

~~We strongly recommend that you read about a topic before it is covered in lectures.~~

<del>Lecture Date</del>	<del>Topics Covered</del>	<del>Reading from Giancoli</del>
<del>#17 Mon 3/18</del>	<del>Motional EMF Dynamos Eddy Currents Magnetic Braking</del>	<del>Sect. 29-3, 29-4, 29-5 &amp; 29-8</del>
<del>#18 Wed 3/20</del>	<del>Displacement Current (<i>Difficult Concept</i>) Synchronous Motors Induction Motors Secret Top, how does it work?</del>	<del>Sect. 32-1 Sect. 27-6 &amp; 31-8</del>
<del>#19 Fri 3/22</del>	<del><b>Vacation Special</b> * How do Magicians levitate women? (with demo) * Electric shock treatment (no demo) * Electrocardiogram Pacemakers (with demo) * Superconductivity Floating Bullet Trains * Aurora Borealis</del>	<del>Sect. 25-10 Page 606 Sect. 25-9 Page 694</del>

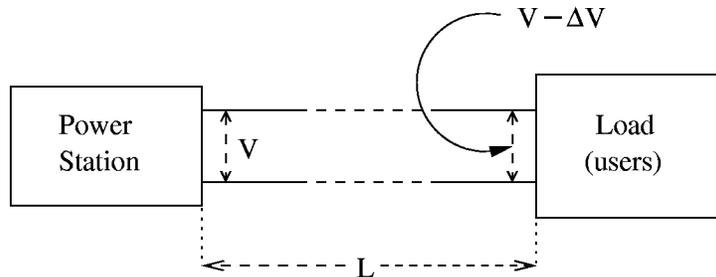
~~Enjoy the "Special", and Take Notes!~~

*Due before 4 PM, Friday, March 22 in 4-339B.*

**Problem 5.1**

*High Voltage Power Transmission.*

A power station transmits 200 MW of electric power, at a voltage  $V$  over a distance  $L$  of 300 miles to the users. The transmission lines are made of 1 inch diameter (about  $5 \text{ cm}^2$  cross sectional area) aluminum cables (see Table 25.1 in Giancoli).



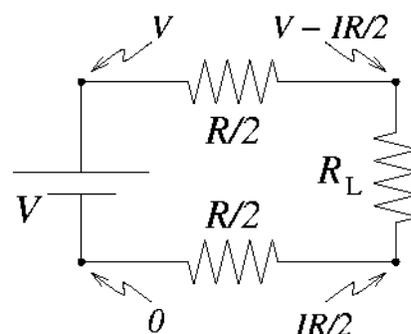
- Find the ohmic resistance  $R$  of the transmission lines at  $+40^\circ \text{C}$ , and at  $-40^\circ \text{C}$  (length  $2L$ ).
- There is a total voltage drop  $\Delta V$  along the two lines, thus the voltage delivered to the user is  $V - \Delta V$  (see diagram). Express this voltage loss  $\Delta V$  in terms of  $R$  and the current  $I$ , and also in terms of the power  $P$ , and the voltage  $V$ .
- It is desirable that  $\Delta V$  be at most 2% of  $V$ . Since  $P$  and  $R$  are a given, this imposes a condition on the voltage  $V$ . What is the minimum voltage  $V$  needed to keep  $\Delta V$  down to 2% of  $V$  at  $+40^\circ \text{C}$ , and  $-40^\circ \text{C}$ ?
- What is the power dissipated in the lines for these two temperatures?
- If the distance between the wires is 8 m (assume they run parallel), what then is the Lorentz force on a 25 m segment of one of these wires (calculate this for both temperatures)?

## MIT 8.02 Spring 2002 Assignment #5 Solutions

### Problem 5.1

*High voltage power transmission.*

The simple circuit diagram at right can be used in the analysis of this power transmission system. The power station acts as a “battery” of voltage  $V$ . Three “resistors”, two of resistances  $R/2$  (the power lines) and one of resistance  $R_L$  (the “load resistance” of the power consumers), are wired to the battery in series. Voltages are as shown at various points about the circuit. We find that the voltage delivered to the power consumers is  $V - \Delta V = V - IR$ , so therefore  $\Delta V = IR$  is the total voltage drop along the power lines and  $R$  is the total resistance of the lines.



(Note: we have taken  $V = 0$  at a particular point, but the choice is totally arbitrary. Voltage differences between points about the circuit are unaffected by this choice.)

(a) The resistivity of aluminum at  $+40^\circ\text{C}$  and at  $-40^\circ\text{C}$  can be found using Giancoli Equation (25-5) (p. 641):

$$\rho_T = \rho_0[1 + \alpha(T - T_0)] \ .$$

Giancoli Table 25-1 (p. 640) gives  $\rho_0 = 2.65 \times 10^{-8} \Omega \cdot \text{m}$  and  $\alpha = 0.00429^\circ\text{C}^{-1}$  for aluminum at  $T_0 = 20^\circ\text{C}$ . This gives us

$$\begin{aligned} \rho_{+40} &= (2.65 \times 10^{-8})[1 + (0.00429)(40 - 20)] = 2.88 \times 10^{-8} \Omega \cdot \text{m} \ , \\ \rho_{-40} &= (2.65 \times 10^{-8})[1 + (0.00429)(-40 - 20)] = 1.97 \times 10^{-8} \Omega \cdot \text{m} \ . \end{aligned}$$

Resistances can then be calculated using Giancoli Equation (25-3) (p. 640). First we convert units:  $2 \times 300 \text{ miles} = 9.66 \times 10^5 \text{ m}$  and  $5 \text{ cm}^2 = 5 \times 10^{-4} \text{ m}^2$ .

$$\begin{aligned} R_{+40} &= (2.88 \times 10^{-8})(9.66 \times 10^5)/(5 \times 10^{-4}) = 56 \Omega \ , \\ R_{-40} &= (1.97 \times 10^{-8})(9.66 \times 10^5)/(5 \times 10^{-4}) = 38 \Omega \ . \end{aligned}$$

(b) By Ohm’s law, the voltage drop along the lines is simply  $\Delta V = IR$ .  $P = VI$  is the total power transmitted by the power station, so  $I = P/V$ , and we may write  $\Delta V = RP/V$ .

(c) Imposing  $\Delta V = RP/V = (0.02)V$  gives us

$$V = \sqrt{RP/(0.02)} \ .$$

Putting in  $P = 2 \times 10^8 \text{ W}$  and our two values for  $R$ , we find

$$V_{\min,+40} = 7.5 \times 10^5 \text{ V} , \quad V_{\min,-40} = 6.2 \times 10^5 \text{ V} .$$

(d) The power dissipated in the lines will be

$$\Delta P = \Delta VI = (0.02)VI = (0.02)P = 4 \text{ MW}$$

for both temperatures.

(e) Giancoli Equation (28-2) (p. 711) gives the force per unit length between two parallel current-carrying wires. Both of our wires carry the same current  $I$ , so (with  $d = 8 \text{ m}$  and  $l = 25 \text{ m}$ )

$$F = \frac{\mu_0 I^2 l}{2\pi d} .$$

Using  $I^2 = (0.02)P/R$  and plugging in known values gives

$$F_{+40} = (2 \times 10^{-7})(0.02) \frac{(2 \times 10^8)(25)}{(56)(8)} = 0.045 \text{ N} ,$$

$$F_{-40} = (2 \times 10^{-7})(0.02) \frac{(2 \times 10^8)(25)}{(38)(8)} = 0.066 \text{ N} .$$

Since the currents in the two wires run in opposite directions, this force between the wires is repulsive.

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