

### ~~6.5 Surprising Bounce – Lecture Demo~~

~~In lectures we dropped a tennis ball on top of a much heavier basketball from a height of about 3 m. The tennis ball bounced up much higher than 3 m. Calculate how high the tennis ball bounced up by approximating the bounce as the result of two one-dimensional collisions in which first the basketball bounced off the floor and then collided with the falling tennis ball. Thus, in this second collision the two balls had velocities in opposite directions. Assume that the collisions were elastic.~~

### ~~6.6 Center of Mass Motion and Internal Energy~~

~~A 6 kg object is moving in 1 dimension with a speed of 350 m/sec on a horizontal frictionless surface. It explodes and breaks apart into one fragment of 2 kg and one of 4 kg with velocities along the initial direction of motion of 250 m/sec and 400 m/sec, respectively.~~

- ~~a) What is the total momentum before and after the object explodes? Is momentum conserved?~~
- ~~b) What is the total kinetic energy before and after the object explodes? Is mechanical energy conserved?~~
- ~~c) What is the center of mass velocity before and after the object explodes? Does it change?~~
- ~~d) Answer the questions under a) and b), but evaluate all quantities in the center-of-mass frame of reference. Do any of your conclusions change?~~

### ~~6.7 Oscillating Masses on Frictionless Table – PIVoT~~

~~Two masses,  $A$  and  $B$  are connected by a spring. The mass of  $A$  is  $m$ , and the mass of  $B$  is  $3m$ ; the mass of the spring is negligible. The spring, with spring constant  $k$ , is compressed so that the potential energy stored in the spring is  $U_0$ . The system is placed on a horizontal frictionless table. The compressed spring is released at time  $t = 0$  at which time the two masses have zero speed.~~

- ~~a) What is the initial total mechanical energy of the system ( $A$ ,  $B$ , and the spring)?~~
- ~~b) How much potential energy is stored in the spring when  $A$  and  $B$  have kinetic energies  $K_A$  and  $K_B$ , respectively?~~
- ~~c) At some point in time the speed of  $A$  is  $v_A$ . What then is the speed of  $B$ ?~~
- ~~d) If  $A$  is moving in the  $+x$  direction, in what direction is  $B$  then moving?~~
- ~~e) What is the period,  $T$ , of the oscillations of the two masses?~~

### ~~6.8 Proton Collision – page 294, problem 38~~

### ~~6.9 Space Shuttle~~

~~A Space Shuttle has to increase its speed by firing its thruster rockets. The exhaust gas speed relative to the Shuttle is about 2.5 km/sec, and the mass of the shuttle is about  $10^5$  kg. What is the mass of the fuel used in increasing the speed from 8.0 km/sec to 8.5 km/sec?~~

### ~~6.10 Saturn V~~

~~The Saturn V rocket was used in the Apollo program. It had an initial mass of 2.85 million kg. The burn rate of the rocket was  $13.8 \times 10^3$  kg/sec (about 14 tons per sec!), which delivered a thrust of 34 million Newtons. Its final mass was 0.77 million kg (770 tons!). Assume that the entire launch is vertically up. In the discussions in lectures (and in Tipler's derivation) air drag was ignored. This, of course is an over-simplification, as air drag must play an important roll. You too may ignore air drag. Use  $g = 10$  m/sec<sup>2</sup>.~~

- ~~a) What was the exhaust speed?~~
- ~~b) How long did the engines burn (this is called the burn time)?~~
- ~~c) What was the acceleration at liftoff?~~
- ~~d) What was the acceleration just before the engines stopped?~~
- ~~e) What was the final speed of the rocket?~~

### ~~6.11 Kerosene Rocket – page 271, problem 54~~

## Problem 6.7

Let  $m_A = m$  and  $m_B = 3m$ .

**(a)** The table is assumed to be frictionless, so the only other force is the spring force which is conservative; hence mechanical energy is conserved. (Gravity and the normal force are present, but if the table is level then these forces precisely balance and thus are irrelevant.) Therefore, the total mechanical energy of the system is constant, and we can evaluate it at  $t = 0$ , at which time both masses are at rest and the spring has potential energy  $U_0$ .

$$E = U_0$$

**(b)** In general if  $K_A$  and  $K_B$  are the kinetic energies of mass A and mass B and  $U_S$  is the potential energy of the spring, then the mechanical energy is

$$E = K_A + K_B + U_S$$

Mechanical energy conservation gives

$$U_0 = K_A + K_B + U_S \quad \implies \quad U_S = U_0 - K_A - K_B$$

**(c)** There are no outside forces acting on this system, so momentum is conserved. (Again, gravity and the normal force are irrelevant since they cancel.) Therefore, the total momentum of the system is constant, and we can evaluate it at  $t = 0$ , at which time both masses are at rest.

$$P = 0$$

Now suppose at some time the velocities are  $\vec{v}_A$  and  $\vec{v}_B$ . Then

$$P = m_A \vec{v}_A + m_B \vec{v}_B = m \vec{v}_A + 3m \vec{v}_B$$

Momentum conservation gives

$$0 = m \vec{v}_A + 3m \vec{v}_B \quad \implies \quad \vec{v}_B = -\frac{1}{3} \vec{v}_A \quad \implies \quad |v_B| = \frac{1}{3} |v_A|$$

**(d)** The result above demonstrates that the particles are always moving in opposite directions. Thus if A is moving in the  $+x$  direction then B is moving in the  $-x$  direction.

**(e)** It is convenient if we choose the center of our coordinate system to coincide with the center of mass of the system. Let  $x_A$  and  $x_B$  label the position of mass A and B respectively. Then by definition of the center of mass

$$0 = m_A x_A + m_B x_B = m x_A + 3m x_B \quad \implies \quad x_B = -\frac{1}{3} x_A$$

Then the force acting on A is

$$m \ddot{x}_A = F = -k(x_A + x_B - l_0) = -k(x_A + \frac{1}{3}x_A - l_0) = -\frac{4k}{3} \cdot (x_A - \frac{3}{4}l_0)$$

Thus the equation of motion is

$$\ddot{x}_A = -\frac{4k}{3m} \cdot (x_A - \frac{3}{4}l_0)$$

This indicates that the motion is simple harmonic with frequency

$$\omega_A = \sqrt{\frac{4k}{3m}}$$

A similar argument for B yields

$$3m \ddot{x}_B = F = -k(x_A + x_B - l_0) = -k(3x_B + x_B - l_0) = -4k \cdot (x_B - \frac{1}{4}l_0) \quad \implies$$

$$\omega_B = \sqrt{4k \cdot \frac{1}{3m}} = \sqrt{\frac{4k}{3m}}$$

Thus the motion is also simple harmonic in  $x_B$  with the same frequency  $\omega$  given by

$$\omega = \sqrt{\frac{4k}{3m}}$$

**~~Problem 6.8~~** (~~Ohanian, page 294, problem 38~~)

~~Collisions in two dimensions are discussed in Section 11.3 beginning on page 281. The relevant results, using conservation of momentum and kinetic energy, are~~

$$~~m_1 v_1 = m_1 v'_1 \cos \theta'_1 + m_2 v'_2 \cos \theta'_2~~$$

$$~~\theta = m_1 v'_1 \sin \theta'_1 = m_2 v'_2 \sin \theta'_2~~$$

$$~~\frac{m_1 v_1^2}{2} = \frac{m_1 v'^2_1}{2} + \frac{m_2 v'^2_2}{2}~~$$