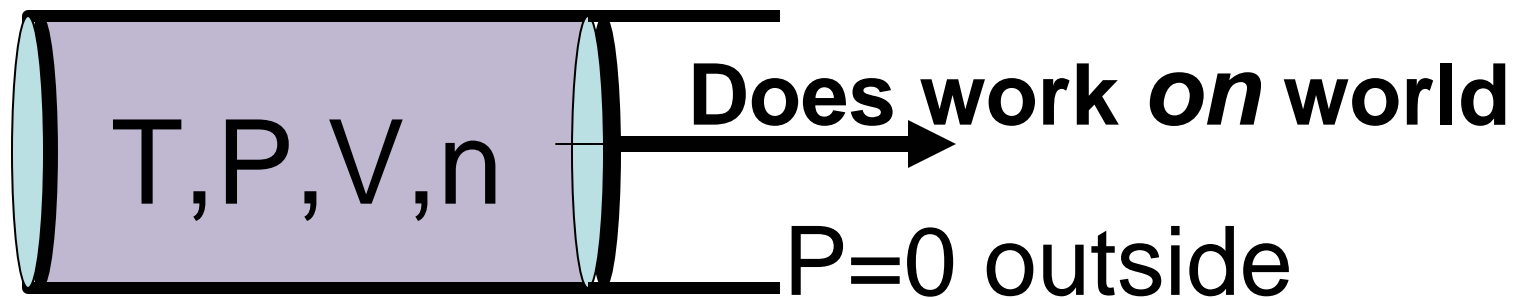


$$PV = nRT = NkT$$

- P is the Absolute pressure
 - Measured from Vacuum = 0
 - Gauge Pressure = Vacuum - Atmospheric
 - Atmospheric = 14.7 lbs/sq in = 10^5 N/m
- V is the volume of the system in m^3
 - often the system is in cylinder with piston
 - Force on the piston does work on world



$PV = nRT = NkT$

chemists vs physicists

- Mole View (more Chemical) = nRT
 - R is *gas constant* 8.315 J/mole/K
- Molecular View (physicists) = NkT
 - N is number of molecules in system
 - K is Boltzmann's Constant 1.38×10^{-23} J/K

Using $PV=nRT$

- Recognize: it relates state variables of a gas
- Typical Problems
 - Lift of hot air balloon
 - Pressure change in heated can of tomato soup
 - Often part of work integral

Heat and Work are Processes

- Processes accompany/cause state changes
 - Work **along particular path** to state B from A
 - Heat added along path to B from A
- Processes are **not state variables**
 - Processes change the state!
 - But Eq. Of State generally obeyed

Ideal Gas Law Derivation: Assumptions

- Gas molecules are hard spheres without internal structure
- Molecules move randomly
- All collisions are elastic
- Collisions with the wall are elastic and instantaneous

Gas Properties

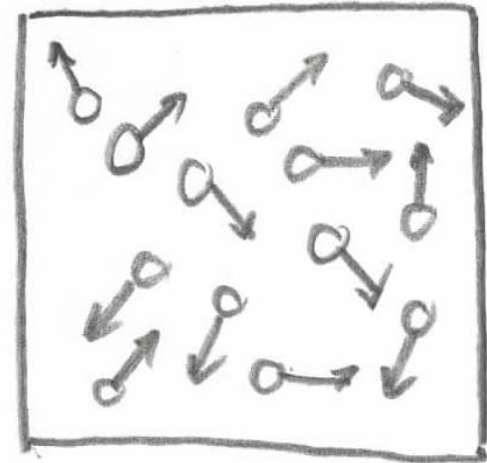
- N number of atoms in volume
- n_m moles in volume
- m is atomic mass ($^{12}\text{C} = 12$)

- mass density
$$\rho = \frac{m_T}{V} = \frac{nm}{V} = \frac{n_m N_A m}{V}$$

- Avogadro's Number
$$N_A = 6.02 \times 10^{23} \text{ molecules} \cdot \text{mole}^{-1}$$

Motion of Molecules

- Assume all molecules have the same velocity (we will drop this latter)
- The velocity distribution is isotropic

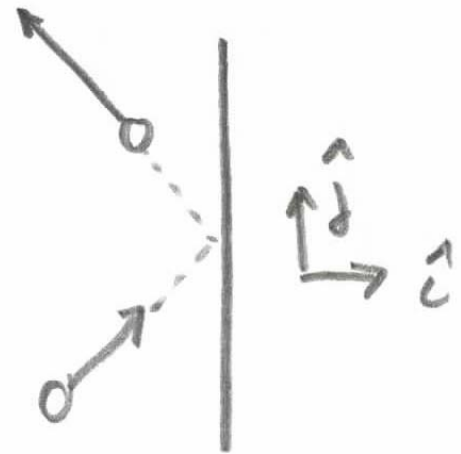


Collision with Wall

- Change of momentum $\hat{\mathbf{i}}: \Delta p_x = -mv_{x,f} - mv_{x,0}$
 $\hat{\mathbf{j}}: \Delta p_y = mv_{y,f} - mv_{y,0}$

- Elastic collision $v_{y,f} = v_{y,0}$

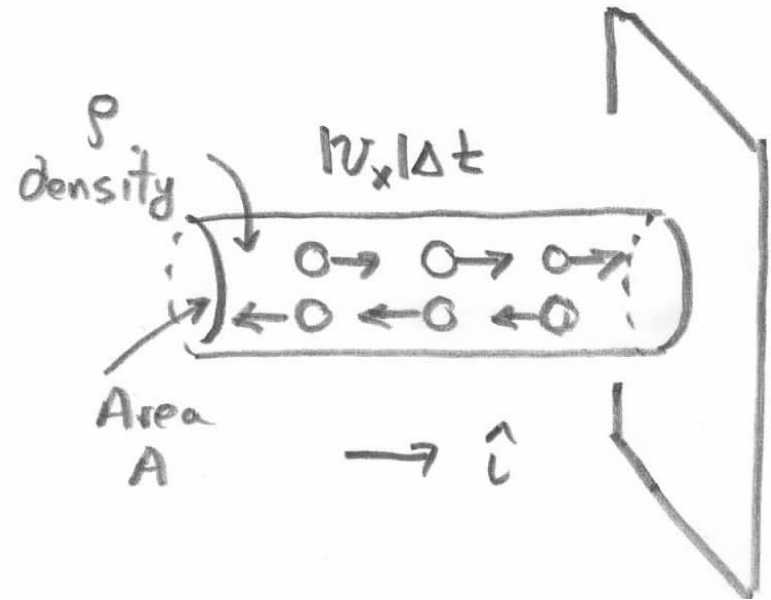
$$v_x \equiv v_{x,f} = v_{x,0}$$



- Conclusion $\hat{\mathbf{i}}: \Delta p_x = -2mv_x$

Momentum Flow Tube

- Consider a tube of cross sectional area A and length $v_x \Delta t$
- In time Δt half the molecules in tube hit wall
- Mass enclosed that hit wall



$$\Delta m = \frac{\rho}{2} \text{Volume} = \frac{\rho}{2} A v_x \Delta t$$

Pressure on the wall

- Newton's Second Law

$$\vec{\mathbf{F}}_{wall, gas} = \frac{\Delta \vec{\mathbf{p}}}{\Delta t} = \frac{-2\Delta m v_x}{\Delta t} \hat{\mathbf{i}} = -\frac{2\rho A v_x^2 \Delta t}{2\Delta t} \hat{\mathbf{i}} = -\rho A v_x^2 \hat{\mathbf{i}}$$

- Third Law $\vec{\mathbf{F}}_{gas, wall} = -\vec{\mathbf{F}}_{wall, gas} = \rho A v_x^2 \hat{\mathbf{i}}$

- Pressure $P_{pressure} = \frac{|\vec{\mathbf{F}}|_{gas, wall}}{A} = \rho v_x^2$

Average velocity

- Replace the square of the velocity with the average of the square of the velocity

$$\left(v_x^2\right)_{ave}$$

- random motions imply

$$\left(v^2\right)_{ave} = \left(v_x^2\right)_{ave} + \left(v_y^2\right)_{ave} + \left(v_z^2\right)_{ave} = 3\left(v_x^2\right)_{ave}$$

- Pressure

$$P_{pressure} = \frac{1}{3} \rho \left(v^2\right)_{ave} = \frac{2}{3} \frac{n_m N_A}{V} \frac{1}{2} m \left(v^2\right)_{ave}$$

Degrees of Freedom in Motion

- Three types of degrees of freedom for molecule
 1. Translational
 2. Rotational
 3. Vibrational
- Ideal gas Assumption: only 3 translational degrees of freedom are present for molecule with no internal structure

Equipartition theorem: Kinetic energy and temperature

- Equipartition of Energy Theorem

$$\frac{1}{2}m(v^2)_{ave} = \frac{(\text{\#degrees of freedom})}{2} kT = \frac{3}{2} kT$$

- Boltzmann Constant $k = 1.38 \times 10^{-23} J \cdot K^{-1}$
- Average kinetic of gas molecule defines kinetic temperature

Ideal Gas Law

- Pressure

$$p_{\text{pressure}} = \frac{2}{3} \frac{n_m N_A}{V} \frac{1}{2} m \left(v^2 \right)_{\text{ave}} = \frac{2}{3} \frac{n_m N_A}{V} \frac{3}{2} kT = \frac{n_m N_A}{V} kT$$

- Avogadro's Number $N_A = 6.022 \times 10^{23} \text{ molecules} \cdot \text{mole}^{-1}$

- Gas Constant $R = N_A k = 8.31 \text{ J} \cdot \text{mole}^{-1} \cdot \text{K}^{-1}$

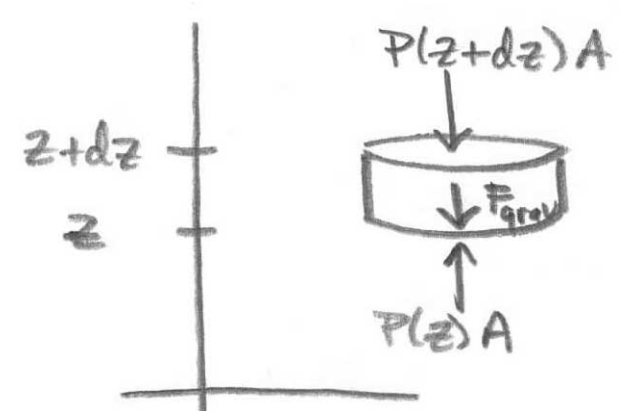
- Ideal Gas Law $pV = n_m RT$

Ideal Gas Atmosphere

- Equation of State $PV = n_m RT$

- Let M_{molar} be the molar mass.

- Mass Density $\rho = \frac{n_m M_{\text{molar}}}{V} = \frac{M_{\text{molar}}}{RT} P$



- Newton's Second Law

$$A \left(P(z) - P(z + \Delta z) \right) - \rho g A \Delta z = 0$$

Isothermal Atmosphere

- Pressure Equation

$$\frac{P(z + \Delta z) - P(z)}{\Delta z} = \rho g$$

- Differential equation

$$\frac{dP}{dz} = -\rho g = -\frac{M_{molar} g}{RT} P$$

- Integration

$$\int_{P_0}^{P(z)} \frac{dp}{p} = - \int_{z=0}^z \frac{M_{molar} g}{RT} dz'$$

- Solution

$$\ln\left(\frac{P(z)}{P_0}\right) = -\frac{M_{molar} g}{RT} z$$

- Exponentiate

$$P(z) = P_0 \exp\left(-\frac{M_{molar} g}{RT} z\right)$$