

~~*It is strongly recommended that you read about a subject before it is covered in lectures.*~~

<del>Lecture Date</del>	<del>Material Covered</del>	<del>Reading</del>
<del>#13 Fri 10/8</del>	<del>Potential Energy - watch <b>PIVoT</b> Energy Considerations to derive SHM - watch <b>PIVoT</b></del>	<del>Page 184 - 194 Page 387 - 389</del>
<del>#14 Wed 10/13</del>	<del>Escape Velocities - Bound and Unbound Orbits - <b>PIVoT</b> Circular Orbits (elliptical orbits will be discussed later in the course). Various Forms of Energy - Power</del>	<del>Page 211 - 228 Page 194 - 201</del>
<del>#15 Fri 10/15</del>	<del>Momentum - Conservation of Momentum - watch <b>PIVoT</b> Center of Mass - watch <b>PIVoT</b></del>	<del>Page 113 - 114 Page 245 - 261</del>

~~*Due Friday, Oct 15, before 4 PM in 4-339B. Solutions will be posted on the Web Sat, Oct 16.*~~

~~**4.1** *Air drag on very small drops. Watch **PIVoT** under resistive force.*~~

~~We release an oil drop of radius  $r$  in air. The density of the oil is  $700 \text{ kg/m}^3$ .  $C_1$  and  $C_2$  for 1 atmosphere air at  $20^\circ \text{ C}$  are  $3.1 \times 10^{-4} \text{ (kg/m)/sec}$  and  $0.85 \text{ kg/m}^3$ , respectively. How small should the oil drop be so that the drag force is dominated by the linear term in the speed (in lectures we called this *Regime I*). In this regime, the terminal velocity is  $mg/C_1 r$ . [ $m$  is the mass of the drop].~~

~~**4.2** *Drag force at very low speeds. Watch **PIVoT** under resistive force.*~~

~~At low speeds (especially in liquids rather than gases), the drag force is proportional to the speed rather than its square, i.e.,  $F = -C_1 r v$ , where  $C_1$  is a constant. At time  $t = 0$ , a small ball of mass  $m$  is projected into a liquid so that it initially has a horizontal velocity of  $u$  in the  $+x$  direction. The initial speed in the vertical direction ( $y$ ) is zero. The gravitational acceleration is  $g$ .~~

- ~~Write down the differential equations of motion in the  $x$  and  $y$  direction.~~
- ~~What is the horizontal component of the ball's velocity at time  $t$ ?~~
- ~~What is the vertical component of the ball's velocity at time  $t$ ?~~
- ~~After how many seconds is the vertical speed 99% of its maximum value? What would that be for the 1/4 inch steel ball bearing that we dropped in Karo Corn Syrup in lectures?~~
- ~~Answer the questions under b) and c) for the limiting case that  $t$  becomes infinitely large.~~

~~**4.3** *SHO* page 405, problem 1. Watch **PIVoT** under simple harmonic motion.~~

~~**4.4** *SHO* page 405, problem 4. Watch **PIVoT** under simple harmonic motion.~~

~~**4.5** *Oscillating Spring.* Watch **PIVoT** under simple harmonic motion.~~

~~A 3 kg mass is attached to a spring. The period of oscillation is 0.4 sec. At  $t = 0$ , the mass has a speed of 3 m/sec towards equilibrium, and its displacement from equilibrium is 0.1 m.~~

- ~~Calculate the position of the mass for all time  $t > 0$ .~~
- ~~When will the mass first go through equilibrium; what then will be its speed, acceleration, kinetic energy, and potential energy?~~
- ~~When will the mass first reach a turning point; what then will be its speed, acceleration, kinetic energy, and potential energy?~~

~~**4.6** *Vertical Spring* page 406, problem 11.~~

$$t = -\frac{\tau}{6} + \frac{\tau}{2} = \frac{\tau}{3} = \frac{2\pi}{3\omega} = \frac{2\pi}{3 \times 2.0 \text{ rad/s}} \approx 1.0 \text{ s}$$

~~(c) All the times for which  $x = 0$  and  $x = \pm A$  were already given above.~~

$$x = 0 \text{ at } t = \frac{\tau}{12} + \frac{n\tau}{2}$$

and

$$x = \pm A \text{ at } t = -\frac{\tau}{6} + \frac{n\tau}{2}$$

~~The part  $\frac{n\tau}{2}$  indicates that the particle will move through a turning point or equilibrium exactly at intervals of  $\frac{\tau}{2}$ . But we must be careful. There are two turning points: half of the times above correspond to  $x = +A$ , and the other half correspond to  $x = -A$ . The time  $t = -\frac{\tau}{6}$  corresponds to  $x = +A$ . So the possible times for  $x = +A$  are~~

$$t = -\frac{\tau}{6} + n\tau$$

~~The possible times for  $x = -A$  are~~

$$t = -\frac{\tau}{6} + \frac{\tau}{2} + n\tau = \frac{\tau}{3} + n\tau$$

## Problem 4.5

(a) The motion is simple harmonic, so the position as a function of time is

$$x = A \cos(\omega t + \delta)$$

and the velocity as a function of time is

$$v = -\omega A \sin(\omega t + \delta)$$

The angular frequency  $\omega$  is given by

$$\omega = \frac{2\pi}{\tau} = \frac{2\pi}{0.4} \approx 15.7 \text{ rad/s}$$

If we choose the direction for  $x$  to point along the same direction as the initial displacement then the initial ( $t = 0$ ) displacement from origin is  $x = 0.1 \text{ m}$ , so

$$0.1 = A \cos \delta \tag{1}$$

The initial ( $t = 0$ ) velocity is  $v = -3$  m/s (it is directed *toward* the origin from the *positive* side), so

$$-3 = -\omega A \sin \delta \quad (2)$$

Using equation (1) and equation (2) we can find  $\delta$ .

$$\tan \delta = \frac{\sin \delta}{\cos \delta} = \frac{3}{0.1 \omega} = \frac{3\tau}{0.1 \cdot 2\pi}$$

Thus

$$\delta = \tan^{-1} \left( \frac{3 \cdot \tau}{0.1 \cdot 2\pi} \right) = \tan^{-1} \left( \frac{3 \cdot 0.4}{0.1 \cdot 2\pi} \right) \approx 1.088$$

(Remember that any value  $\delta = 1.088 + 2n\pi$  will do.) Then using equation (1) gives

$$A = \frac{0.1}{\cos \delta} \approx 0.216 \text{ m}$$

Therefore the motion is given by

$$x = 0.216 \cdot \cos(15.7 t + 1.088)$$

**(b)** Equilibrium corresponds to  $x = 0$ .

$$x = A \cos(\omega t + 1.088) = 0 \quad \implies$$

$$\begin{aligned} \cos(\omega t + 1.088) &= 0 \\ \omega t + 1.088 &= \frac{\pi}{2} + n\pi \\ t &= \frac{1}{\omega} \left( \frac{\pi}{2} + n\pi - 1.088 \right) \\ t &= \frac{\tau}{2\pi} (0.48 + n\pi) \\ t &= 7.7 \times 10^{-2} \cdot \tau + \frac{n\tau}{2} \end{aligned}$$

The first time the mass will pass through equilibrium is

$$t = 7.7 \times 10^{-2} \cdot \tau = 3.1 \times 10^{-2} \text{ s}$$

The total energy is conserved.

$$E = K + U$$

with

$$K = \frac{1}{2}mv^2 \quad \text{and} \quad U = \frac{1}{2}kx^2 = \frac{m\omega^2 x^2}{2} = \frac{2\pi^2 m}{\tau^2} x^2$$

At  $t = 0$  the energy is given by

$$E = \frac{1}{2}mv_0^2 + \frac{2\pi^2 m}{\tau^2} x_0^2 = \frac{1}{2}m \cdot (3)^2 + \frac{2\pi^2 m}{\tau^2} \cdot (0.1)^2 \approx 17.2 \text{ J}$$

By energy conservation, this will be the total energy at any other time.

At equilibrium,  $x = 0$ , and the potential energy is

$$U = \frac{1}{2}k(0)^2 = 0$$

Thus the total energy, which is now just the kinetic energy, is

$$E = K = \frac{1}{2}mv^2 \quad \implies \quad |v| = \sqrt{\frac{2E}{m}}$$

Thus the kinetic energy is

$$K = E \approx 17.2 \text{ J}$$

and the speed is

$$|v| = \sqrt{\frac{2E}{m}} \approx 3.4 \text{ m/s}$$

The acceleration at  $x = 0$  is given by

$$a = \frac{d^2x}{dt^2} = -\omega^2 x \quad \implies \quad a = 0$$

(c) The turning points corresponds to  $x = \pm A$ . These points also correspond to  $v = 0$ .

$$v = -\omega A \sin(\omega t + 1.088) = 0 \quad \implies$$

$$\begin{aligned}
\sin(\omega t + 1.088) &= 0 \\
\omega t + 1.088 &= n\pi \\
t &= \frac{1}{\omega}(n\pi - 1.088) \\
t &= \frac{\tau}{2\pi}(n\pi - 1.088) \\
t &= -0.18 \cdot \tau + \frac{n\tau}{2}
\end{aligned}$$

The first time the mass will pass through a turning point is

$$t = -0.18 \cdot \tau + \frac{\tau}{2} \approx 0.13 \text{ s}$$

By substituting  $t = 0.13 \text{ s}$  into the equation of motion we find that the corresponding position is  $x = -0.216 \text{ m}$ . (We knew it had to be  $x = \pm 0.216 \text{ m}$  by the above.)

At a turning point,  $v = 0$ , and the kinetic energy is

$$K = \frac{1}{2}m(0)^2 = 0$$

Thus the total energy, which is now just the potential energy, is

$$U = E \approx 17.2 \text{ J}$$

The acceleration at  $x = -0.216 \text{ m}$  is given by

$$a = \frac{d^2x}{dt^2} = -\omega^2 A \quad \implies \quad a = 53.2 \text{ m/s}^2$$

### **Problem 4.6** (~~Ohanian, page 406, problem 11~~)

~~Springs and gravity are discussed in example 2 on page 386. The key point in that example is that gravity only lowers the equilibrium position of the spring. The motion is still simple harmonic with the same frequency. The equilibrium length is moved down by an amount~~

$$\Delta x = \frac{mg}{k} \approx 0.27 \text{ m}$$

~~Thus, holding the spring at the unstretched position is actually holding it at a displacement of  $\Delta x$  up from equilibrium. At this point, determining the motion will be very similar to problem 4.5 part (a). Label the vertical direction with  $x$ ; define the increasing~~