

The corresponding phasor diagram is shown in Figure 14.10.3. Notice that all the phasors lie on a circular arc of radius R , with each successive phasor differed in phase by $\Delta\beta$.

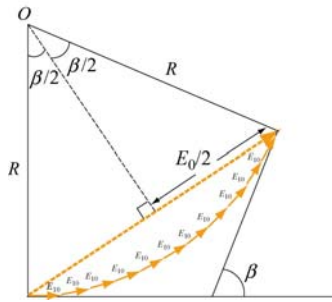


Figure 14.10.3 Phasor diagram for determining the time-independent portion of E .

From the figure, we see that

$$\sin \frac{\beta}{2} = \frac{E_0/2}{R} \quad (14.10.12)$$

Since the length of the arc is $NE_{10} = R\beta$, we have

$$E_0 = 2R \sin\left(\frac{\beta}{2}\right) = 2 \frac{NE_{10}}{\beta} \sin\left(\frac{\beta}{2}\right) = E_{10} \left[\frac{\sin(\beta/2)}{\Delta\beta/2} \right] \quad (14.10.13)$$

where $\beta = N\Delta\beta$. The result is completely consistent with that obtained in Eq. (14.6.11) using the algebraic approach. The intensity is proportional to E_0^2 , and again we write it as

$$I = \frac{I_0}{N^2} \left[\frac{\sin(\beta/2)}{\Delta\beta/2} \right]^2 = I_0 \left[\frac{\sin(\beta/2)}{\beta/2} \right]^2 \quad (14.10.14)$$

which reproduces the result shown in Eq. (14.6.15).

14.11 Solved Problems

14.11.1 Double-Slit Experiment

In Young's double-slit experiment, suppose the separation between the two slits is $d=0.320$ mm. If a beam of 500-nm light strikes the slits and produces an interference pattern. How many maxima will there be in the angular range $-45.0^\circ < \theta < 45.0^\circ$?

Solution:

On the viewing screen, light intensity is a maximum when the two waves interfere constructively. This occurs when

$$d \sin \theta = m\lambda, \quad m=0, \pm 1, \pm 2, \dots \quad (14.11.1)$$

where λ is the wavelength of the light. At $\theta = 45.0^\circ$, $d = 3.20 \times 10^{-4}$ m and $\lambda = 500 \times 10^{-9}$ m, we get

$$m = \frac{d \sin \theta}{\lambda} = 452.5 \quad (14.11.2)$$

Thus, there are 452 maxima in the range $0 < \theta < 45.0^\circ$. By symmetry, there are also 452 maxima in the range $-45.0^\circ < \theta < 0$. Including the one for $m = 0$ straight ahead, the total number of maxima is

$$N = 452 + 452 + 1 = 905 \quad (14.11.3)$$

14.11.2 Phase Difference

In the double-slit interference experiment shown in Figure 14.2.3, suppose $d = 0.100$ mm and $L = 1.00$ m, and the incident light is monochromatic with a wavelength $\lambda = 500$ nm.

- (a) What is the phase difference between the two waves arriving at a point P on the screen when $\theta = 0.800^\circ$?
- (b) What is the phase difference between the two waves arriving at a point P on the screen when $y = 4.00$ mm?
- (c) If $\phi = 1/3$ rad, what is the value of θ ?
- (d) If the path difference is $\delta = \lambda/4$, what is the value of θ ?

Solutions:

- (a) The phase difference ϕ between the two wavefronts is given by

$$\phi = \frac{2\pi}{\lambda} \delta = \frac{2\pi}{\lambda} d \sin \theta \quad (14.11.4)$$

With $\theta = 0.800^\circ$, we have

$$\phi = \frac{2\pi}{(5.00 \times 10^{-7} \text{ m})} (1.00 \times 10^{-4} \text{ m}) \sin(0.800^\circ) = 17.5 \text{ rad} \quad (14.11.5)$$

(b) When θ is small, we make use of the approximation $\sin \theta \approx \tan \theta = y/L$. Thus, the phase difference becomes

$$\phi \approx \frac{2\pi}{\lambda} d \left(\frac{y}{L} \right) \quad (14.11.6)$$

For $y = 4.00 \text{ mm}$, we have

$$\phi = \frac{2\pi}{(5.00 \times 10^{-7} \text{ m})} (1.00 \times 10^{-4} \text{ m}) \left(\frac{4.00 \times 10^{-3} \text{ m}}{1.00 \text{ m}} \right) = 5.03 \text{ rad} \quad (14.11.7)$$

(c) For $\phi = 1/3 \text{ rad}$, we have

$$\frac{1}{3} \text{ rad} = \frac{2\pi}{\lambda} d \sin \theta = \frac{2\pi}{(5.00 \times 10^{-7} \text{ m})} (1.00 \times 10^{-4} \text{ m}) \sin \theta \quad (14.11.8)$$

which gives

$$\theta = 0.0152^\circ \quad (14.11.9)$$

(d) For $\delta = d \sin \theta = \lambda/4$, we have

$$\theta = \sin^{-1} \left(\frac{\lambda}{4d} \right) = \sin^{-1} \left[\frac{5.00 \times 10^{-7} \text{ m}}{4(1.00 \times 10^{-4} \text{ m})} \right] = 0.0716^\circ \quad (14.11.10)$$

14.11.3 Constructive Interference

Coherent light rays of wavelength λ are illuminated on a pair of slits separated by distance d at an angle θ_1 , as shown in Figure 14.11.1.

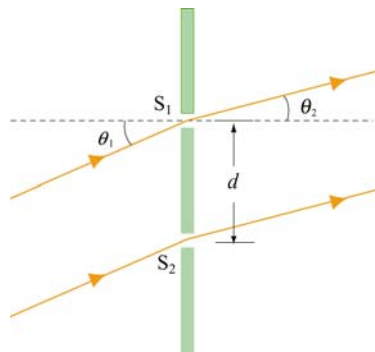


Figure 14.11.1

If an interference maximum is formed at an angle θ_2 at a screen far from the slits, find the relationship between θ_1 , θ_2 , d and λ .

Solution:

The path difference between the two rays is

$$\delta = d \sin \theta_1 - d \sin \theta_2 \quad (14.11.11)$$

The condition for constructive interference is $\delta = m\lambda$, where $m = 0, \pm 1, \pm 2, \dots$ is the order number. Thus, we have

$$d(\sin \theta_1 - \sin \theta_2) = m\lambda \quad (14.11.12)$$

14.11.4 Intensity in Double-Slit Interference

Let the intensity on the screen at a point P in a double-slit interference pattern be 60.0% of the maximum value.

- (a) What is the minimum phase difference (in radians) between sources?
 (b) In (a), what is the corresponding path difference if the wavelength of the light is $\lambda = 500 \text{ nm}$?

Solution:

(a) The average intensity is given by

$$I = I_0 \cos^2 \left(\frac{\phi}{2} \right) \quad (14.11.13)$$

where I_0 is the maximum light intensity. Thus,

$$0.60 = \cos^2 \left(\frac{\phi}{2} \right) \quad (14.11.14)$$

which yields

$$\phi = 2 \cos^{-1} \left(\sqrt{\frac{I}{I_0}} \right) = 2 \cos^{-1}(\sqrt{0.60}) = 78.5^\circ = 1.37 \text{ rad} \quad (14.11.15)$$

(b) The phase difference ϕ is related to the path difference δ and the wavelength λ by

$$\delta = \frac{\lambda \phi}{2\pi} = \frac{(500 \text{ nm})(1.37 \text{ rad})}{2\pi} = 109 \text{ nm} \quad (14.11.16)$$

14.11.5 Second-Order Bright Fringe

A monochromatic light is incident on a single slit of width 0.800 mm, and a diffraction pattern is formed at a screen which is 0.800 m away from the slit. The second-order bright fringe is at a distance 1.60 mm from the center of the central maximum. What is the wavelength of the incident light?

Solution:

The general condition for destructive interference is

$$\sin \theta = m \frac{\lambda}{a} \approx \frac{y}{L} \quad (14.11.17)$$

where small-angle approximation has been made. Thus, the position of the m -th order dark fringe measured from the central axis is

$$y_m = m \frac{\lambda L}{a} \quad (14.11.18)$$

Let the second bright fringe be located halfway between the second and the third dark fringes. That is,

$$y_{2b} = \frac{1}{2}(y_2 + y_3) = \frac{1}{2}(2+3) \frac{\lambda L}{a} = \frac{5\lambda L}{2a} \quad (14.11.19)$$

The approximate wavelength of the incident light is then

$$\lambda \approx \frac{2a y_{2b}}{5L} = \frac{2(0.800 \times 10^{-3} \text{ m})(1.60 \times 10^{-3} \text{ m})}{5(0.800 \text{ m})} = 6.40 \times 10^{-7} \text{ m} \quad (14.11.20)$$

14.11.6 Intensity in Double-Slit Diffraction

Coherent light with a wavelength of $\lambda = 500 \text{ nm}$ is sent through two parallel slits, each having a width $a = 0.700 \mu\text{m}$. The distance between the centers of the slits is $d = 2.80 \mu\text{m}$. The screen has a semi-cylindrical shape, with its axis at the midline between the slits.

(a) Find the direction of the interference maxima on the screen. Express your answers in terms of the angle away from the bisector of the line joining the slits.

(b) How many bright fringes appear on the screen?

14.12 Conceptual Questions

1. In Young's double-slit experiment, what happens to the spacing between the fringes if
 - (a) the slit separation is increased?
 - (b) the wavelength of the incident light is decreased?
 - (c) if the distance between the slits and the viewing screen is increased?
2. In Young's double-slit experiment, how would the interference pattern change if white light is used?
3. Explain why the light from the two headlights of a distant car does not produce an interference pattern.
4. What happens to the width of the central maximum in a single-slit diffraction if the slit width is increased?
5. In a single-slit diffraction, what happens to the intensity pattern if the slit width becomes narrower and narrower?
6. In calculating the intensity in double-slit interference, can we simply add the intensities from each of the two slits?

14.13 Additional Problems

14.13.1 Double-Slit Interference

In the double-slit interference experiment, suppose the slits are separated by $d = 1.00$ cm and the viewing screen is located at a distance $L = 1.20$ m from the slits. Let the incident light be monochromatic with a wavelength $\lambda = 500$ nm.

- (a) Calculate the spacing between the adjacent bright fringes.
- (b) What is the distance between the third-order fringe and the center line?

14.13.2 Interference-Diffraction Pattern

In the double-slit Fraunhofer interference-diffraction experiment, if the slits of width 0.010 mm are separated by a distance 0.20 mm, and the incident light is monochromatic with a wavelength $\lambda = 600$ nm. How many bright fringes are there in the central diffraction maximum?

14.13.3 Three-Slit Interference

Suppose a monochromatic coherent light source of wavelength λ passes through three parallel slits, each separated by a distance d from its neighbor.

(a) Show that the positions of the interference minima on a viewing screen a distance $L \gg d$ away is approximately given by

$$y = n \frac{\lambda L}{3d}, \quad n = 1, 2, 4, 5, 7, 8, 10, \dots$$

where n is not a multiple of 3.

(b) Let $L = 1.20 \text{ m}$, $\lambda = 450 \text{ nm}$ and $d = 0.10 \text{ mm}$. What is the spacing between the successive minima?

14.13.4 Intensity of Double-Slit Interference

In the double-slit interference experiment, suppose the slits are of different size, and the fields at a point P on the viewing screen are

$$E_1 = E_{10} \sin \omega t, \quad E_2 = E_{20} \sin(\omega t + \phi)$$

Show that the intensity at P is

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi$$

where I_1 and I_2 are the intensities due to the light from each slit.

14.13.5 Secondary Maxima

In a single-slit diffraction pattern, we have shown in 14.6 that the intensity is

$$I = I_0 \left[\frac{\sin(\beta/2)}{\beta/2} \right]^2 = I_0 \left[\frac{\sin(\pi a \sin \theta / \lambda)}{\pi a \sin \theta / \lambda} \right]^2$$

(a) Explain why the condition for the secondary maxima is not given by $\beta/2 = (m+1/2)\pi$, $m = 1, 2, 3, \dots$

(b) By differentiating the expression above for I , show that the condition for secondary maxima is

$$\frac{\beta}{2} = \tan\left(\frac{\beta}{2}\right)$$

(c) Plot the curves $y = \beta/2$ and $y = \tan(\beta/2)$. Using a calculator which has a graphing function, or mathematical software, find the values of β at which the two curves intersect, and hence, the values of β for the first and second secondary maxima. Compare your results with $\beta/2 = (m+1/2)\pi$.

14.13.6 Interference-Diffraction Pattern

If there are 7 fringes in the central diffraction maximum in a double-slit interference pattern, what can you conclude about the slit width and separation?