

In-Class Problem 16: Consider a planet of mass m_1 in orbit around a extremely massive star of mass m_2 . The period of the orbit is T . Assume that there is a uniform distribution of dust, of density ρ throughout the space surrounding the star and extending well beyond the planet with $\frac{4\pi^2}{T^2} > \frac{4}{3}G\pi\rho$. The gravitational effect of this dust cloud is to add an attractive centripetal force on the planet with magnitude

$$\vec{\mathbf{F}}_{dust} = -\frac{4}{3}G\pi\rho r m_1 \hat{\mathbf{r}}$$

in addition to the gravitational attraction between the star and the planet. You may neglect any drag forces due to collisions with the dust particles.

- Briefly describe how you intend to model the motion of the object? What directions are you choosing for analyzing the components of your forces and state why?
- Find an expression for the radius of the orbit of the planet.
- If there were no dust present, would the radius of the circular orbit be greater, equal, or less than your result from part a). Briefly explain your reasoning.

Several billion years later, the dust cloud has vanished, but now assume that there is a repulsive force acting on the planet that is given by

$$\vec{\mathbf{F}}_{repulsive} = \frac{k}{r^3} \hat{\mathbf{r}},$$

in addition to the gravitational force between the star and the planet. The constant $k > 0$ and satisfies $G > \frac{2v}{m_2} \sqrt{k/m_1}$.

- Show that there are two possible circular orbits for the planet that have the same velocity v . Find the radii of these orbits.

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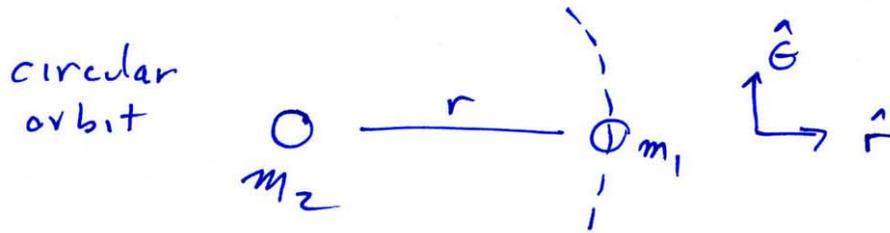
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Solution: star with uniform dust cloud



dust cloud of density ρ , $m_{\text{dust}} = \rho \frac{4}{3} \pi r^3$

$$\vec{F}_{\text{dust}} = -G \frac{m_1 m_2}{r^2} \hat{r} = -G m_1 \rho \frac{4}{3} \pi r^3 \frac{1}{r^2} \hat{r}$$

$$= -\frac{4}{3} G m_1 \rho \pi r \hat{r}$$

$$\vec{F}_{\text{grav}} = -G \frac{m_1 m_2}{r^2} \hat{r}$$

$$\frac{\vec{F}}{m} = \vec{a}$$

$$-\frac{G m_1 m_2}{r^2} - G m_1 \rho \frac{4}{3} \pi r = -m_1 r \left(\frac{2\pi}{T} \right)^2$$

$$\Rightarrow \frac{+G m_2}{r^2} + G \rho \frac{4}{3} \pi r = \frac{r 4\pi^2}{T^2}$$

$$\frac{G m_2}{r^2} = r \left(\frac{4\pi^2}{T^2} - G \rho \frac{4}{3} \pi \right)$$

$$\left(\frac{G m_2}{\left(\frac{4\pi^2}{T^2} - G \rho \frac{4\pi}{3} \right)} \right)^{1/3} = r_{\text{dust}}$$

note: $\frac{4\pi^2}{T^2} > G \rho \frac{4\pi}{3}$

with no dust present $r_{\text{no dust}} = \left(\frac{G m_2}{4\pi^2/T^2} \right)^{1/3}$

$r_{\text{dust}} > r_{\text{no dust}}$

With repulsive force and gravitation:

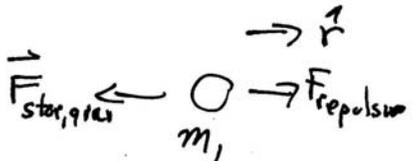


Diagram showing a mass m_1 with forces $\vec{F}_{\text{star, grav}}$ (pointing left) and $\vec{F}_{\text{repulsive}}$ (pointing right), and a velocity vector \vec{r} (pointing up).

$$-\frac{G m_1 m_2}{r^2} + \frac{k}{r^3} = -\frac{m_1 v^2}{r}$$

$$-G m_1 m_2 r + k = -m_1 v^2 r^2$$

$$r^2 - \frac{G m_2 r}{v^2} + \frac{k}{m_1 v^2} = 0$$

$$r = \left(\frac{G m_2}{v^2} \pm \left(\frac{G^2 m_2^2}{v^4} - \frac{4k}{m_1 v^2} \right)^{1/2} \right) / 2.$$

note: $\frac{G m_2^2}{v^4} > \frac{4k}{m_1 v^2}$