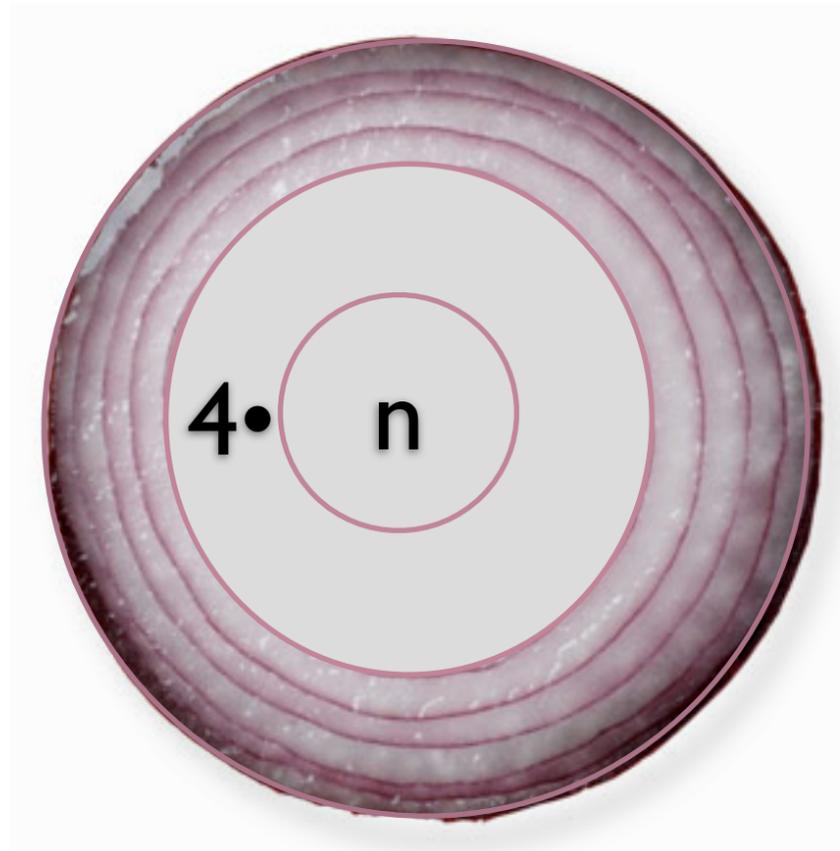
How do we take off the layer of “•4” : divide by 4

Alternately, “What is the opposite of multiplying by four?”

How do we solve...



layers



$$4n + 6 = 2010$$

$$\quad -6 \quad \quad -6$$

$$\frac{4n}{4} = \frac{2004}{4}$$

$$n=501$$

$$n=501$$

Alternately, "What is the opposite of multiplying by four?"

Time-in: Back to the problem

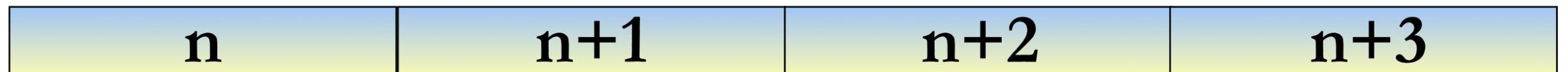


Alright, we've been looking at simple (one-step) equations, now we need to look at how to solve multi-step equations.

Remember

What four consecutive numbers
have a sum of 2010?

Analytic Solution (with help from the graphic):



$$n + n+1 + n+2 + n+3 = 2010$$

$$4n + 6 = 2010$$

How do we solve this?

multiplication
•4

addition
+6

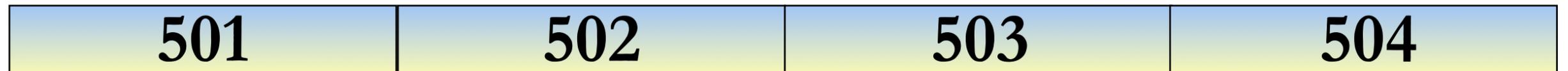
... which comes first?

We are back to the slide where we were stuck, now equipped to solve the problem.

Discuss

What four consecutive numbers have a sum of 2010?

Analytic Solution (with help from the graphic):



$$n + n+1 + n+2 + n+3 = 2010$$

$$4n + 6 = 2010$$

$$4n + 6 = 2010$$

$$\begin{array}{r} -6 \quad -6 \\ \hline \end{array}$$

$$\frac{4n}{4} = \frac{2004}{4}$$

Since $n=501$

$$n=501$$

Inserting our solution, we can now fully answer the question (next slide)

When will the water level be back down to normal?



After a substantial rain, the Loch Laggan dam water level raised so that at noon, the water was 11 feet above normal.

An hour later the water has dropped by 8 inches.

When will the water level be back down to normal?

(Small Group Discussion) Above is another multi-step equation problem. Have students develop a solution and then share the results. The following slides show a numeric, graphic, and then analytic solutions.

When will the water level be back down to normal?



Numeric

time (hours after noon, the first day)	water-level (inches above normal)
0 hrs	132 in
1	124

Notice that I started at inches (so that I wouldn't have to use fractions/decimals of feet) and since I don't know if it will happen 12 or 24 hours, I decided to start with a zero-hour.

When will the water level be back down to normal?



Numeric

time (hours after noon, the first day)	water-level (inches above normal)
0 hrs	132 in
1	124
2	116
3	108
...	...
x hours	0 inches

What we're trying to find:



The algebra is introduced to show our goal. Although this table could be continued to get an answer, but we are going to use the CONSTANT pattern to speed up the process (and then we'll use that slope to write an equation).

When will the water level be back down to normal?



Numeric
constant change: -8 in/hr

time (hours after noon, the first day)	water-level (inches above normal)
0 hrs	132 in
+1 hr ↓ 1	124 ↓ -8 in
2	116
3	108
...	...
x hours	0 inches

The constant change (slope) is a drop in 8 inches every hour.

When will the water level be back down to normal?



**Numeric
constant change: -8 in/hr**

time (hours after noon, the first day)	water-level (inches above normal)
0 hrs	132 in
+1 hr ↓ 1	124 ↓ -8 in
2	116
3	108
+10 hr ↓ ↓ -80 in
13	28
14	20
15	12
16	4
+0.5 hr ↓ 16.5	0 ↓ -4 in
x hours	0 inches

By continuing the pattern using the slope, we can skip steps (because slope allows us to skip steps). We can move ahead 10 hours and drop 80 inches and when necessary, we can drop 4 inches by only going up half an hour.

When will the water level be back down to normal?



Numeric
constant change: -8 in/hr

The water level will be back to normal 16.5 hours after noon the first day, which brings us to 4:30 am the next morning.

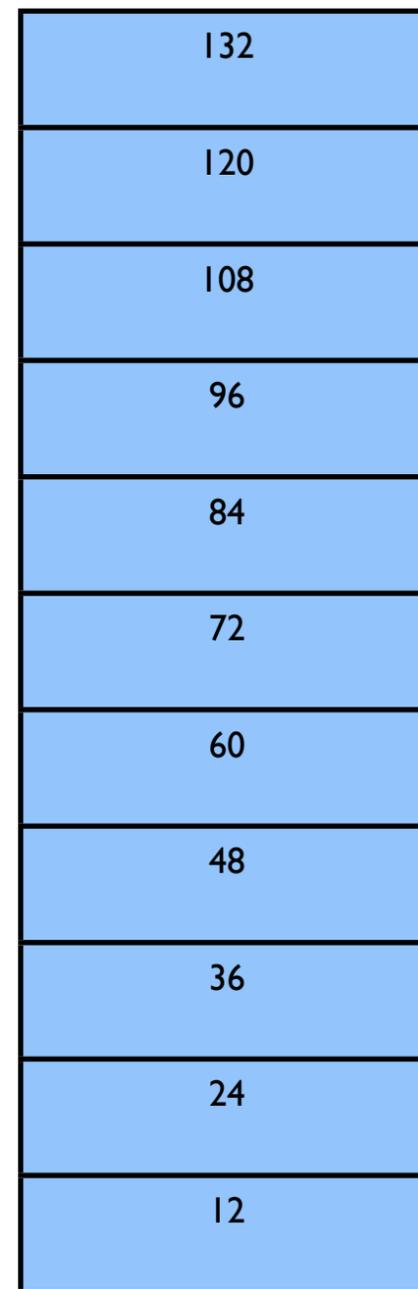
time (hours after noon, the first day)	water-level (inches above normal)
0 hrs	132 in
+1 hr ↓ 1	124 ↓ -8 in
2	116
3	108
+10 hr ↓ ↓ -80 in
13	28
14	20
15	12
16	4
+0.5 hr ↓ 16.5	0 ↓ -4 in
x hours	0 inches

Our answer is 16.5 hours after noon the first day, so 12 hours later is midnight and the remaining 4.5 hours bring us to 4:30 in the morning.

When will the water level be back down to normal?



Graphic



water gauge
132 inches high

Our answer is 16.5 hours after noon the first day, so 12 hours later is midnight and the remaining 4.5 hours bring us to 4:30 in the morning.

When will the water level be back down to normal?



Graphic

$$\frac{132 \text{ inches}}{8 \text{ inches}} = 16.5$$

132
120
108
96
84
72
60
48
36
24
12

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
16.5

Our answer is 16.5 hours after noon the first day, so 12 hours later is midnight and the remaining 4.5 hours bring us to 4:30 in the morning.

When will the water level be back down to normal?



time (hours after noon, the first day)	water-level (inches above normal)
0 hrs	132 in
+1 hr ↓ 1	124 ↓-8 in
2	116
3	108
...	...
x hours	0 inches

$$132 - 8x = 0$$

Analytic

constant change: -8 in/hr

We start at 132 inches and have a constant change (slope) is a drop in 8 inches every hour and need to get back to 0 inches, the normal level.

When will the water level be back down to normal?



time (hours after noon, the first day)	water-level (inches above normal)
0 hrs	132 in
+1 hr ↓ 1	124 ↓ -8 in
2	116
3	108
...	...
x hours	0 inches

$$132 - 8x = 0$$

starting value ↑

Analytic
constant change: -8 in/hr

We start at 132 inches and have a constant change (slope) of -8 inches every hour and need to get back to 0 inches, the normal level.

When will the water level be back down to normal?



time (hours after noon, the first day)	water-level (inches above normal)
0 hrs	132 in
+1 hr ↓ 1	124 ↓ -8 in
2	116
3	108
...	...
x hours	0 inches

$$132 - 8x = 0$$

constant
change ↑

Analytic
constant change: -8 in/hr

We start at 132 inches and have a constant change (slope) of -8 inches every hour and need to get back to 0 inches, the normal level.

When will the water level be back down to normal?



time (hours after noon, the first day)	water-level (inches above normal)
0 hrs	132 in
+1 hr ↓	124 ↓ -8 in
2	116
3	108
...	...
x hours	0 inches

$$132 - 8x = 0$$

↑ ending value

Analytic

constant change: -8 in/hr

We start at 132 inches and have a constant change (slope) of -8 inches every hour and need to get **back to 0 inches, the normal level.**

When will the water level be back down to normal?



time (hours after noon, the first day)	water-level (inches above normal)
0 hrs	132 in
+1 hr ↓ 1	124 ↓ -8 in
2	116
3	108
...	...
x hours	0 inches

Solve: $132 - 8x = 0$

Analytic

constant change: -8 in/hr

Solve the equation. What is the first step? Think layers.

When will the water level be back down to normal?



Analytic

constant change: -8 in/hr

time (hours after noon, the first day)	water-level (inches above normal)
0 hrs	132 in
+1 hr ↓ 1	124 ↓ -8 in
2	116
3	108
...	...
x hours	0 inches

$$132 - 8x = 0$$

Solve:

1. start with x
2. multiply by -8
3. add 132 to that value
4. get an answer of 0

Solve the equation. What is the first step? Think layers.

When will the water level be back down to normal?



time (hours after noon, the first day)	water-level (inches above normal)
0 hrs	132 in
+1 hr ↓ 1	124 ↓ -8 in
2	116
3	108
...	...
x hours	0 inches

$$132 - 8x = 0$$

Solve:

1. start with x
2. multiply by -8
3. add 132 to that value
4. get an answer of 0

Analytic

constant change: -8 in/hr

$$\boxed{x} \cdot (-8) + 132 = 0$$

Solve the equation.

When will the water level be back down to normal?



time (hours after noon, the first day)	water-level (inches above normal)
0 hrs	132 in
+1 hr ↓ 1	124 ↓ -8 in
2	116
3	108
...	...
x hours	0 inches

$$132 - 8x = 0$$

Analytic

constant change: -8 in/hr

$$\boxed{x} \cdot (-8) + 132 = 0$$

Solve the equation.

When will the water level be back down to normal?



time (hours after noon, the first day)	water-level (inches above normal)
0 hrs	132 in
+1 hr ↓ 1	124 ↓ -8 in
2	116
3	108
...	...
x hours	0 inches

$$\begin{array}{rcl} 132 - 8x & = & 0 \\ -132 & & -132 \\ \hline -8x & = & -132 \end{array}$$

Analytic

constant change: -8 in/hr

$$\boxed{x} \cdot (-8) + 132 = 0$$

Solve the equation.

When will the water level be back down to normal?



time (hours after noon, the first day)	water-level (inches above normal)
0 hrs	132 in
+1 hr ↓ 1	124 ↓ -8 in
2	116
3	108
...	...
x hours	0 inches

Analytic

constant change: -8 in/hr

$$\boxed{x} \cdot (-8) + 132 = 0$$

$$132 - 8x = 0$$

$$\frac{-132}{-8x} = \frac{-132}{-8x}$$

$$-8x = -132$$

$$\frac{-8x}{-8} = \frac{-132}{-8}$$

Solve the equation.

When will the water level be back down to normal?



time (hours after noon, the first day)	water-level (inches above normal)
0 hrs	132 in
+1 hr ↓ 1	124 ↓ -8 in
2	116
3	108
...	...
x hours	0 inches

Analytic
constant change: -8 in/hr

$$x \cdot (-8) + 132 = 0$$

$$\begin{aligned} 132 - 8x &= 0 \\ \frac{-132}{-8x} &= \frac{-132}{-8x} \\ \frac{-132}{-8} &= \frac{-132}{-8} \\ x &= \frac{33}{2} \end{aligned}$$

Solve the equation.

When will the water level be back down to normal?



time (hours after noon, the first day)	water-level (inches above normal)
0 hrs	132 in
+1 hr ↓ 1	124 ↓ -8 in
2	116
3	108
...	...
x hours	0 inches

Analytic
constant change: -8 in/hr

$x = 16.5$ hours after noon the first day

$$\begin{aligned} 132 - 8x &= 0 \\ -132 & \quad -132 \\ \hline -8x &= -132 \\ \frac{-8x}{-8} &= \frac{-132}{-8} \\ x &= \frac{33}{2} \\ x &= 16.5 \end{aligned}$$

Solve the equation.

Homework Options for Teacher

- 1. Do a google search for:
multi-step equation practice or
multi-step equation filetype:pdf**
- 2. Your State Assessment practice materials on multi-step linear equations.**
- 3. Local textbook resources for multi-step linear equations.**
- 4. Develop locally-applicable multi-step equation problems.**

On #4, look around your town and find some situations that have a starting value, a constant change, and some output value. For instance, number of \$1.50 parking tickets the city would need to assign in order to exceed \$2000 when they already have \$450-- find a picture that matches some content and put it up for students with a simple and direct question. Make the picture noticeable for students (local application).

Disclaimer

**All photos contained are used under
creative commons rights.**

World's Largest Ball of Twine by auvet

<http://www.flickr.com/photos/auvet/860982521/>

**Shrek is owned by DreamWorks
Animation**

Loch Laggan Dam by graeme_smith

http://www.flickr.com/photos/graeme_smith/1147054638/