

~~We strongly recommend that you read about a topic before it is covered in lectures.~~

Lecture Date	Topics Covered	Reading from Giancoli
#20 Mon 4/1	Inductance RL Circuits Magnetic Field Energy	Chapter 30 through Sect. 30-4
Tue 4/2	Due Date Electric Motor! Testing 1-5 PM, 26-110	
#21 Wed 4/3	Magnetic Materials Dia-, Para-, and Ferromagnetism Prize Ceremony Motor Contest	Lecture Supplement Sect. 28-8, 28-9, 28-10
#22 Fri 4/5	Hysteresis Electromagnets Bohr Magneton Maxwell's Equations	Sect. 28-8 & 28-9 Sect. 32-3 & 40-7

~~Due before 4 PM, Friday, April 5 in 4-339B.~~

~~Problem 6.1~~

~~Flip Coil.~~

~~Giancoli 29-62.~~

~~Problem 6.2~~

~~Displacement Current.~~

~~Giancoli 32-4.~~

~~Problem 6.3~~

~~Self-inductance of a toroid.~~

~~Giancoli 30-48.~~

~~Problem 6.4~~

~~Magnetic Field Energy and Self-Inductance.~~

A long straight solid cylindrical conducting wire with radius R carries a steady uniform current I .

- ~~Calculate the magnetic field energy inside a length ℓ of the wire.~~
- ~~What is the contribution of the interior portion of the conductor to the total self-inductance?~~

~~Problem 6.5~~

~~RL Circuit.~~

~~A coil with resistance 0.05Ω and self-inductance 0.09 H is connected across a 12-volt car battery of negligible internal resistance.~~

- ~~How long after the switch is closed will the current reach 95 percent of its final value?~~
- ~~At that time how much energy (in Joules) is stored in the magnetic field?~~
- ~~How much energy has been delivered by the battery up to that time?~~

By Giancoli Equation (30-4) (p. 758), the toroid's self-inductance is then

$$\bar{L} = \frac{N\Phi_B}{I} \approx \frac{\mu_0 N^2 d^2}{8r_0} ,$$

as we were to show. This is indeed consistent with the result for a solenoid of length $l = 2\pi r_0$ and cross-sectional area $A = \pi(d/2)^2$ (see Giancoli Example 30-3, p. 759), as it should be:

$$L = \frac{\mu_0 N^2 A}{l} = \frac{\mu_0 N^2 \pi (d/2)^2}{2\pi r_0} = \frac{\mu_0 N^2 d^2}{8r_0} .$$

Our toroid is just a solenoid bent into a donut, and to the extent that we can ignore terms of order d/r_0 , the bending has no effect.

(b) Plugging in the given numbers,

$$\bar{L} \approx \frac{(4\pi \times 10^{-7})(550)^2(0.020)^2}{8(0.25)} = 76 \mu\text{H} .$$

Problem 6.4

Magnetic field energy and self-inductance.

(a) From the results of problem 5.2, we have the following expression for the magnetic field magnitude inside the wire as a function of distance r from the wire's axis:

$$B = \frac{\mu_0 I r}{2\pi R^2} .$$

Using Giancoli Equation (30-7) (p. 761), we can express the magnetic field energy density (that is, Joules per cubic meter) as

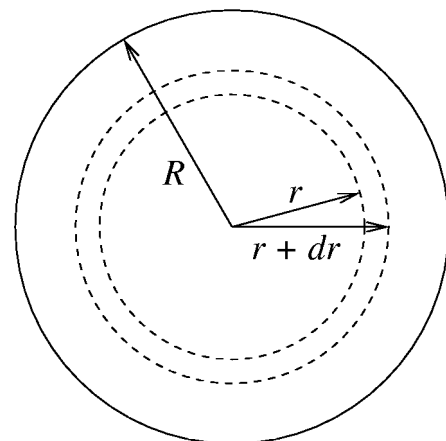
$$u = \frac{B^2}{2\mu_0} = \frac{1}{2\mu_0} \left(\frac{\mu_0 I r}{2\pi R^2} \right)^2 = \frac{\mu_0 I^2 r^2}{8\pi^2 R^4} .$$

Consider now a cylindrical shell of length ℓ , radius r , and thickness dr within the wire. Such a shell has volume

$$d(\text{volume}) = 2\pi r \ell dr ,$$

and in the limit $dr \rightarrow 0$ the magnetic field energy density is constant throughout the shell. The total magnetic field energy inside a length ℓ of the wire can thus be found by integrating:

$$U_{\text{in}} = \int u d(\text{volume}) = \int_0^R \left(\frac{\mu_0 I^2 r^2}{8\pi^2 R^4} \right) (2\pi r \ell dr) = \left(\frac{\mu_0 I^2 \ell}{4\pi R^4} \right) \int_0^R r^3 dr = \frac{\mu_0 I^2 \ell}{16\pi} .$$



(b) From the relationship between stored magnetic field energy and self-inductance (Giancoli Equation (30-6), p. 761), we can identify the self-inductance due to the interior of the wire:

$$U_{\text{in}} = \frac{1}{2} L_{\text{in}} I^2 = \frac{\mu_0 I^2 \ell}{16\pi} \implies L_{\text{in}} = \frac{\mu_0 \ell}{8\pi} .$$

The wire therefore has a self-inductance per unit length $\mu_0/8\pi$ associated with its internal magnetic field.

~~Problem 6.5~~

~~RL circuit.~~

~~Giancoli Equation (30-9) (p. 762) gives us an expression for the time-dependent current in an LR circuit connected to a battery of voltage V_0 at time $t = 0$:~~

$$I(t) = \frac{V_0}{R} (1 - e^{-t/\tau}) , \quad \tau = L/R .$$

~~We will make good use of this! For our circuit, the time constant is $\tau = (0.09)/(0.05) = 1.8$ s.~~

~~(a) The final value of the current is simply V_0/R . Let $\alpha = 0.05$; the condition that the current has reached 95% of its final value can then be expressed as~~

$$\begin{aligned} I &= \frac{V_0}{R} (1 - e^{-t/\tau}) = (1 - \alpha) \frac{V_0}{R} \\ &\implies e^{-t/\tau} = \alpha \\ &\implies t = -\tau \ln \alpha = 5.4 \text{ s} . \end{aligned}$$

~~(b) Using Giancoli Equation (30-6) (p. 761), we have for the energy stored in the magnetic field~~

$$U = \frac{1}{2} L I^2 = \frac{1}{2} L (1 - \alpha)^2 \frac{V_0^2}{R^2} = \frac{1}{2} (0.09) (0.95)^2 \frac{(12)^2}{(0.05)^2} \simeq 2300 \text{ J} .$$

~~(c) The power delivered by the battery at any instant is~~

$$P(t) = V_0 I(t) = \frac{V_0^2}{R} (1 - e^{-t/\tau}) .$$

~~To find the total energy W delivered by the battery up to the time t found in part (a) we integrate:~~

$$\begin{aligned} W &= \int_0^t P(t') dt' = \frac{V_0^2}{R} \int_0^t (1 - e^{-t'/\tau}) dt' = \frac{V_0^2}{R} [t' + \tau e^{-t'/\tau}]_0^t \\ &= \frac{V_0^2}{R} [t - (1 - \alpha)\tau] = \frac{(12)^2}{(0.05)} [5.4 - (0.95)(1.8)] \simeq 10600 \text{ J} . \end{aligned}$$