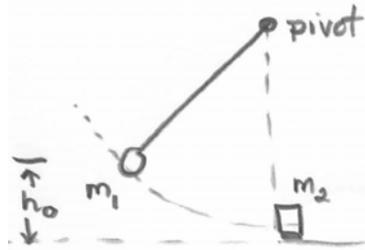
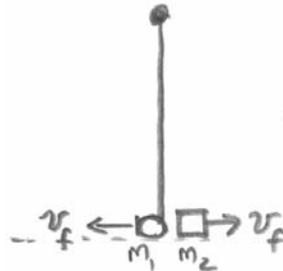


**Problem 29: Pendulums and Collisions**

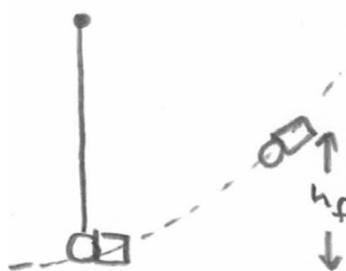
A simple pendulum consists of a bob of mass  $m_1$  that is suspended by a massless string. The bob is pulled out and released from a height  $h_0$  as measured from the bottom of the swing and swings downward in a circular orbit. At the bottom of the swing, the bob collides with a block of mass  $m_2$  that is initially at rest on a frictionless table. Assume the pivot point is frictionless.



- What is the velocity of the bob immediately before the collision at the bottom of the swing?
- Assume the collision is perfectly elastic. The block moves along the table and the bob moves in the opposite direction but with the same speed as the block. What is the mass,  $m_2$ , of the block?

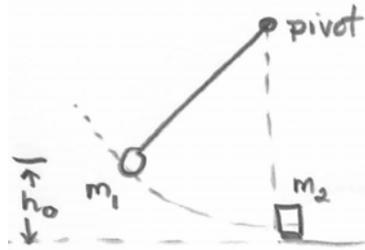


- Suppose the collision is completely inelastic due to some putty that is placed on the block. What is the velocity of the combined system immediately after the collision? (Assume that the putty is massless.)
- After the completely inelastic collision, the bob and block continue in circular motion. What is the maximum height,  $h_f$ , that the combined system rises after the collision?

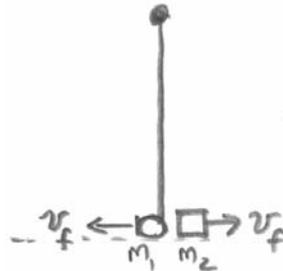


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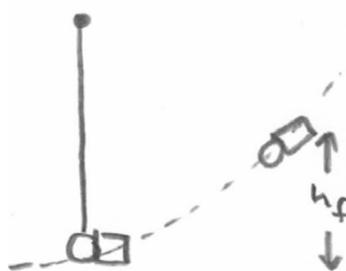
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The speed of the bob considered a point mass can be found applying conservation of energy:

$$v_0 = \sqrt{2gh_0}$$

Applying conservation of momentum and conservation of energy we have

$$m_1 v_0 = (m_2 - m_1) v_f \quad m_1 v_0^2 = (m_1 + m_2) v_f^2$$

$$\Rightarrow m_1 v_0^2 = \frac{(m_1 + m_2)}{(m_1 - m_2)^2} m_1^2 v_0^2 \Rightarrow m_2 (m_2 - 3m_1) = 0$$

The physically sensible solution is  $m_2 = 3m_1$  (The other one will have  $v_f < 0$ ) In this case  $v_f = v_0/2$

In the inelastic case  $m_1 v_0 = (m_2 + m_1) v_f'$

$$v_f' = v_0 \frac{m_1}{m_2 + m_1} = \frac{v_0}{4}$$

Applying conservation of energy after the collision we find

$$(m_1 + m_2) g h_{\max} = \frac{1}{2} (m_1 + m_2) v_f'^2 \Rightarrow h_{\max} = \frac{v_0^2}{32g} = \frac{1}{16} h_0$$