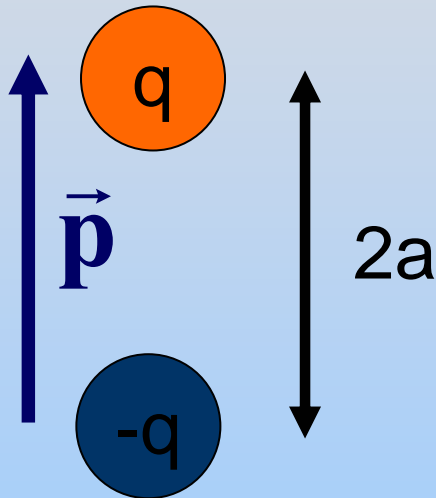


Electric Dipoles

A Special Charge Distribution

Electric Dipole

Two equal but opposite charges $+q$ and $-q$, separated by a distance $2a$



Dipole Moment

$$\begin{aligned}\vec{p} &\equiv \text{charge} \times \text{displacement} \\ &= q \times 2a \hat{j} = 2qa \hat{j}\end{aligned}$$

\vec{p} points from negative to positive charge

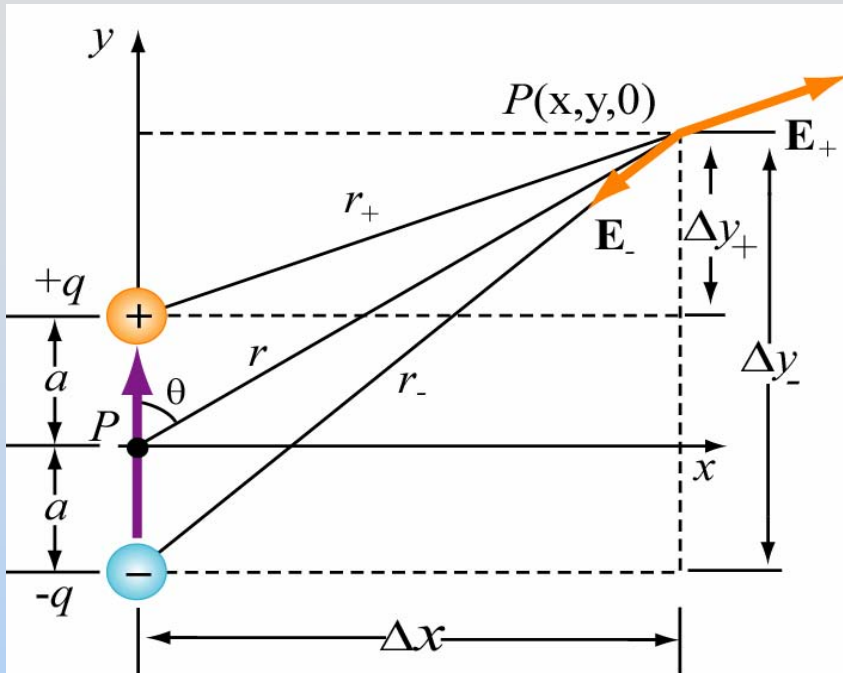
Why Dipoles?



Nature Likes To Make Dipoles!

Dipoles *make* Fields

Electric Field Created by Dipole



Thou shalt use components!

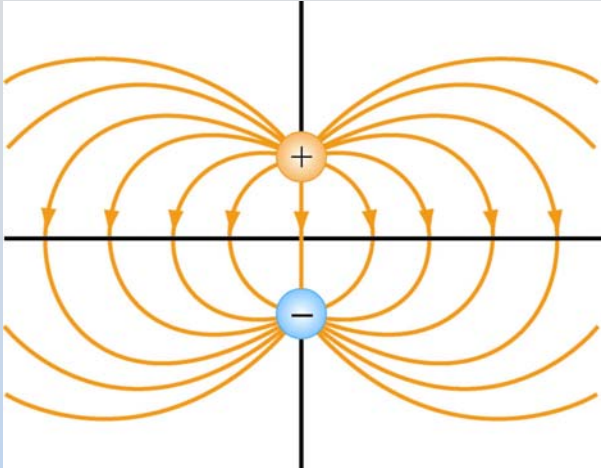
$$\frac{\hat{\mathbf{r}}}{r^2} = \frac{\vec{\mathbf{r}}}{r^3} = \frac{\Delta x}{r^3} \hat{\mathbf{i}} + \frac{\Delta y}{r^3} \hat{\mathbf{j}}$$

$$E_x = k_e q \left(\frac{\Delta x}{r_+^3} - \frac{\Delta x}{r_-^3} \right) = k_e q \left(\frac{x}{[x^2 + (y-a)^2]^{3/2}} - \frac{x}{[x^2 + (y+a)^2]^{3/2}} \right)$$

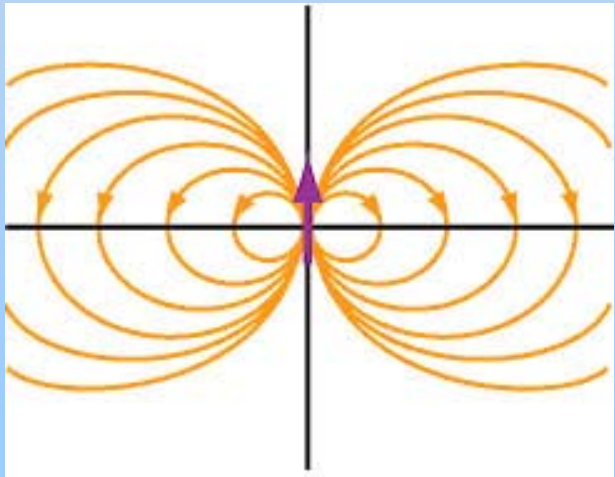
$$E_y = k_e q \left(\frac{\Delta y_+}{r_+^3} - \frac{\Delta y_-}{r_-^3} \right) = k_e q \left(\frac{y-a}{[x^2 + (y-a)^2]^{3/2}} - \frac{y+a}{[x^2 + (y+a)^2]^{3/2}} \right)$$

PRS Question: Dipole Fall-Off

Point Dipole Approximation



Finite Dipole



Point Dipole

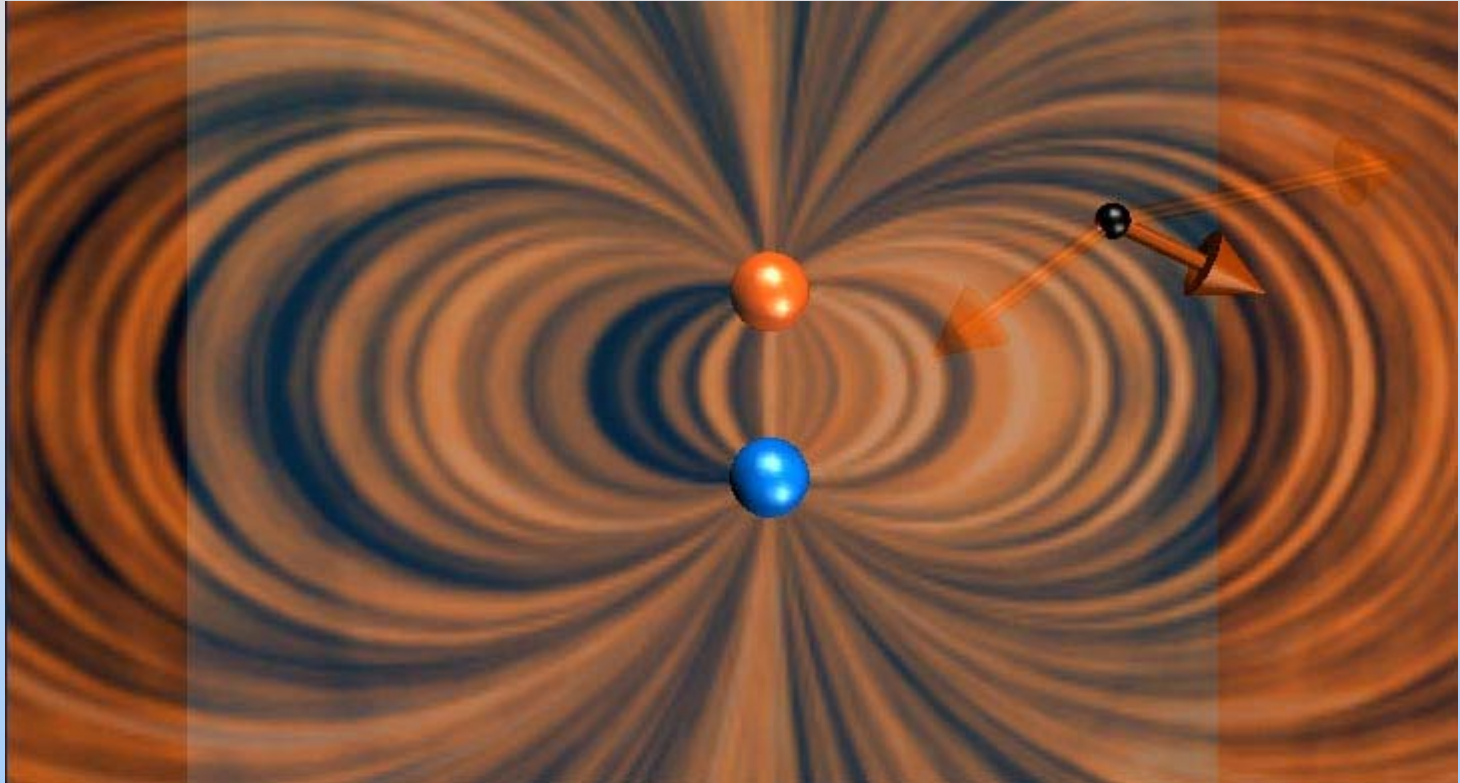
Take the limit $r \gg a$

You can show...

$$E_x \rightarrow \frac{3p}{4\pi\epsilon_0 r^3} \sin\theta \cos\theta$$

$$E_y \rightarrow \frac{p}{4\pi\epsilon_0 r^3} (3\cos^2\theta - 1)$$

Shockwave for Dipole

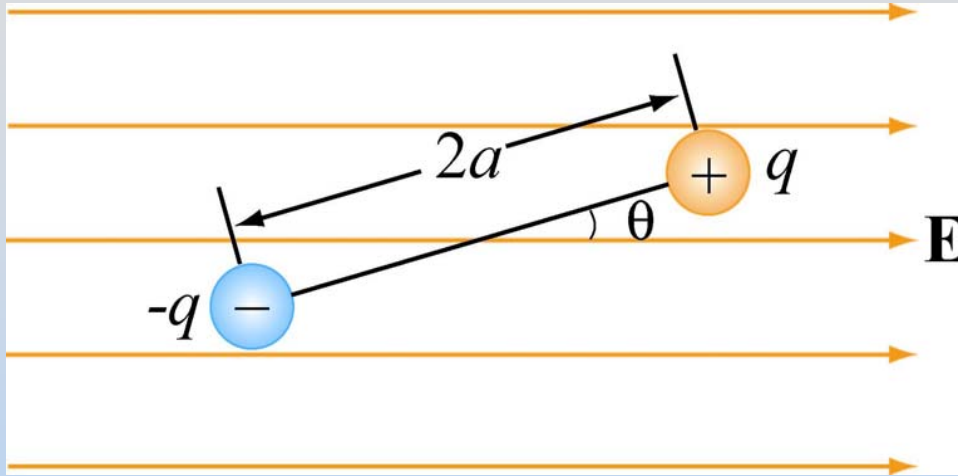


<http://ocw.mit.edu/ans7870/8/8.02T/f04/visualizations/electrostatics/06-DipoleField3d/06-dipField320.html>

Dipoles *feel* Fields

Demonstration: Dipole in Field

Dipole in Uniform Field



$$\vec{E} = E\hat{i}$$

$$\vec{p} = 2qa(\cos\theta\hat{i} + \sin\theta\hat{j})$$

$$\text{Total Net Force: } \vec{F}_{net} = \vec{F}_+ + \vec{F}_- = q\vec{E} + (-q)\vec{E} = 0$$

$$\text{Torque on Dipole: } \boxed{\vec{\tau} = \vec{r} \times \vec{F} = \vec{p} \times \vec{E}}$$

$$\tau = rF_+ \sin(\theta) = (2a)(qE)\sin(\theta) = pE \sin(\theta)$$

\vec{p} tends to align with the electric field

Torque on Dipole

Total Field (dipole + background) shows torque:

<http://ocw.mit.edu/ans7870/8/8.02T/f04/visualizations/electrostatics/43-torqueondipolee/43-torqueondipolee320.html>

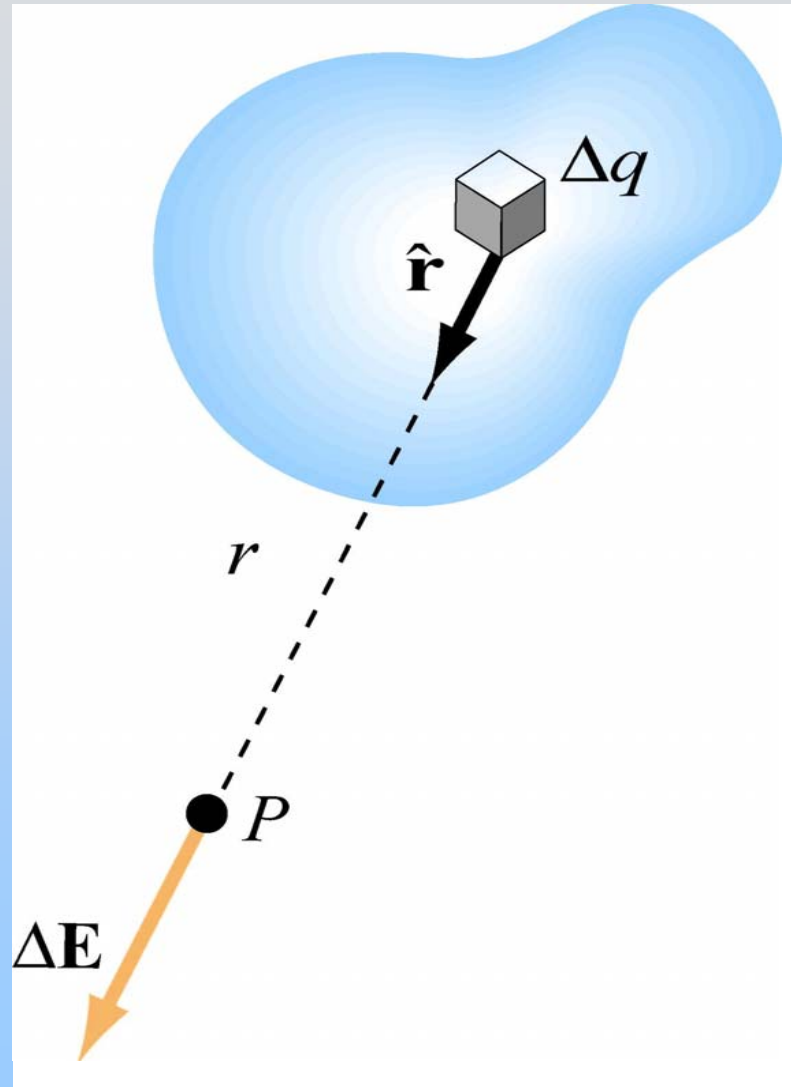


- Field lines transmit tension
- Connection between dipole field and constant field “pulls” dipole into alignment

PRS Question: Dipole in Non-Uniform Field

Continuous Charge Distributions

Continuous Charge Distributions



Break distribution into parts:

$$Q = \sum_i \Delta q_i \rightarrow \int_V dq$$

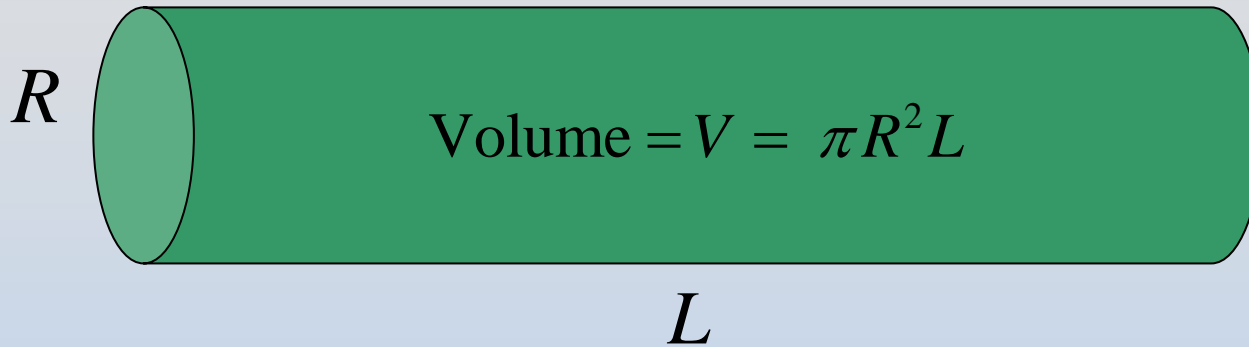
E field at P due to Δq

$$\Delta \vec{\mathbf{E}} = k_e \frac{\Delta q}{r^2} \hat{\mathbf{r}} \rightarrow d\vec{\mathbf{E}} = k_e \frac{dq}{r^2} \hat{\mathbf{r}}$$

Superposition:

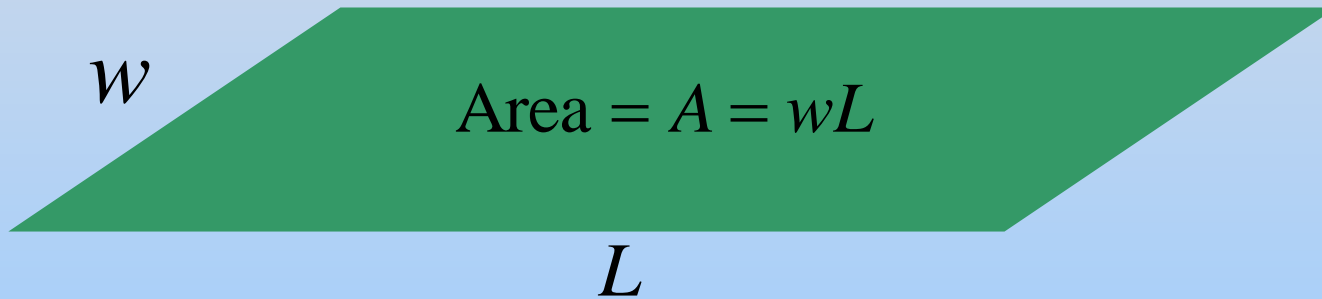
$$\vec{\mathbf{E}} = \sum \Delta \vec{\mathbf{E}} \rightarrow \int d\vec{\mathbf{E}}$$

Continuous Sources: Charge Density



$$dQ = \rho dV$$

$$\rho = \frac{Q}{V}$$



$$dQ = \sigma dA$$

$$\sigma = \frac{Q}{A}$$

Length = L



$$dQ = \lambda dL$$

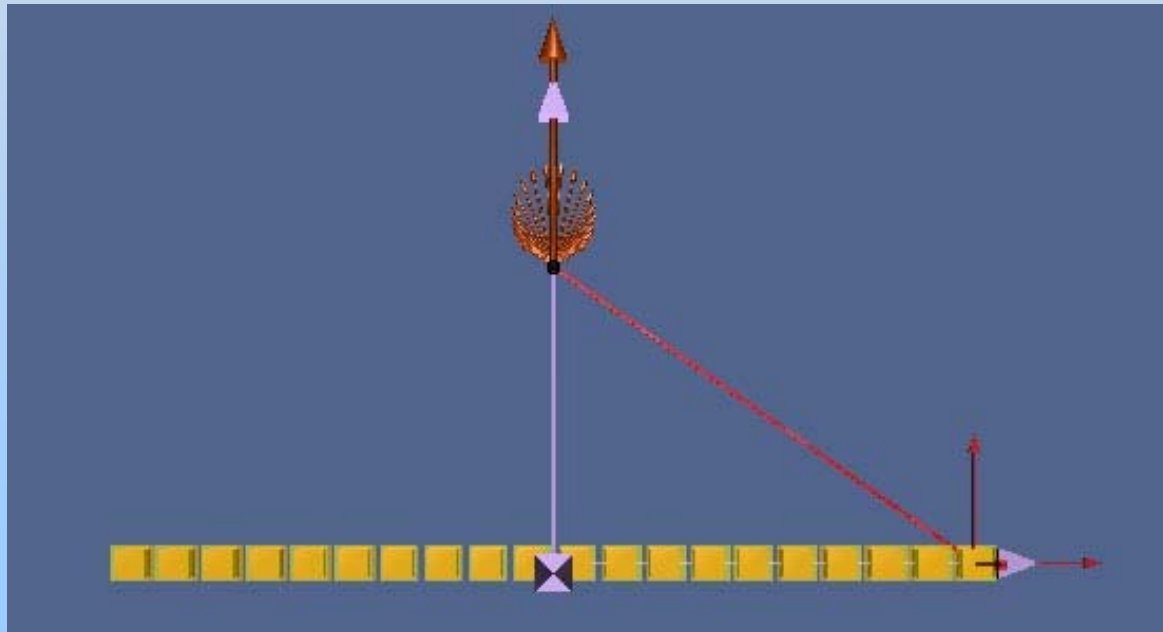
$$\lambda = \frac{Q}{L}$$

Examples of Continuous Sources: Line of charge

Length = L

$$dQ = \lambda dL$$

$$\lambda = \frac{Q}{L}$$

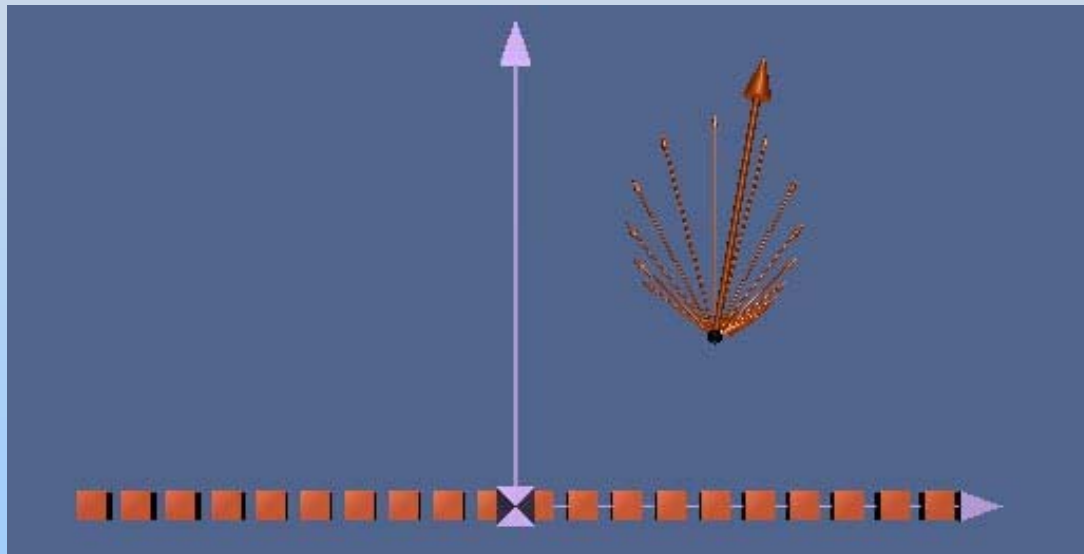


<http://ocw.mit.edu/ans7870/8/8.02T/f04/visualizations/electrostatics/07-LineIntegration/07-LineInt320.html>

Examples of Continuous Sources: Line of charge

Length = L

L



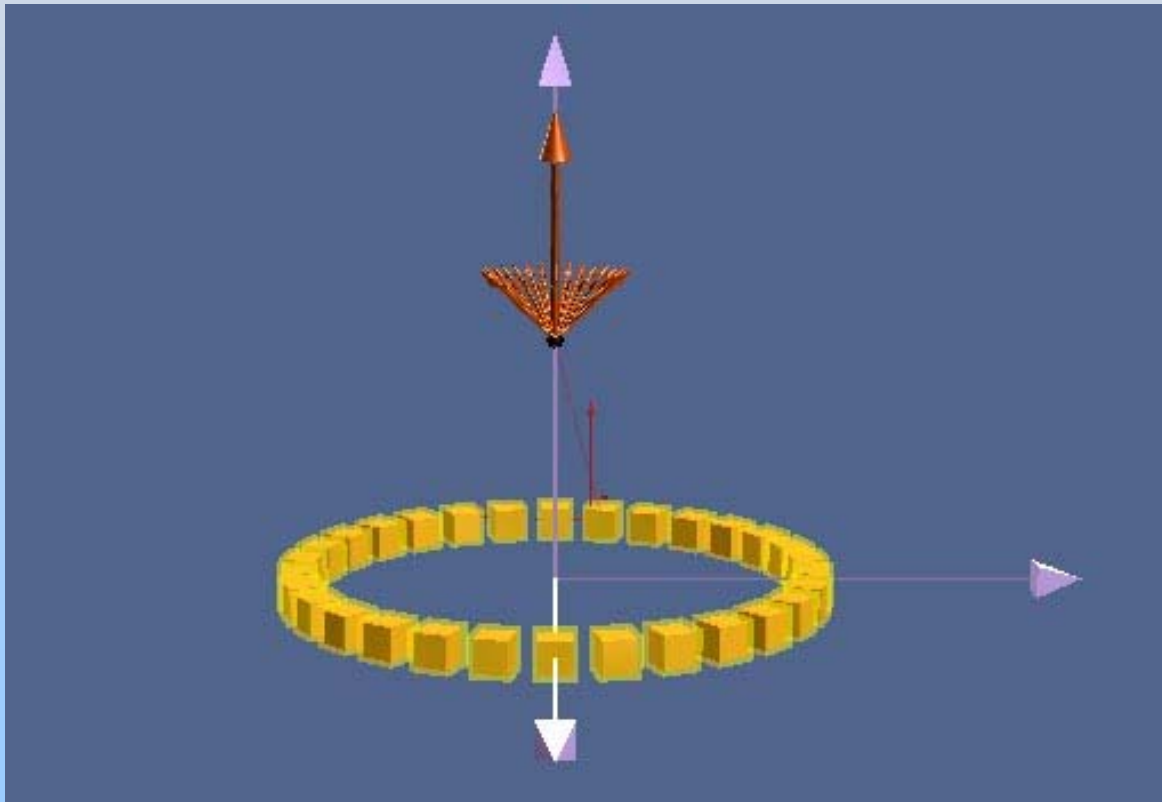
$$dQ = \lambda dL$$

$$\lambda = \frac{Q}{L}$$

<http://ocw.mit.edu/ans7870/8/8.02T/f04/visualizations/electrostatics/08-LineField/08-LineField320.html>

Examples of Continuous Sources: Ring of Charge

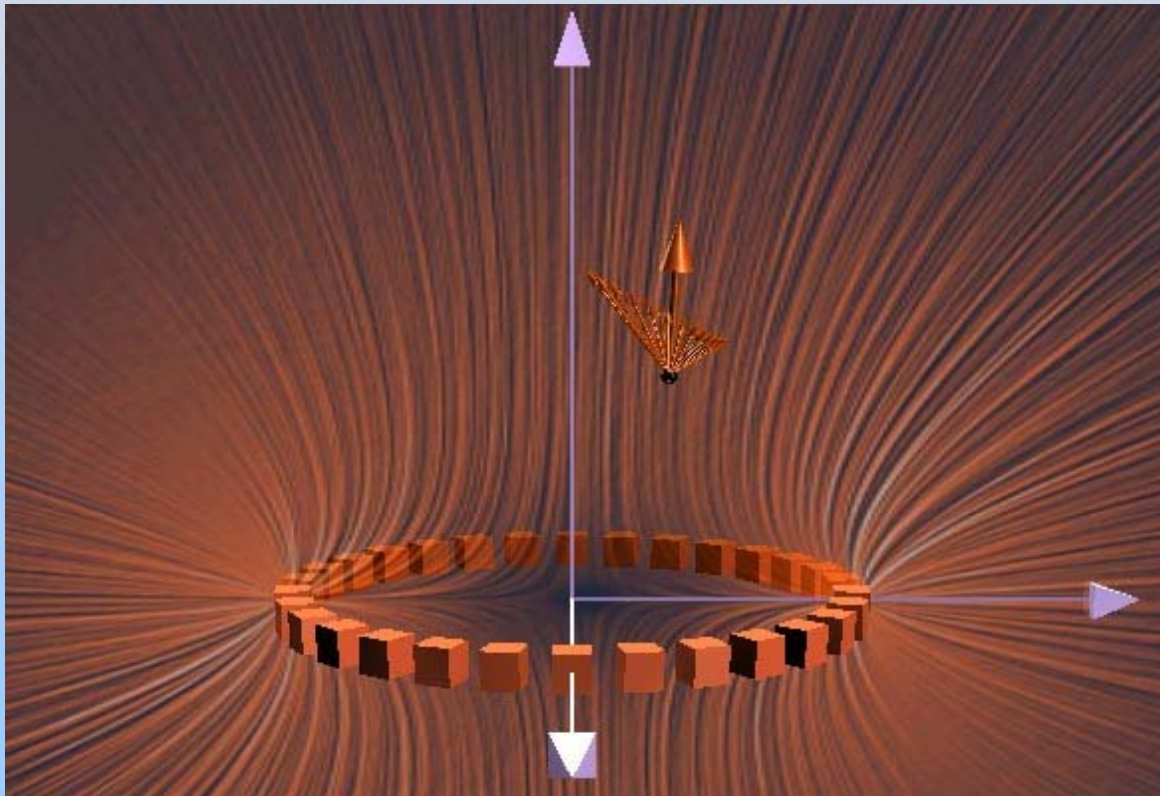
$$dQ = \lambda dL \quad \lambda = \frac{Q}{2\pi R}$$



<http://ocw.mit.edu/ans7870/8/8.02T/f04/visualizations/electrostatics/09-RingIntegration/09-ringInt320.html>

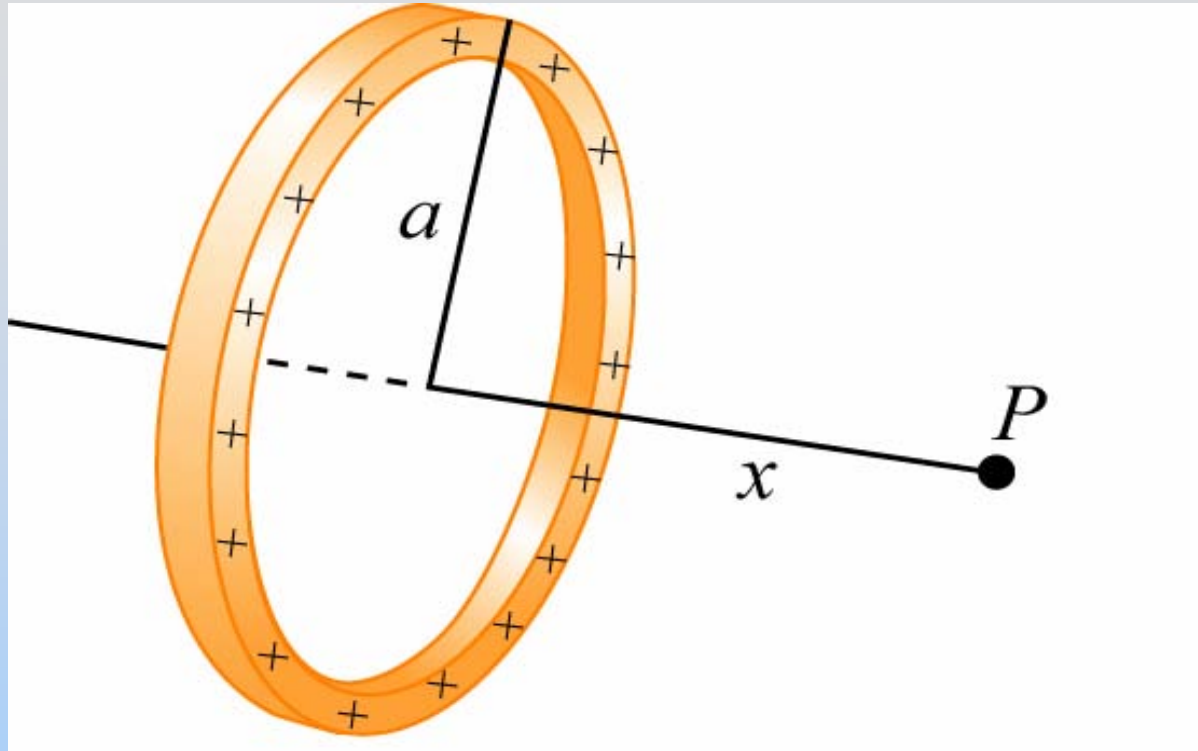
Examples of Continuous Sources: Ring of Charge

$$dQ = \lambda dL \quad \lambda = \frac{Q}{2\pi R}$$



<http://ocw.mit.edu/ans7870/8/8.02T/f04/visualizations/electrostatics/10-RingField/10-ringField320.html>

Example: Ring of Charge



P on axis of ring of charge, x from center
Radius a , charge density λ .

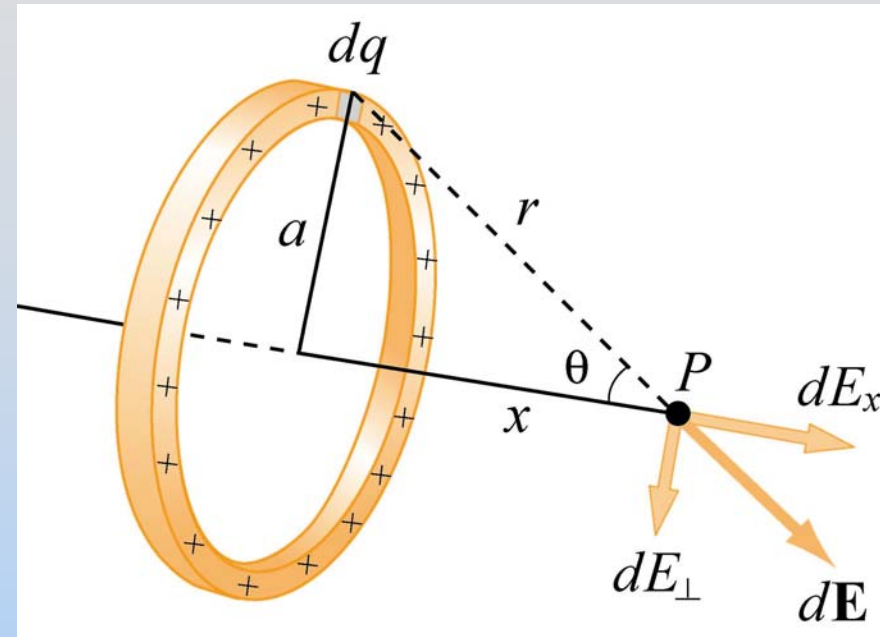
Find \mathbf{E} at P

Ring of Charge

1) Think about it

$E_{\perp} = 0$ Symmetry!

<http://ocw.mit.edu/ans7870/8/8.02T/f04/visualizations/electrostatics/09-RingIntegration/09-ringInt320.html>



2) Define Variables

$$dq = \lambda dl = \lambda (a d\varphi)$$

$$r = \sqrt{a^2 + x^2}$$

Ring of Charge

3) Write Equation

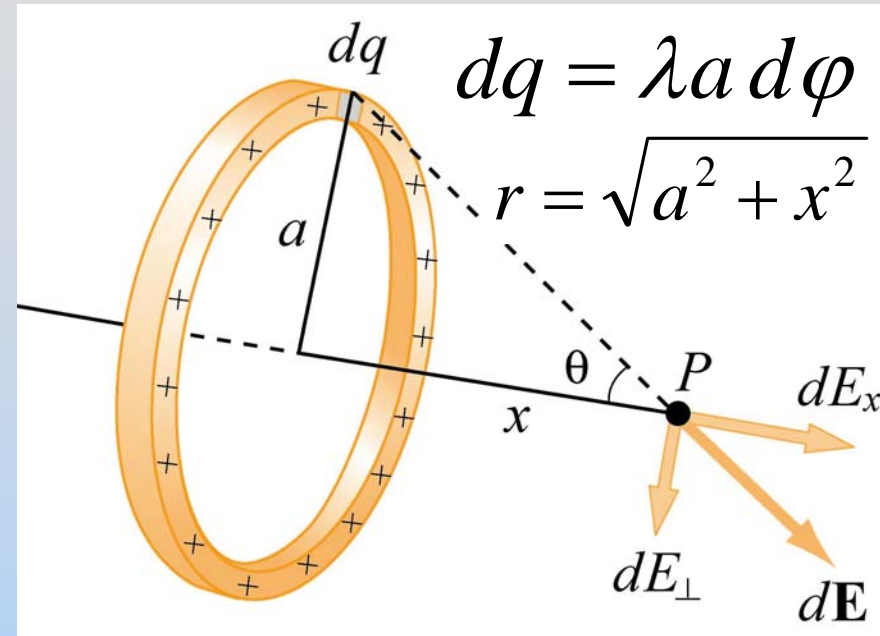
$$d\vec{\mathbf{E}} = k_e dq \frac{\hat{r}}{r^2} = k_e dq \frac{\vec{r}}{r^3}$$

a) My way

$$dE_x = k_e dq \frac{x}{r^3}$$

b) Another way

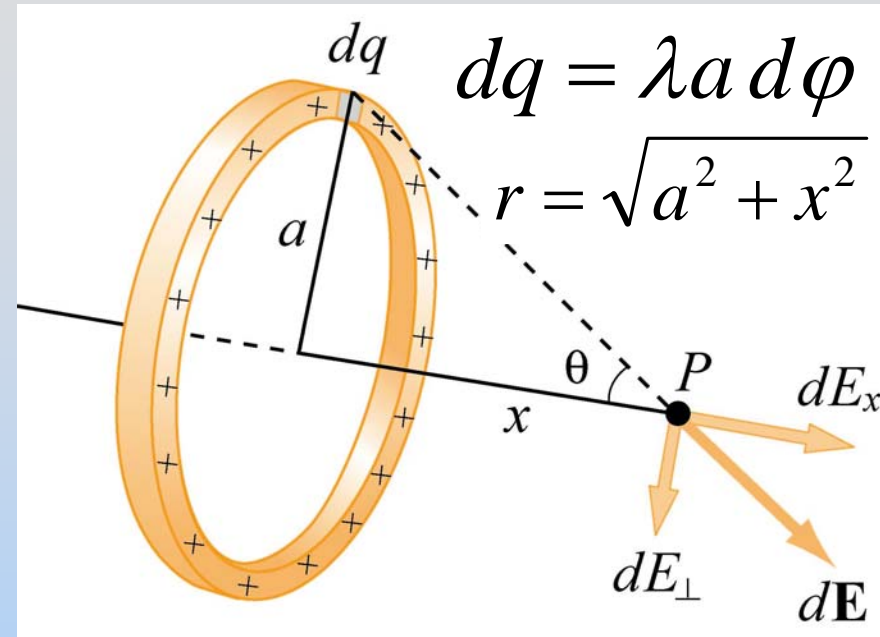
$$dE_x = |d\vec{\mathbf{E}}| \cos(\theta) = k_e dq \frac{1}{r^2} \cdot \frac{x}{r} = k_e dq \frac{x}{r^3}$$



Ring of Charge

4) Integrate

$$\begin{aligned} E_x &= \int dE_x = \int k_e dq \frac{x}{r^3} \\ &= k_e \frac{x}{r^3} \int dq \end{aligned}$$



Very special case: everything except dq is constant

$$\begin{aligned} \int dq &= \int_0^{2\pi} \lambda a d\phi = \lambda a \int_0^{2\pi} d\phi = \lambda a 2\pi \\ &= Q \end{aligned}$$

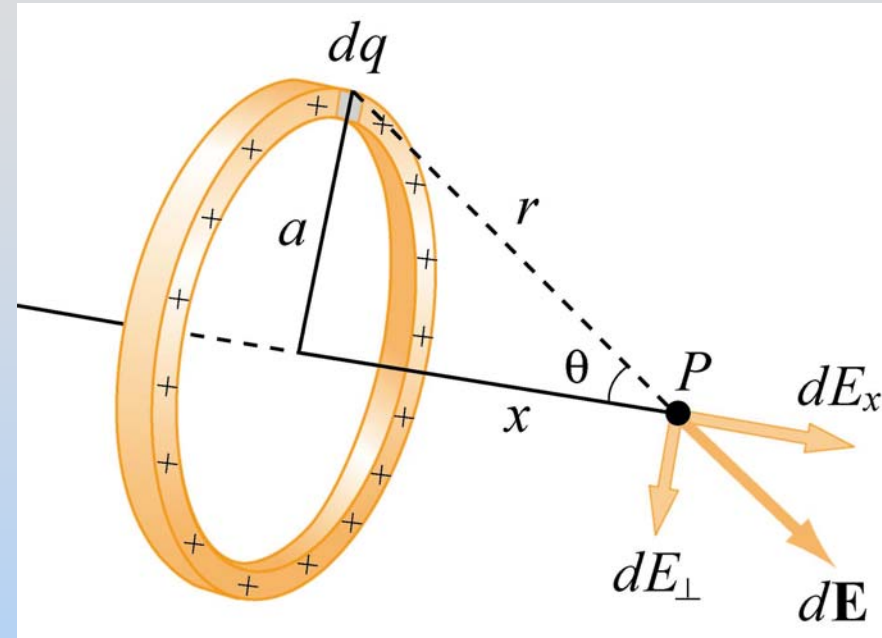
Ring of Charge

5) Clean Up

$$E_x = k_e Q \frac{x}{r^3}$$

$$E_x = k_e Q \frac{x}{(a^2 + x^2)^{3/2}}$$

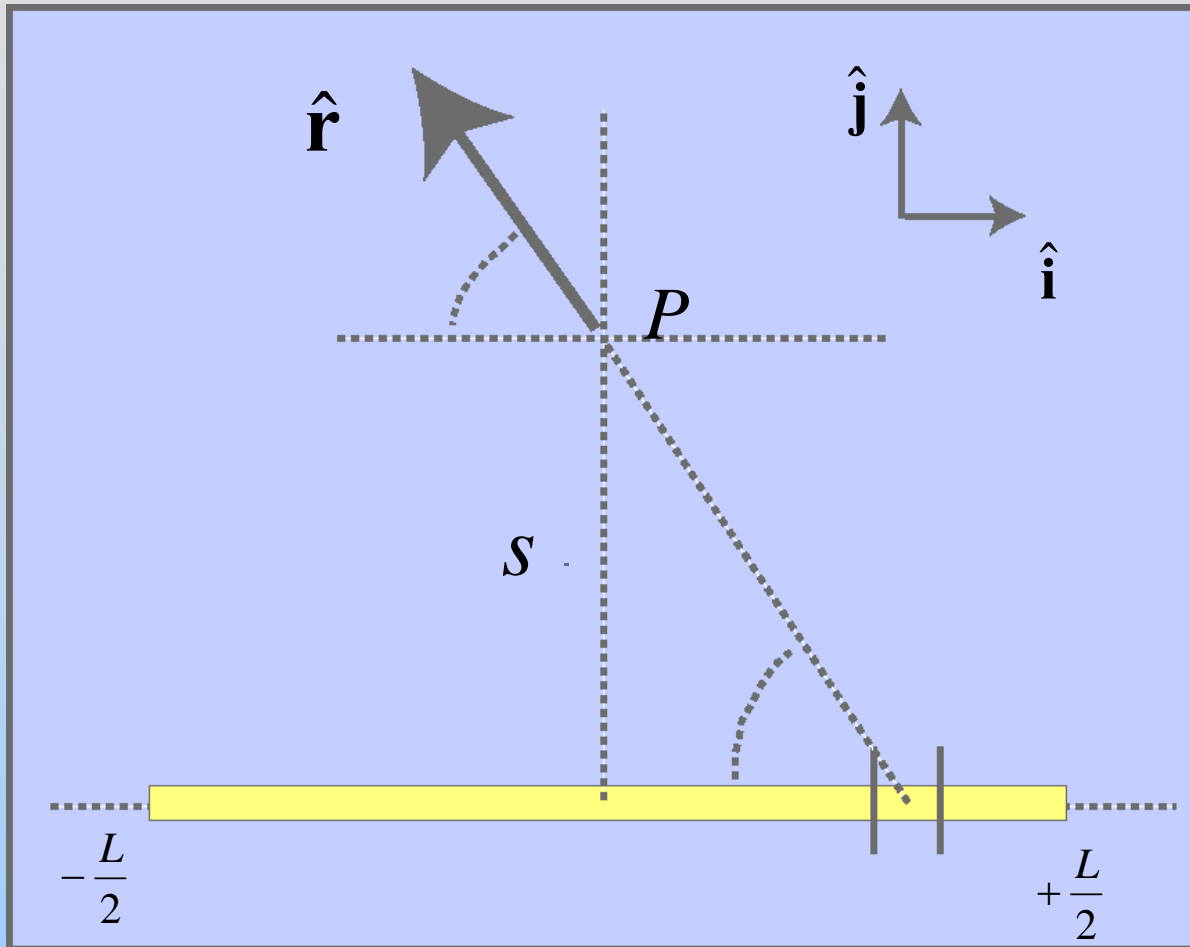
$$\vec{\mathbf{E}} = k_e Q \frac{x}{(a^2 + x^2)^{3/2}} \hat{\mathbf{i}}$$



6) Check Limit $a \rightarrow 0$

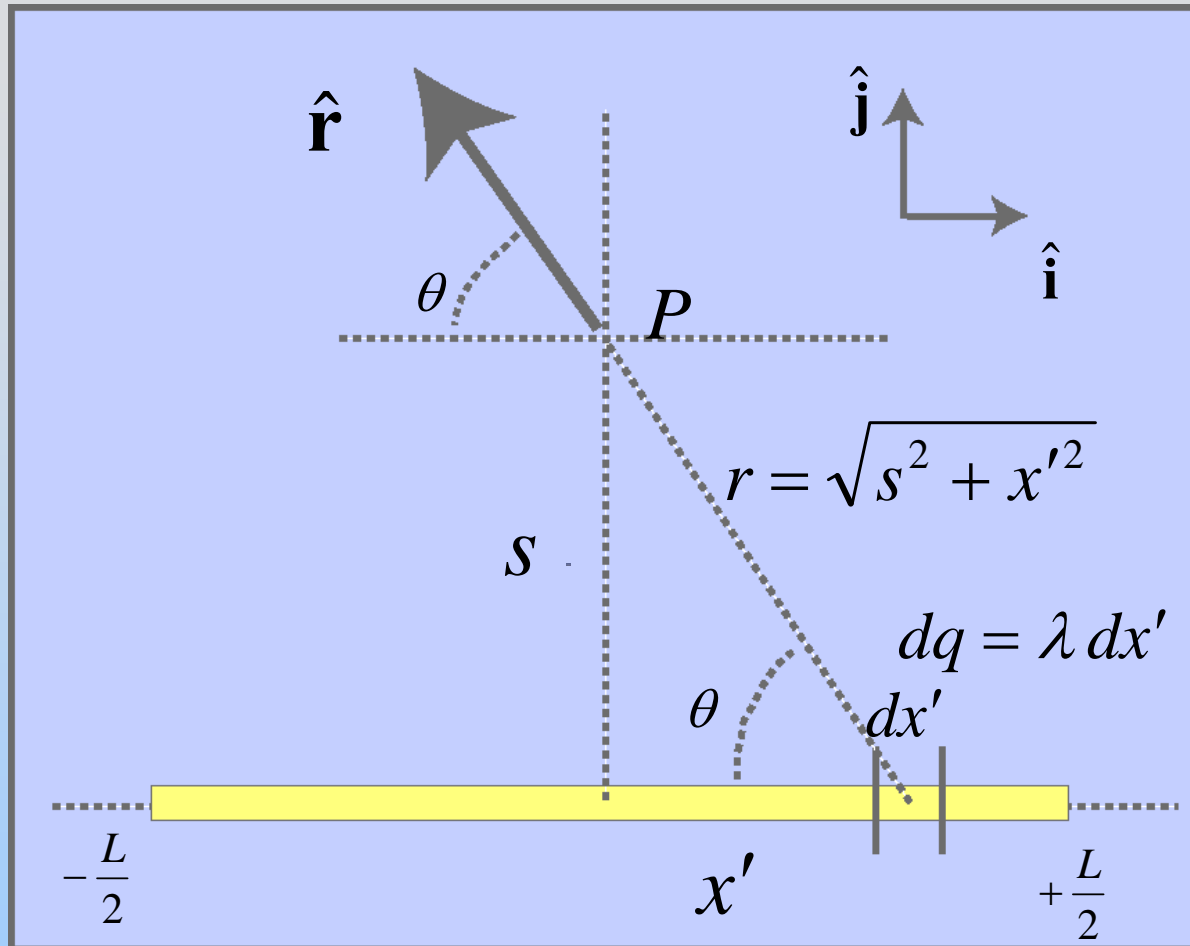
$$E_x \rightarrow k_e Q \frac{x}{(x^2)^{3/2}} = \frac{k_e Q}{x^2}$$

In-Class: Line of Charge



Point P lies on perpendicular bisector of uniformly charged line of length L , a distance s away. The charge on the line is Q . What is \mathbf{E} at P ?

Hint: <http://ocw.mit.edu/ans7870/8/8.02T/f04/visualizations/electrostatics/07-LineIntegration/07-LineInt320.html>



Typically give the integration variable (x') a “primed” variable name.

E Field from Line of Charge

$$\vec{\mathbf{E}} = k_e \frac{Q}{s(s^2 + L^2 / 4)^{1/2}} \hat{\mathbf{j}}$$

Limits:

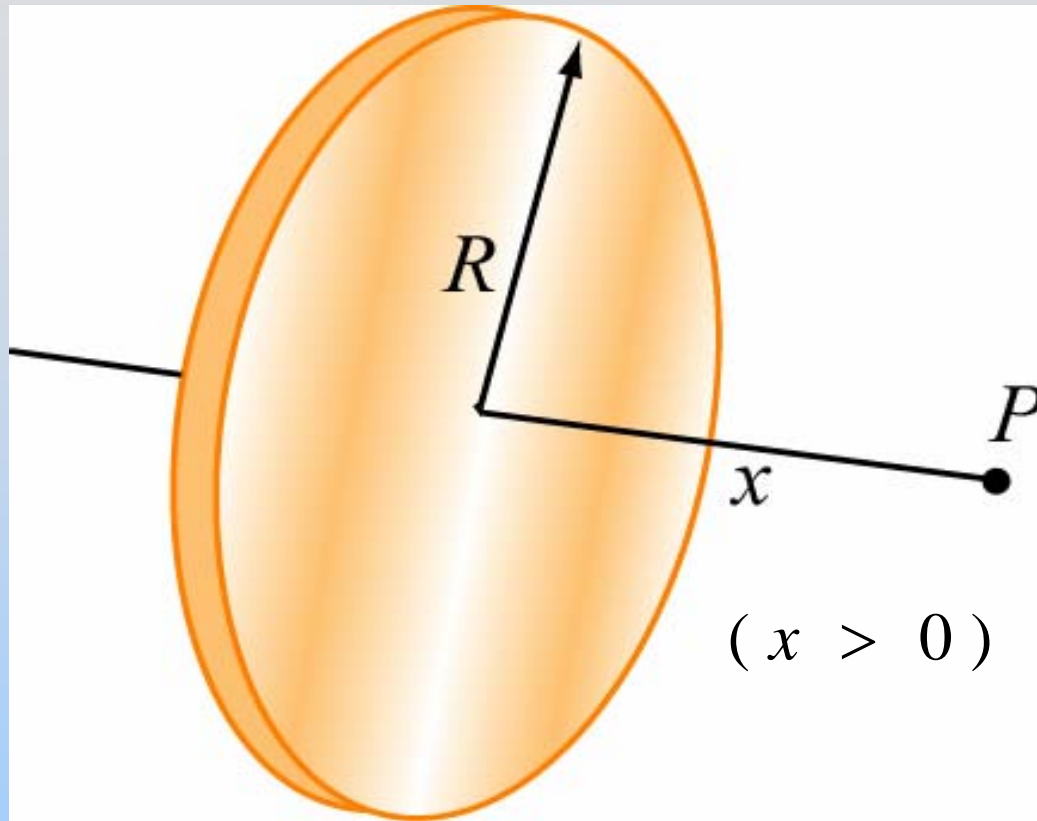
$$\lim_{s \gg L} \vec{\mathbf{E}} \rightarrow k_e \frac{Q}{s^2} \hat{\mathbf{j}}$$

Point charge

$$\lim_{s \ll L} \vec{\mathbf{E}} \rightarrow 2k_e \frac{Q}{Ls} \hat{\mathbf{j}} = 2k_e \frac{\lambda}{s} \hat{\mathbf{j}}$$

Infinite charged line

In-Class: Uniformly Charged Disk



P on axis of disk of charge, x from center
Radius R , charge density σ .

Find \mathbf{E} at P

Disk: Two Important Limits

$$\vec{\mathbf{E}}_{disk} = \frac{\sigma}{2\epsilon_0} \left[1 - \frac{x}{(x^2 + R^2)^{1/2}} \right] \hat{\mathbf{i}}$$

Limits:

$$\lim_{x \gg R} \vec{\mathbf{E}}_{disk} \xrightarrow{***} \frac{1}{4\pi\epsilon_0} \frac{Q}{x^2} \hat{\mathbf{i}} \quad \text{Point charge}$$

$$\lim_{x \ll R} \vec{\mathbf{E}}_{disk} \rightarrow \frac{\sigma}{2\epsilon_0} \hat{\mathbf{i}} \quad \text{Infinite charged plane}$$

E for Plane is Constant????

- 1) Dipole: E falls off like $1/r^3$
- 2) Point charge: E falls off like $1/r^2$
- 3) Line of charge: E falls off like $1/r$
- 4) Plane of charge: E constant