

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Physics 8.01 TEAL

Fall Term 2004

Exam 2: Equation Summary

One Dimensional Kinematics:

$$\vec{v} = d\vec{r} / dt, \quad \vec{a} = d\vec{v} / dt$$

$$v_x(t) - v_{x,0} = \int_{t'=0}^{t'=t} a_x(t') dt' \quad x(t) - x_0 = \int_{t'=0}^{t'=t} v_x(t') dt'$$

Constant Acceleration:

$$x(t) = x_0 + v_{x,0}(t-t_0) + \frac{1}{2} a_x(t-t_0)^2 \quad v_x(t) = v_{x,0} + a_x(t-t_0)$$
$$y(t) = y_0 + v_{y,0}(t-t_0) + \frac{1}{2} a_y(t-t_0)^2 \quad v_y(t) = v_{y,0} + a_y(t-t_0)$$

where $x_0, v_{x,0}, y_0, v_{y,0}$ are the initial position and velocities components at $t = t_0$

Newton's Second Law: Force, Mass, Acceleration

$$\vec{F} \equiv m\vec{a} \quad \vec{F}^{total} = \vec{F}_1 + \vec{F}_2 \quad F_x^{total} = ma_x \quad F_y^{total} = ma_y \quad F_z^{total} = ma_z$$

Newton's Third Law:

$$\vec{F}_{1,2} = -\vec{F}_{2,1}$$

Force Laws:

Universal Law of Gravity: $\vec{F}_{1,2} = -G \frac{m_1 m_2}{r_{1,2}^2} \hat{r}_{1,2}$, attractive

Gravity near surface of earth: $\vec{F}_{grav} = m_{grav} \vec{g}$, towards earth

Contact force: $\vec{F}_{contact} = \vec{N} + \vec{f}$, depends on applied forces

Static Friction: $0 \leq f_s \leq f_{s,max} = \mu_s N$ direction depends on applied forces

Kinetic Friction: $f_k = \mu_k N$ opposes motion

Hooke's Law: $F = k|\Delta x|$, restoring

Kinematics Circular Motion:

arc length: $s = R\theta$; **angular velocity:** $\omega = d\theta/dt$ **tangential velocity:** $v = R\omega$; **angular acceleration:** $\alpha = d\omega/dt = d^2\theta/dt^2$; **tangential acceleration** $a_\theta = R\alpha$.

Period: $T = \frac{2\pi R}{v} = \frac{2\pi R}{R\omega} = \frac{2\pi}{\omega}$; **frequency:** $f = \frac{1}{T} = \frac{\omega}{2\pi}$,

Radial Acceleration: $|a_r| = R\omega^2$; $|a_r| = \frac{v^2}{R}$; $|a_r| = 4\pi^2 R f^2$; $|a_r| = \frac{4\pi^2 R}{T^2}$

Center of Mass: $\vec{R}_{cm} = \frac{\sum_{i=1}^{i=N} m_i \vec{r}_i}{\sum_{i=1}^{i=N} m_i} \rightarrow \frac{\int_{body} dm \vec{r}}{\int_{body} dm}$;

Velocity of Center of Mass: $\vec{V}_{cm} = \frac{\sum_{i=1}^{i=N} m_i \vec{v}_i}{\sum_{i=1}^{i=N} m_i} \rightarrow \frac{\int_{body} dm \vec{v}}{\int_{body} dm}$

Torque: $\vec{\tau}_S = \vec{r}_{S,P} \times \vec{F}_P$ $|\vec{\tau}_S| = |\vec{r}_{S,P}| |\vec{F}_P| \sin \theta = r_\perp F = r F_\perp$

Static Equilibrium: $\vec{F}_{total} = \vec{F}_1 + \vec{F}_2 + \dots = \vec{0}$; $\vec{\tau}_S^{total} = \vec{\tau}_{S,1} + \vec{\tau}_{S,2} + \dots = \vec{0}$.

Kinetic Energy: $K = \frac{1}{2}mv^2$; $\Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2$

Work: $W = \int_{r_0}^{r_f} \vec{F} \cdot d\vec{r}$; **Work-Change in Kinetic Energy:** $W^{total} = \Delta K$

Power: $P = \vec{F} \cdot \vec{v} = dK/dt$

Potential Energy: $\Delta U = -W_{conservative} = -\int_A^B \vec{F}_c \cdot d\vec{r}$

Potential Energy Functions with Zero Points:

Constant Gravity: $U(y) = mgy$ with $U(y_0 = 0) = 0$.

Inverse Square Gravity: $U_{gravity}(r) = -\frac{Gm_1 m_2}{r}$ with $U_{gravity}(r_0 = \infty) \equiv 0$.

Hooke's Law: $U_{spring}(x) = \frac{1}{2}kx^2$ with $U_{spring}(x = 0) \equiv 0$.

Work- Change in Mechanical Energy: $\vec{F}^{total} = \vec{F}_c^{total} + \vec{F}_{nc}^{total}$;

$W^{total} = W_c^{total} + W_{nc}^{total} = \Delta K$; $W_{nc} = \Delta K + \Delta U^{total} = \Delta E_{mech}$, $E_{mech} = K + U^{total}$

$W_{nc} = \Delta E_{mech} = E_f - E_0 = (K_f + U_f^{total}) - (K_0 + U_0^{total})$

Problem 3: spring and loop

A mass m is pushed against a spring with spring constant k and held in place with a catch. The spring compresses an unknown distance x . When the catch is removed, the mass leaves the spring and slides along a frictionless circular loop of radius r . When the mass reaches the top of the loop, the force of the loop on the mass (the normal force) is equal to twice the weight of the mass.

- Using conservation of energy, find the kinetic energy at the top of the loop. Express your answer as a function of k , m , x , g , and R .
- Using Newton's second law, derive the equation of motion for the mass when it is at the top of the loop.
- How far was the spring compressed?

Problem 4: inclined plane, friction, spring

An object of mass m slides down a plane which is inclined at an angle θ . The object starts out at rest a distance s from an unstretched spring that lies at the bottom of the plane. The spring has a constant k .

- Assume the incline plane is frictionless. How far will the spring compress when the mass first comes to rest?
- Now assume that the incline plane has a coefficient of kinetic friction μ_k . How far will the spring compress when the mass first comes to rest?
- In case b), how much energy has been lost to heat?

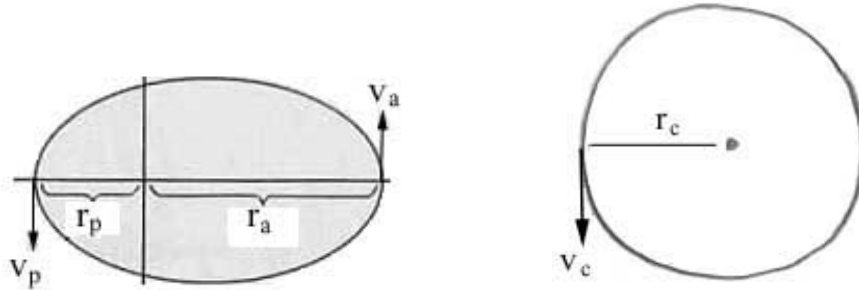
Problem 5: elliptic orbit

A satellite of mass m is in an elliptical orbit around a planet of mass m_p which is located at one focus of the ellipse. The satellite has a velocity v_a at the distance r_a when it is furthest from the planet. The distance of closest approach is r_p .

- What is the magnitude of the velocity v_p of the satellite when it is closest to the planet?
- If the satellite were in a circular orbit of radius $r_c = r_p$, is its velocity v_c greater than, equal to, or less than the velocity v_p of the original elliptic orbit? Justify your answer.

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Solution:

Choose zero for the gravitational potential energy to be when the satellite and planet are separated by an infinite distance, $U(r = \infty) = 0$. When the satellite is at closest approach the energy is

$$E_p = K_p + U_p = \frac{1}{2} m_1 v_p^2 - \frac{G m_1 m_2}{r_p}.$$

When the satellite is at the furthest distance from the planet, the energy is

$$E_a = K_a + U_a = \frac{1}{2} m_1 v_a^2 - \frac{G m_1 m_2}{r_a}.$$

Mechanical energy is conserved so

$$\frac{1}{2} m_1 v_p^2 - \frac{G m_1 m_2}{r_p} = \frac{1}{2} m_1 v_a^2 - \frac{G m_1 m_2}{r_a}.$$

Therefore the velocity of the satellite at closest approach is

$$v_p = \sqrt{v_a^2 - 2Gm_2 \left(\frac{1}{r_a} - \frac{1}{r_p} \right)}.$$

b) The velocity of a satellite undergoing uniform circular motion can be found from the force equation,

$$-\frac{Gm_1m_2}{r_c^2} = -\frac{m_1v_c^2}{r_c}.$$

So the velocity is

$$v_c = \sqrt{\frac{Gm_2}{r_c}}.$$

Suppose we give the satellite that is in a circular motion a small increase in velocity in the tangential direction, $v_{new} = v_c + \Delta v$. This implies that the energy increases. The satellite will no longer travel in a circular orbit since for the same radius r_c ,

$$\frac{m_1v_{new}^2}{r_c} > \frac{Gm_1m_2}{r_c^2}.$$

The satellite will move away from the planet entering into an elliptical orbit. So any velocity v_p greater than v_c will form an elliptic orbit.