

~~Is this heat positive or negative? Explain whether it is added to the system or removed.~~

~~e) What is the pressure P_e of the gas after the expansion is finished?~~

~~(3) When the gas has reached point c it expands from V_e to V_d while no heat is allowed to flow into or out of the system. The expansion of the gas lowers the temperature and pressure from an initial temperature T_2 to a final temperature T_1 . During this process the quantity $PV^\gamma = \text{constant}$.~~

~~f) What is the pressure P_d and volume V_d of the state d of the gas after the expansion is finished?~~

~~g) What is the change in internal energy of the gas during this change of state?~~

~~h) What is the work done by the gas during this expansion?~~

~~(4) The gas is now compressed isothermally from d to a at constant T_1 from volume V_d back to V_a .~~

~~i) Find the work done by the system on the surroundings W_{ad} and the amount of heat Q_{ad} that flows between the system and the surroundings. Are these quantities positive or negative? Explain whether heat is added to the system or removed from the heat source at T_1 .~~

Total Cycle:

~~j) What is the total work W_{cycle} done by the gas during this cycle?~~

~~k) What is the total heat Q_{cycle} (from T_2) drawn from the higher temperature heat source during this cycle?~~

~~l) What is the efficiency of this cycle $\epsilon_{\max} = W_{cycle} / Q_{cycle}$ (from T_2)?~~

Problem 4 Heat pump

A reversible heat engine can be run in the other direction, in which case it does negative work W_{cycle} on the world while “pumping” heat Q_{cycle} (into T_2) into a reservoir at an upper temperature, T_2 , from a lower temperature, T_1 . The heat gain of this cycle, defined to be

$$g \equiv Q_{\text{cycle}}(\text{into } T_2) / W_{\text{cycle}} = (1 / \varepsilon_{\text{max}})$$

where $\varepsilon_{\text{max}} = (T_2 - T_1) / T_2$ is the maximum thermodynamic efficiency of a heat engine. The refrigerator performance is defined to be

$$K \equiv Q_{\text{cycle}}(\text{from } T_1) / W_{\text{cycle}} = T_1 / (T_2 - T_1)$$

Consider that you have a large swimming pool and plan to heat your house with a heat pump that pumps heat from the pool into your house. A large plate in the water will remain at 0°C due to the formation of ice. You pick T_2 to be 50°C , which will be the temperature of the (large) radiators used to heat your house. Assume that your heat pump has the maximum efficiency allowed by thermodynamics.

- What is the heat gain and the refrigerator performance for this cycle? Be careful to use units of Kelvin for temperature.
- If your house formerly burned 1200 gallons of oil in a winter (at \$2.00/gallon), how much will the electricity cost (at \$0.10 per kilowatt-hour) to replace this heat using the heat pump? A gallon of oil has mass 3.4 kg and contains $1.4 \times 10^8 \text{ J} \cdot \text{gal}^{-1}$.
- The ice cube that appears in your pool over the winter will be how many meters on each side? (It takes $3.35 \times 10^6 \text{ J}$ to melt one kg of ice; it takes up this much heat when freezing.) The density of ice is $0.931 \times 10^3 \text{ kg} \cdot \text{m}^{-3}$.

This would be great for cooling your house in the summer – even if the pool warmed up enough to swim in it, you could still cool your house by running the heat pump in reverse as an air conditioner! More practically, you might be able to use ground water (and the dirt around it) as the heat sink.

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a) The heat gain with $T_2 = 323 \text{ K}$, $T_1 = 273 \text{ K}$

$$g = (1/\epsilon_{\max}) = \frac{T_2}{\Delta T} = \frac{T_2}{T_2 - T_1} = \frac{323 \text{ K}}{(323 - 273) \text{ K}} = 6.46$$

The refrigerator performance

$$K = \frac{T_1}{T_2 - T_1} = \frac{273 \text{ K}}{323 \text{ K} - 273 \text{ K}} = 5.46$$

b) if you burned 1200 gallons of oil, the energy content

$$\Delta U = (1200 \text{ gal}) \left(1.4 \times 10^8 \frac{\text{J}}{\text{gal}} \right) = 1.68 \times 10^{10} \text{ J}$$

Since you are gaining heat according to

$$Q_{\text{cycle}} (\text{into } T_2) = (g) W_{\text{cycle}}$$

you only need to put in

$$\begin{aligned} W_{\text{cycle}} &= \frac{Q_{\text{cycle}} (\text{into } T_2)}{g} = \frac{\Delta U}{g} = \Delta U \frac{(T_2 - T_1)}{T_2} \\ &= (1.68 \times 10^{10} \text{ J}) \frac{(50 \text{ K})}{(323 \text{ K})} = 2.6 \times 10^{10} \text{ J} \end{aligned}$$

$$\$(cost) = (W_{cycle}) \left(\frac{\$0.10}{kW-hr} \right) = (W_{cycle}) \left(\frac{\$0.10}{3.6 \times 10^6 J} \right)$$

since $1 kW-hr = (10^3 W)(3.6 \times 10^3 sec) = 3.6 \times 10^6 J$

$$\$(cost)_{elec} = \frac{(2.6 \times 10^{10} J)(\$0.10)}{3.6 \times 10^6 J} = \$7.2 \times 10^2 \approx \$72.$$

The cost of the oil at \$2/gal is

$$\$(cost)_{oil} = (1200 gal)(\$2/gal) = \$2400$$

b) The heat withdrawn from the water is

$$Q_{cycle}(\text{from } T_1) = K W_{cycle} = \frac{T_1}{(T_2 - T_1)} W_{cycle}$$

$$= \left(\frac{273 K}{50 K} \right) (2.6 \times 10^{10} J) = 1.42 \times 10^{10} J$$

(less than the energy content of the oil).

So ice will develop

The amount of ice in kilograms

$$is \quad m_{ice} = \frac{AQ}{(\Delta Q)_{fusion} \Delta m} = \frac{1.42 \times 10^{10} J}{3.35 \times 10^6 J \cdot kg^{-1}}$$

$$= 4.23 \times 10^4 kg$$

assume the density of ice is

$$\rho_{ice} = 0.931 \times 10^3 \text{ kg/m}^3$$

then

$$\rho_{ice} = \frac{m_{ice}}{V_{ice}} \Rightarrow V_{ice} = \frac{m_{ice}}{\rho_{ice}} = \frac{4.23 \times 10^4 \text{ kg}}{0.931 \times 10^3 \text{ kg/m}^3}$$
$$= 4.55 \times 10^1 \text{ m}^3$$

if the ice forms a cube, the dimension on each side

$$s = (V_{ice})^{1/3} = (4.55 \times 10^1 \text{ m}^3)^{1/3}$$
$$= 3.57 \text{ m}$$