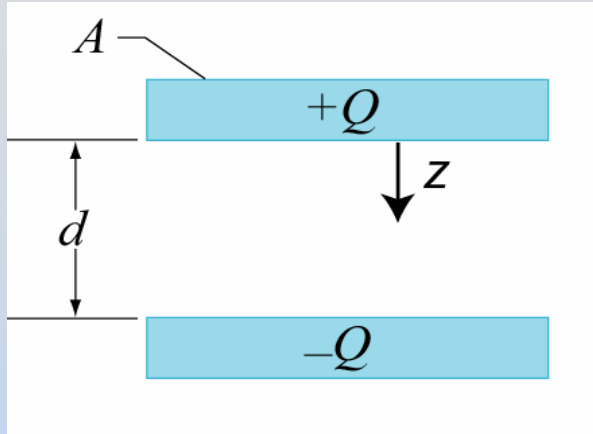


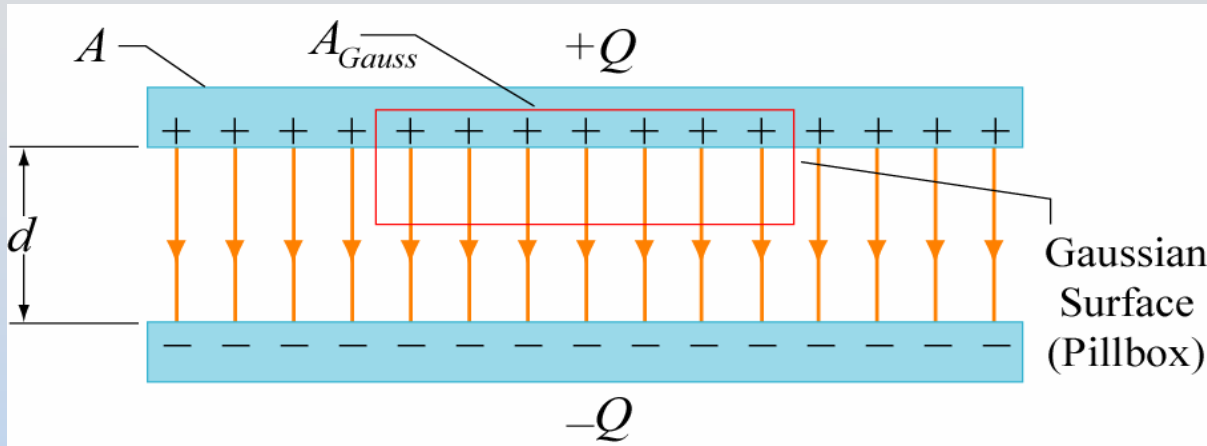
Q: Parallel Plate Capacitor



A parallel plate capacitor consists of two conducting plates of area A , separated by a distance d , with charge $+Q$ placed on the upper plate and $-Q$ on the lower plate. The z -axis is defined as pictured.

- a) What is the direction and magnitude of the electric field \mathbf{E} in each of the following regions of space: above & below the plates, in the plates and in between the plates.
- b) What is the electric potential $V(z)$ in these same five regions. Take the electric potential to be zero at $z=0$ (the lower surface of the top plate).
- c) What is the electric potential difference between the upper and lower plate, $\Delta V = V(0) - V(d)$?
- d) What is the capacitance of this capacitor?
- e) If this capacitor is now submerged into a vat of liquid dielectric (of dielectric constant κ), what now is the potential $V(z)$ everywhere?

A: Parallel Plate Capacitor



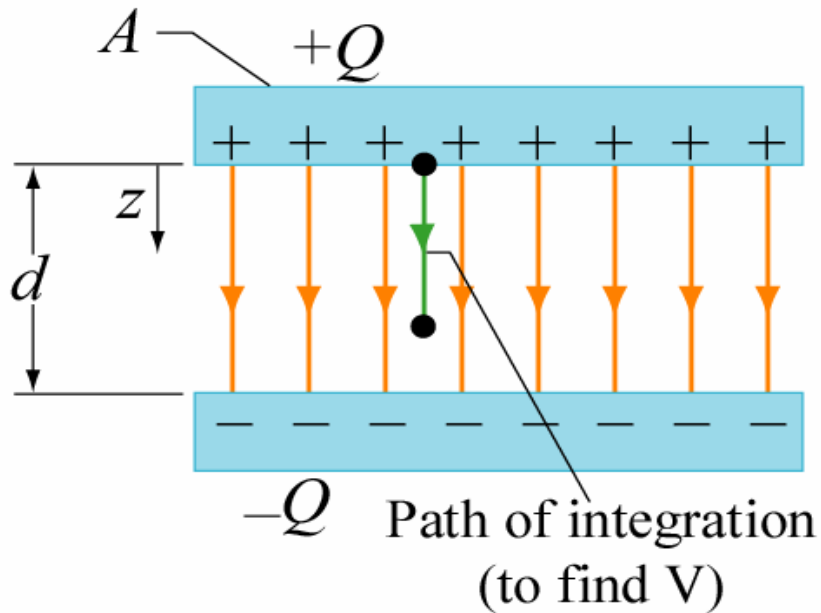
(a) Charges are attracted, so live on inner surface only

Conductors have $E=0$ inside, and by Gauss's law the only place $E \neq 0$ is between the plates:

$$\oiint_S \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\epsilon_0} \quad E(A_{Gauss}) = \frac{\sigma A_{Gauss}}{\epsilon_0} \quad \boxed{E = \frac{\sigma}{\epsilon_0} = \frac{Q}{A\epsilon_0} \text{ down}}$$

Note that you only need to consider one plate – the other plate was already used ($\pm Q$ to inner surfaces)

A: Parallel Plate Capacitor



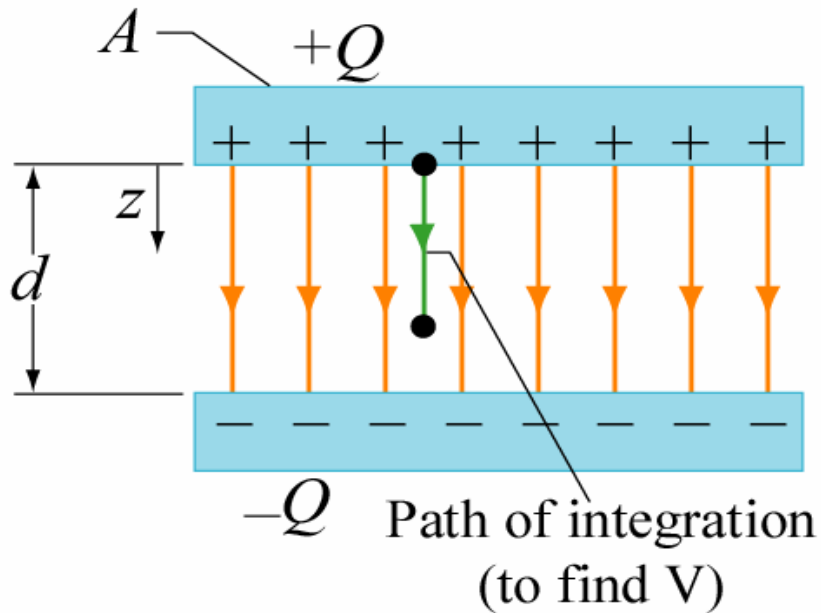
(b) Start where potential is known $V(z = 0) = 0$

Above and inside the top conductor $\mathbf{E} = 0$ so V is constant $\rightarrow V = 0$

Between plates: $V_{in}(z) = \Delta V = -\int_0^z \vec{\mathbf{E}} \cdot d\vec{\mathbf{S}} = -Ez = -\frac{Q}{A\epsilon_0} z$

In the bottom plate and below ($\mathbf{E}=0$): $V_{below} = -\frac{Q}{A\epsilon_0} d$

A: Parallel Plate Capacitor



$$(c) \Delta V = V(0) - V(d) = \frac{Qd}{A\epsilon_0}$$

$$(d) C = \frac{Q}{|\Delta V|} = \frac{\epsilon_0 A}{d}$$

(e) The dielectric constant is now everywhere κ .

This reduces the electric field & potential by $1/\kappa$
 V above and inside top conductor still 0

$$V_{in}(z) = -\frac{Q}{\kappa A \epsilon_0} z$$

$$V_{below}(z) = -\frac{Q}{\kappa A \epsilon_0} d$$