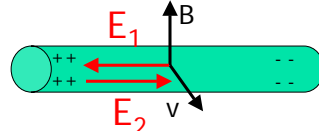


## Moving rod in uniform B

- Let's move a conducting rod in a uniform B

- Charges move with velocity  $v$  // x axis
- B // y axis



- What happens?

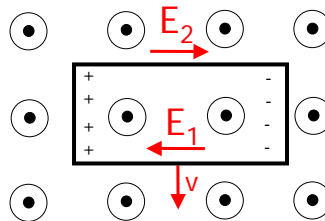
- Lorentz force:  $\vec{F}_{Lorentz} = q \frac{\vec{v}}{c} \times \vec{B} = q \vec{E}_1$
- Electric field  $E_1$  causes separation of charges on the wire
- Separation of charges creates an opposite electric field  $E_2$  that exactly compensates  $E_1$  and equilibrium is established:

$$\vec{E}_2 = -\frac{\vec{v}}{c} \times \vec{B}$$

## Moving a loop in uniform B

- Now move a rectangular loop of wire in B

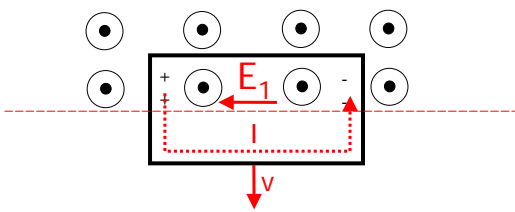
- Same velocity
- Same B



- What happens?

- Lorentz force  $\rightarrow E_1$
- $E_1 \rightarrow$  separation of charges on the wire
- Separation of charges creates opposite electric field  $E_2 = -E_1$ :

## What if B is non uniform?

- Now move the rectangular loop of wire in non uniform B
    - Velocity  $v$
    - $B = B_0$  above - - -
    - $B = 0$  below - - -
- 
- What happens?
    - Lorentz force  $\rightarrow E_1$
    - $E_1 \rightarrow$  separation of charges on the wire
    - Separation of charges creates charges to flow in the loop (no opposing force in the bottom part!)
  - This phenomenon is called **electromagnetic induction**

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## Comments on induction

Please notice the following:

- End of electrostatics!

$$\oint_{loop} \vec{E} \cdot d\vec{l} \neq 0 \text{ or } \nabla \times \vec{E} \neq 0$$

- The current flowing in top leg of the loop will feel a force  $F_B$  from B pointing up
  - Lenz's law

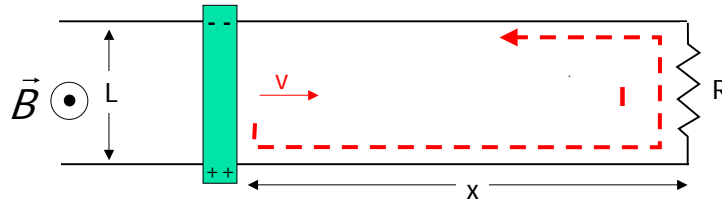
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## Induced emf

- Consider a sliding conducting bar on rails closed on a resistor R in a region of constant magnetic field B



- Charge separation in the bar will induce current → e.m.f.

$$e.m.f. = \frac{1}{q} W(- \rightarrow +) = \frac{1}{q} \int_{-}^{+} \vec{F} \cdot d\vec{s} = \frac{1}{c} \int_{-}^{+} (\vec{v} \times \vec{B}) \cdot d\vec{s} = \frac{vBL}{c}$$

- Current flowing in the loop:  $I = \frac{vBL}{cR}$

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## Faraday's law

- EMF in the loop:  $e.m.f. = \frac{vBL}{c} = \frac{BL}{c} \frac{dx}{dt}$
- Magnetic flux in the rectangle is defined as:  $\Phi_B = Blx$
- Combine the two keeping in mind that given the direction of v, flux decreases with time:

→ Faraday's law: 
$$e.m.f. = -\frac{1}{c} \frac{\partial \Phi_B}{\partial t}$$

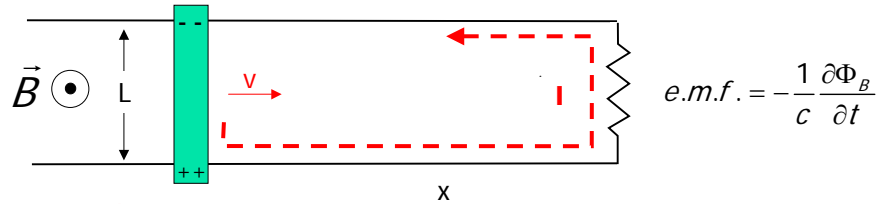
- The minus sign is important: **Lentz's law**
  - It indicates that the direction of the current is such to oppose the changes in flux of B: ~"electromagnetic inertia"

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## Thoughts on Lenz's law



### Lenz's law:

The current generated in wire opposes changes in flux of  $B$

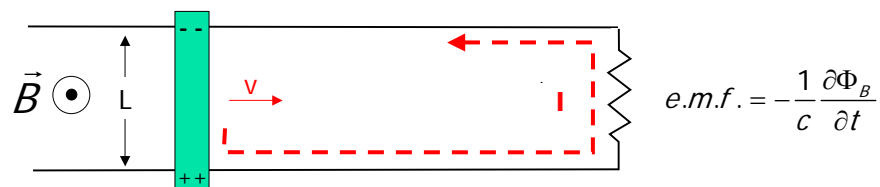
- $v$  is L→R:
  - Flux of  $B$  decreases over time → e.m.f. is created with direction that compensates this change: counterclockwise
- $v$  is R→L:
  - Flux of  $B$  increases over time → e.m.f. is created with direction that compensates this change: clockwise

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## Another way of looking at Lenz



When current flows in magnetic field it feels a force

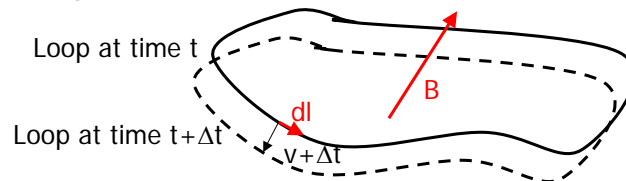
Lenz's law: the force will be will try to slow down the bar

- If  $I$  clockwise:
  - It creates a  $B$  pointing into the board →  $\mathbf{I} \times \mathbf{B}$  points to the left
- If  $I$  counterclockwise:
  - It creates a  $B$  pointing out of the board →  $\mathbf{I} \times \mathbf{B}$  points to the right

NB: the – sign in Lenz's law is what allows conservation of energy

## General proof of Faraday's law

- Consider a loop of arbitrary shape moving with velocity  $\mathbf{v}$  through a static magnetic field  $\mathbf{B}$



- At time  $t$ , the flux through the loop is:  $\Phi_B = \int_S \vec{B} \cdot d\vec{a}$
- How does it change when  $t \rightarrow t + \Delta t$ ?  

$$\Delta\Phi_B = \Phi_B(t + \Delta t) - \Phi_B(t) = \Phi_{\text{ribbon}} = \int_{\text{ribbon}} \vec{B} \cdot d\vec{a}$$
- On the ribbon:  

$$d\vec{a} = (\vec{v} \Delta t) \times d\vec{l}$$

## Proof of Faraday's law(2)

- This means that:

$$\Delta\Phi_B = \int_{\text{ribbon}} \vec{B} \cdot d\vec{a} = \int_{\text{ribbon}} \vec{B} \cdot (\vec{v} \Delta t) \times d\vec{l} = \int_{\text{ribbon}} \Delta t \vec{B} \cdot (\vec{v} \times d\vec{l})$$

- Using the identity  $\vec{a} \cdot (\vec{b} \times \vec{c}) = (\vec{a} \times \vec{b}) \cdot \vec{c}$  we obtain:

$$\Delta\Phi_B = \int_{\text{loop}} \Delta t \vec{B} \cdot (\vec{v} \times d\vec{l}) = \Delta t \int_{\text{loop}} \vec{B} \times \vec{v} \cdot d\vec{l} = -\Delta t \int_{\text{ribbon}} \vec{v} \times \vec{B} \cdot d\vec{l}$$

$$\text{For } \Delta t \rightarrow 0: \quad \frac{\partial\Phi_B}{\partial t} = -c \int_{\text{loop}} \left( \frac{\vec{v}}{c} \times \vec{B} \right) \cdot d\vec{l}$$

Since  $\mathbf{v}/c \times \mathbf{B}$  is the magnetic force for unit charge

→ its line integral on the loop is the work necessary to move a unit charge around the wire: e.m.f!

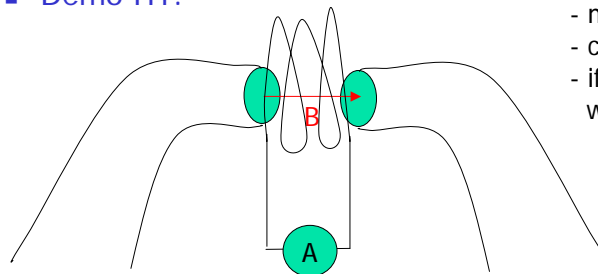
$$\rightarrow e.m.f. = -\frac{1}{c} \frac{\partial\Phi_B}{\partial t}$$

## Work from B???

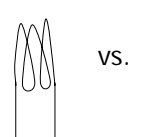
- Faraday's law:  $e.m.f. = -\frac{1}{c} \frac{\partial \Phi_B}{\partial t}$
- This means that  $\vec{v}/c \times \vec{B}$  integrated over the loop is the work that we have to do to move a unit charge around the loop
- But last time we proved that B cannot do work
- Are these 2 statements inconsistent???
- No, the work done to move the charges is not done by B
- It's done by whoever is moving the loop in B

## Verification of Faradya's law

- Faraday's law:  $e.m.f. = -\frac{1}{c} \frac{\partial \Phi_B}{\partial t}$
- What does it mean?
  - E.m.f. Is produced when the flux of B changes over time
  - → area of the loop cannot be null!
- Demo H1:



- move loop in B
- current flows in wire
- if we used instead a wire with 0 area: no I



# “Relativity”

- What if loop is static and B changes?
  - Relativity tells me that we should get the same result
    - Same problem from another reference frame
- Does this make sense?
  - Charges do not move in the other reference frame
  - What causes the force? The induced electric field
- Since e.m.f. is the work necessary to move a unit charge around the loop:

$$e.m.f. = \oint_C \vec{E} \cdot d\vec{l}$$

- Demo H3: magnet bar moving in the loop

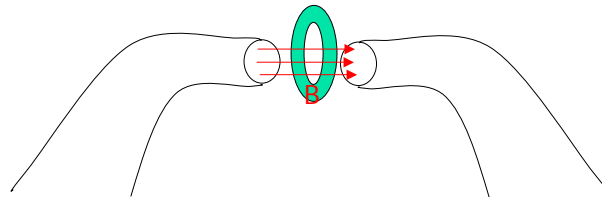
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# Application of Lenz's law

- H5: disk falling in a magnetic field B
  - Create B with e electromagnets (solenoid on Fe core)



- What happen if we drop a disk of conductor?
  - With and without B
- What if we drop a full disk
- What if we drop a disk with a cut?

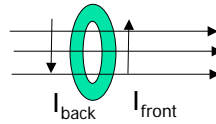
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## Explanation

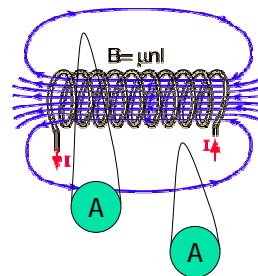
- **Falling Loop:**
  - B perpendicular to loop is limited in space  $\rightarrow$  flux changes during fall  $\rightarrow$  induced I
  - $\rightarrow$  loop will levitate (Eddy currents)



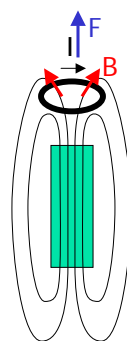
- **Falling Disk**
  - Will it slow down?
- **Falling open ring**
  - Will it levitate?

## More demos on Faraday's law

- **H8: current generated by a solenoid**
  - Where to put the loop of wire to have current?
  - Remember: B of solenoid is 0 outside
  - Switch I on and off



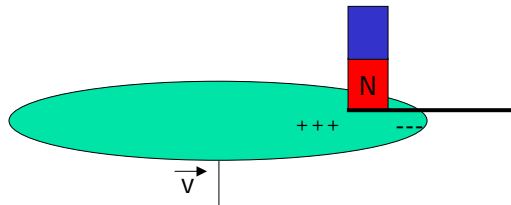
- **H22 Levitating rings**





## More demos on Faraday's law

- H15a: current generated by a solenoid
  - Spinning disk of conductor
  - Magnet sitting on top separated by a plastic sheet
  - When disk starts spinning, magnet levitates
  - Why?



## Faraday's law in differential form

- Faraday's law in integral form:  $e.m.f. = -\frac{1}{c} \frac{\partial \Phi_B}{\partial t}$
- Right term (apply Stokes):  $e.m.f. = \oint_C \vec{E} \cdot d\vec{l} = \int_S \vec{\nabla} \times \vec{E} \cdot d\vec{a}$
- Left term:  $-\frac{1}{c} \frac{\partial \Phi_B}{\partial t} = -\frac{1}{c} \frac{\partial}{\partial t} \int_S \vec{B} \cdot d\vec{a}$   
 $\rightarrow \int_S \left( \vec{\nabla} \times \vec{E} + \frac{1}{c} \frac{\partial \vec{B}}{\partial t} \right) \cdot d\vec{a} = 0$
- Since this is valid for any surface:  $\vec{\nabla} \times \vec{E} = -\frac{1}{c} \frac{\partial \vec{B}}{\partial t}$ 
  - curl E is not longer zero: bye bye electrostatics!
  - Explicit link between E and B, as in relativity!

## Another step toward Maxwell's equations...

- All the equations in differential form that we found so far:

$$\left\{ \begin{array}{ll} \vec{\nabla} \cdot \vec{E} = 4\pi\rho & \leftarrow \text{Relates E and charge density } (\rho) \text{ - Gauss} \\ \vec{\nabla} \cdot \vec{B} = 0 & \leftarrow \text{Magnetic field lines are closed} \\ \vec{\nabla} \times \vec{E} = -\frac{1}{c} \frac{\partial \vec{B}}{\partial t} & \leftarrow \text{Change in B creates E - Faraday} \\ \vec{\nabla} \times \vec{B} = \frac{4\pi}{c} \vec{J} & \leftarrow \text{Relates B and its sources (J) - Ampere} \end{array} \right.$$

- Another step toward Maxwell's equations: one last missing ingredient... Can you guess what?
  - Symmetry will guide you... Hint:
  - Or vector calculus... Hint: take the divergence of Faraday's law...

## Summary and outlook

- Today:

- Faraday's (and Lenz's) law:

- Integral form:  $e.m.f. = -\frac{1}{c} \frac{\partial \Phi_B}{\partial t}$

- Differential form:  $\vec{\nabla} \times \vec{E} = -\frac{1}{c} \frac{\partial \vec{B}}{\partial t}$

- Next time:

- Mutual and self inductance