

**Problem 1: (Second Law Applications: synchronous satellite)**

A synchronous satellite goes around the earth once every 24 h, so that its position appears stationary with respect to a ground station. The mass of the earth is  $m_e = 5.98 \times 10^{24} \text{ kg}$ . The mean radius of the earth is  $r_e = 6.37 \times 10^6 \text{ m}$ . The universal constant of gravitation is  $G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2 / \text{kg}^2$ . Your goal is to find the radius of the orbit of a synchronous satellite that circles the earth.

- Describe what motion models this problem. Write down a vector description of the acceleration. Clearly indicate your choice of unit vectors and your reasoning for choosing them.
- What is the radius of the orbit of a synchronous satellite that circles the earth? Approximately how many earth radii is this distance?

**Problem 2: (Circular motion: banked turn)**

~~A car of mass  $m$  is going around a circular turn of radius  $R$  which is banked at an angle  $\theta$  with respect to the ground. Assume there is a coefficient of static friction  $\mu_s$  between the wheels and the road. Let  $g$  be the magnitude of the acceleration due to gravity. You may neglect kinetic friction. In each part below show your force diagrams.~~



- ~~Describe the motion of the car if the car is traveling very slowly or extremely fast. Is it possible for the car to travel at speeds such that it can undergo circular motion? If so, what coordinate system best suits this problem. In particular, describe the acceleration vector in terms of your coordinate system unit vectors.~~
- ~~Derive an expression for the minimum velocity necessary to keep the car moving in a circle without slipping down the embanked turn. Express your answer in terms of the given quantities.~~
- ~~Derive an expression for the maximum velocity necessary to keep the car moving in a circle without slipping up the embanked turn. Express your answer in terms of the given quantities.~~
- ~~Derive an expression for the velocity necessary to keep the car moving in a circle without slipping up or down the embanked turn such that the static friction force vanishes. Express your answer in terms of the given quantities.~~

**Problem 1: (Second Law Applications: synchronous satellite)**

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**Answer:**

a) The motion of the satellite is uniform circular motion. Given  $\vec{\omega}$  the angular speed of the satellite around the revolution axis (that we will consider passing through the center of mass of the earth as this is a very good approximation) and  $\vec{r}$  its distance from the center of the earth we have:

$$\frac{d}{dt} \vec{r} = \vec{\omega} \times \vec{r}$$
$$\frac{d^2}{dt^2} \vec{r} = \vec{\omega} \times (\vec{\omega} \times \vec{r}) = -\omega^2 \vec{r}$$

b) Using Newton's second law we easily write:

$$-m_s \omega^2 \vec{r} = -G m_s m_e \frac{\vec{r}}{r^2}$$

from which:

$$r = \left( G \frac{m_e}{\omega^2} \right)^{\frac{1}{3}} = 4.22 \cdot 10^7 \text{ m} \sim 6.6 R_{\text{earth}}$$