

Problem 2.2

Electric field and potential of a charged cylinder.

Consider a very long cylinder of radius a with its axis along the z -axis of a cartesian coordinate system. The cylinder is uniformly charged; its constant volume charge density is ρ (positive).

- (a) Using symmetry arguments show that \vec{E} should be radial both inside and outside the cylinder.
- (b) Using Gauss's Law find the magnitude of E as a function of r , for $0 \leq r \leq a$, and for $r > a$. Do the two results match at $r = a$?

Hint: for $0 \leq r \leq a$, figure out first how much charge $Q(r)$ is contained within a cylindrical gaussian surface of radius r and length ℓ . Of course, the axis of the gaussian surface should coincide with that of the cylinder; if it didn't, it wouldn't help you to solve the problem!

- (c) Using your answers in part (b) calculate the potential difference, ΔV , between a point at a distance r from the z -axis and a point on the z -axis. Make a qualitative plot of ΔV as a function of r for all r 's ($0 \leq r < \infty$).

Problem 2.3

Electric Field, Potential, and Electrostatic Potential Energy.

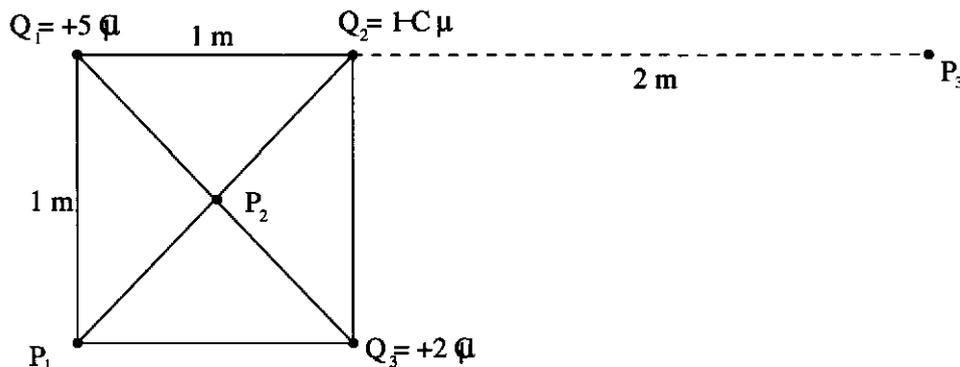
Giancoli 23-66.

Problem 2.4

Electric Field, Potential, and Electrostatic Potential Energy.

This problem contains (just about) all the new concepts introduced so far.

Point charges Q_1 , Q_2 , and Q_3 reside on three corners of a square with sides of 1 m; the distance from Q_2 to P_3 is 2 m (see diagram).



- (a) What is the electric potential, V , in P_1 , P_2 , and P_3 ? (Normalize the potential to be zero at ∞).
- (b) Are there points or surfaces in space (other than infinity) where V is zero?
- (c) There are two points where the electric field equals zero. Could you guess approximately where these points are? State your reasons.
- (d) Sketch roughly the electric field lines.
- (e) What is the electrostatic potential energy of the system?
- (f) Suppose we release the three charges so that they can move freely in empty space. How much energy is released in the form of kinetic energy? Does it matter in what sequence one releases the charges? *Don't be too hasty in answering this last question!*

collection of point charges will be a sum of many terms, *one term for each possible pairing of charges in the system*:

$$U_{\text{total}} = W_{12} + W_{13} + W_{23} + W_{14} + W_{24} + W_{34} + \dots$$

We know how to express these W 's in terms of charges and charge separations. Consider the pairing of charge i with charge j , separated by a distance d_{ij} :

$$W_{ij} = \frac{1}{4\pi\epsilon_0} \frac{Q_i Q_j}{d_{ij}} .$$

Note that although we have imagined constructing the system in a particular charge-by-charge order, the result for the potential energy of the configuration depends only on the final configuration and not on the order in which the charges were brought into place.

For our cube, the quantity QV_{corner} gives the sum of the potential energy terms for 7 pairings in all: all of those pairings that involve one particular charge. The quantity $8QV_{\text{corner}}$ thus adds up the total potential energy of the configuration, but counts all pairs *twice* (convince yourself!) The correct value for the total potential energy of the cubic configuration is then

$$U_{\text{total}} = 4QV_{\text{corner}} \simeq (22.8) \frac{1}{4\pi\epsilon_0} \frac{Q^2}{l} .$$

Problem 2.4

Electric field, potential, and electrostatic potential energy.

From equation (23-6a) of Giancoli (p. 599), we have at any position \mathbf{r} the electrostatic potential as

$$V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \left(\frac{Q_1}{|\mathbf{r}_1 - \mathbf{r}|} + \frac{Q_2}{|\mathbf{r}_2 - \mathbf{r}|} + \frac{Q_3}{|\mathbf{r}_3 - \mathbf{r}|} \right) ,$$

or

$$V(\mathbf{r}) = \frac{10^{-6}}{4\pi\epsilon_0} \left(\frac{5}{|\mathbf{r}_1 - \mathbf{r}|} - \frac{1}{|\mathbf{r}_2 - \mathbf{r}|} + \frac{2}{|\mathbf{r}_3 - \mathbf{r}|} \right) \quad (\text{SI units}).$$

(Here, \mathbf{r} , \mathbf{r}_1 , etc. are position *vectors*, and thus $|\mathbf{r}_1 - \mathbf{r}|$ and the like are *magnitudes of vector differences*.)

At P_1 :

$$|\mathbf{r}_1 - \mathbf{r}_{P_1}| = |\mathbf{r}_3 - \mathbf{r}_{P_1}| = 1 \text{ m}, \quad |\mathbf{r}_2 - \mathbf{r}_{P_1}| = \sqrt{2} \text{ m} \\ \Rightarrow V = 5.7 \times 10^4 \text{ Volts.}$$

At P_2 :

$$|\mathbf{r}_1 - \mathbf{r}_{P_2}| = |\mathbf{r}_2 - \mathbf{r}_{P_2}| = |\mathbf{r}_3 - \mathbf{r}_{P_2}| = \frac{1}{\sqrt{2}} \text{ m} \\ \Rightarrow V = 7.6 \times 10^4 \text{ Volts.}$$

At P_3 :

$$|\mathbf{r}_1 - \mathbf{r}_{P_3}| = 3 \text{ m}, \quad |\mathbf{r}_2 - \mathbf{r}_{P_3}| = 2 \text{ m}, \quad |\mathbf{r}_3 - \mathbf{r}_{P_3}| = \sqrt{5} \text{ m} \\ \Rightarrow V = 1.86 \times 10^4 \text{ Volts.}$$

(b) Our expression for V is the sum of three terms, two positive and one negative. One can see that if we get close enough to the negative charge (Q_2), the negative term can be large enough to cancel the two positive terms. Thus there exists some “egg-shaped” surface surrounding Q_2 where the potential is zero (see sketch below).

(c) There are two points where \mathbf{E} is zero, as shown in the sketch below. One is “sort of” between the two positive charges, where the two fields balance out (this is modified by the presence of the negative charge). The other is to the upper right of the negative charge, and is where the repulsion of the two positive charges is just balanced by the attraction of the negative charge (for a positive test charge).

(d) See sketch of field lines below. The ratio of the number of field lines for the three charges is 5:2:1 in 3-D space. The sketch is 2-dimensional; it is only meant to be illustrative.

(e) The electrostatic potential energy of the system is (see Section 23-8 of Giancoli)

$$U = \frac{1}{4\pi\epsilon_0} \left(\frac{Q_1Q_2}{|\mathbf{r}_1 - \mathbf{r}_2|} + \frac{Q_2Q_3}{|\mathbf{r}_2 - \mathbf{r}_3|} + \frac{Q_3Q_1}{|\mathbf{r}_3 - \mathbf{r}_1|} \right) \\ = \frac{10^{-12}}{4\pi\epsilon_0} \left(-\frac{5}{1} - \frac{2}{1} + \frac{10}{\sqrt{2}} \right) \quad (\text{SI units}) \\ = +6.4 \times 10^{-4} \text{ joules.}$$

(f) The energy above is the amount of energy it takes to bring the three charges in from infinity to their locations as shown. If we could release these charges in such a way that they all flew off to ∞ , all infinitely far away from each other, then the energy $U > 0$ above would be converted to an equal amount of kinetic energy of the three charges at infinity. However, because one of the charges is negative, the charges will not all fly off to ∞ when released, and the amount of kinetic energy they gain upon release can vary enormously. For example, suppose you release Q_3 first. It will start to move in the direction of the net \mathbf{E} -field due to the other two charges. So it will take off in the direction a little South of due East. What happens thereafter is impossible to evaluate based solely upon the given information. It is quite possible that Q_3 will make it all the way out to ∞ , but we cannot exclude the possibility that it will end up on Q_2 . The trajectory of Q_3 will not only depend on the \mathbf{E} -field configuration due to the other two charges, but also on Q_3 's mass. Different masses will experience different accelerations and will therefore have different trajectories. If the mass of Q_3 were immensely large, Q_3 would closely follow the field line going through its point of origin. However, if its mass were small, it would immediately leave that field line.

Now release Q_2 (suppose that Q_3 has in fact flown off to ∞). This charge will be attracted to Q_1 and will smash directly into it. If these were truly point charges, the kinetic energy of

Q_2 would become infinite as $|\mathbf{r}_1 - \mathbf{r}_2| \rightarrow 0$.

In contrast, if you release Q_2 first, it will become bound to Q_1 , performing a complicated orbit about Q_1 . If Q_3 is released at the right time in the orbit of Q_2 about Q_1 , Q_3 might fly off to ∞ , leaving Q_1 and Q_2 to orbit one another. Thus this question has an infinite number of answers—all greater than zero—but is otherwise ill-defined.

