

Ampere's law

- In electrostatics, the electric field E and its sources (charges) are related by Gauss's law:

$$\int_{\text{Surface}} \vec{E} \cdot d\vec{A} = 4\pi Q_{\text{encl}}$$

- Why useful? When symmetry applies, E can be easily computed
- Similarly, in magnetism the magnetic field B and its sources (currents) are related by Ampere's law:

$$\oint_c \vec{B} \cdot d\vec{s} = \frac{4\pi}{c} I_{\text{encl}}$$

- Why useful? When symmetry applies, E can be easily computed
- NB: This is a line integral!

NB: no demonstration has been given so far for Ampere's law.

Application of Ampere's law:

B created by current in a wire

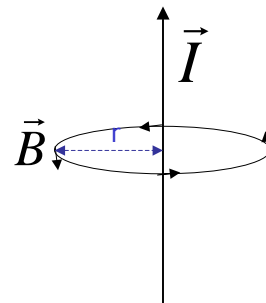
- Long, straight wire in which flows a current I
- Calculate magnetic field B created by I

- Solution:

- Apply Ampere's law:

$$\oint_c \vec{B} \cdot d\vec{s} = B(r)2\pi r = \frac{4\pi}{c} I_{\text{encl}} \Rightarrow \boxed{\vec{B} = \frac{2I}{cr} \hat{\phi}}$$

- Direction: right hand rule
- NB: $B_{\text{wire}} \sim 1/r$. Does this look familiar?
 - Remember E created by a line of charge:
 - Coincidence? Not at all...



$$E(r) = \frac{2\lambda}{r}$$

Force between 2 wires

- Force on wire 1 due to magnetic field B created by wire 2:

$$\vec{F}_1 = \frac{I_1}{c} L \hat{n} \times \vec{B}_2$$

- Magnetic field created by wire 2: $\vec{B}_2 = \frac{2I_2}{cr} \hat{\phi}$

- Total force F: $F = \frac{2I_2 I_1}{c^2 r} L$

- Usually we quote the force/unit length: $\frac{F}{L} = \frac{2I_2 I_1}{c^2 r}$

- Direction? $\vec{F} \propto I_1 \times \hat{\phi}_2$ Using right hand rule:
 - I_1 and I_2 parallel: attractive
 - I_1 and I_2 anti-parallel: repulsive

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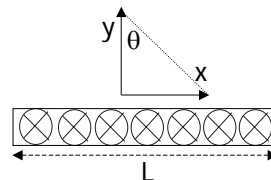
Can we test this experimentally? Demo G8, G9

Another application of Ampere's law:

B created by sheet of current

- Calculate the magnetic field B created by current flowing in a sheet of conductor

- Current // -z axis (into the page)
- Width of sheet of conductor: L
- Current in a metal sheet ~ N parallel wires



- Solution:

- B from a wire is known: $\vec{B} = \frac{2I}{cr} \hat{\phi}$
- Just apply superposition...
 - Direction: for $y > 0$: B // +x; for $y < 0$: B // -x
 - Magnitude: integrate $dB = B$ field from each infinitesimal wire

$$B = \frac{2I}{Lc} (2\theta) \quad \text{When } L \gg y, \theta \rightarrow \pi/2 \Rightarrow B = \frac{2\pi I}{Lc}$$

NB: magnitude of B does not depend on y. As for E of sheet of charges

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Another application of Ampere's law: B created by plane of current

Calculation:

$$B = \int dB_x \quad (\text{only component // } \hat{x} \text{ survives because of symmetry})$$

$$= \int_{x=-L/2}^{x=L/2} \left(\frac{2dl}{cr} \right) \cos \theta$$

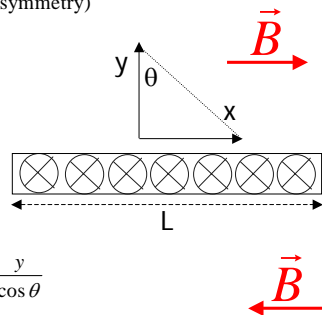
$$= \int_{x=-L/2}^{x=L/2} \frac{2 \left(\frac{I}{L} dx \right)}{cr} \cos \theta$$

$$= \frac{2I}{Lc} \int_{x=-L/2}^{x=L/2} \frac{dx}{r} \cos \theta$$

$$= \frac{2I}{Lc} \int_{-\theta}^{\theta} \frac{y d\theta}{\frac{y}{\cos \theta}} \cos \theta$$

$$x = y \tan \theta \Rightarrow dx = \frac{y d\theta}{\cos^2 \theta}; \quad r = \frac{y}{\cos \theta}$$

$$\Rightarrow \vec{B} = \pm \frac{4I}{Lc} \theta \hat{x} \quad + \text{ for } y > 0; - \text{ for } y < 0$$



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More on B from sheet of current

- If we define current per unit length $K = I/L$: $\vec{B} = \pm \frac{2\pi K}{c} \hat{x}$

- What is the change of B across the sheet of current?

$$\Delta B = \frac{4\pi K}{c}$$

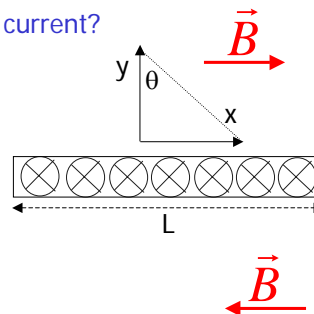
- Does it ring a bell?

- Yes, ΔE across a plane of charge!

$$\Delta E = 4\pi\sigma$$

- Another similarity between electric and magnetic fields.

...This must be more than a pure coincidence...



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Ampere's law in SI

- In SI Ampere's law takes the form: $\oint_C \vec{B} \cdot d\vec{s} = \mu_0 I_{encl}$
 - where $\mu_0 = 4 \cdot 10^{-7} \text{ N/A}^2$ is the magnetic permeability of free space
- Be careful not to mix cgs and SI formulae!
 - To convert cgs \rightarrow SI: multiply by $\mu_0 c / (4\pi)$
 - Examples:
 - Magnetic field created by a wire: $\vec{B} = \frac{2I}{cr} \hat{\phi} \Rightarrow \vec{B} = \frac{\mu_0 I}{2\pi r} \hat{\phi}$
 - Force between 2 wires:
 - NB: factor $1/c$ missing in F_{Lorentz} in SI

$$\frac{F}{L} = \frac{2I_2 I_1}{c^2 r} \Rightarrow \frac{F}{L} = \frac{\mu_0 I_2 I_1}{2\pi r}$$

Divergence of B

- Consider the B produced by a wire of current: $\vec{B} = \frac{2I}{cr} \hat{\phi}$
- Calculate its divergence in Cartesian coordinates:

Given $r = \sqrt{x^2 + y^2}$ and $\hat{\phi} = \hat{y} \cos \phi - \hat{x} \sin \phi = \frac{x\hat{y}}{\sqrt{x^2 + y^2}} - \frac{y\hat{x}}{\sqrt{x^2 + y^2}} \Rightarrow$

$$\vec{B} = \frac{2I}{cr} \left(\frac{x\hat{y}}{\sqrt{x^2 + y^2}} - \frac{y\hat{x}}{\sqrt{x^2 + y^2}} \right) \Rightarrow \vec{\nabla} \cdot \vec{B} = \frac{2I}{cr} \left(\frac{2yx}{(x^2 + y^2)^2} - \frac{2xy}{(x^2 + y^2)^2} \right) = 0$$
- This is a general property of the magnetic field: $\vec{\nabla} \cdot \vec{B} = 0$
- Similar equation for E: $\vec{\nabla} \cdot \vec{E} = 4\pi\rho$
 - The divergence of E is related to the density of electric charges
 - The divergence of B must be related to the density of magnetic charges
 - \rightarrow Magnetic monopole don't exist

(There may be magnetic monopoles leftover from the Early Universe, but never observed experimentally so far)

Thoughts on B

- What exactly is a magnetic field B?
 - Why does it have so much in common with electric field E?
 - Why should there be a field that acts only on moving charges?
- Answer: Special Relativity
 - Relativity: the physics must be the same in all reference frames
 - A charge at rest for observer 1 appears in motion to observer 2 that moves with a certain velocity w.r.t. observer 1:
 - Observer 1 will measure an electric field
 - Observer 2 will measure a magnetic field
 - Calculating attractive or repulsive force acting on a test charge in the 2 reference frames will lead to the same conclusion

Summary and outlook

- Today:
 - Magnetic Field B
 - Magnetic Force acting on charges in motion
 - Ampere's Law
- Next time:
 - Quick Introduction to Special Relativity
 - Goals:
 - Understand how and why Magnetism and Electricity are related
 - Finally play with some really cool physics!