

Problem 4: Spring-mass harmonic oscillator

Consider an ideal spring that has an unstretched length l_0 . Assume the spring has a constant k . Suppose the spring is attached to a mass m that lies on a horizontal frictionless surface. The spring-mass system is compressed a distance of x_0 from equilibrium and then released with an initial speed v_0 toward the equilibrium position.

- a) What is the period of oscillation for this system?
- b) How long will it take for the mass to first return to the equilibrium position?
- c) How long will it take for the mass to first become completely extended?
- d) Draw a graph of the position and velocity of the mass as a function of time. Carefully label your axes and clearly specify any special values.

~~**Problem 5: Analysis Experiment 06 Post**~~

~~a) Enter the values from your experiment into the table below.~~

a_{up} (m/s²)	a_{down} (m/s²)	$U(h_1)$ (mJ)	$U(h_2)$ (mJ)

~~Use these numbers, along with $M = 0.75$ kg, to calculate the friction force on the cart. Compare it with the value you obtained from the potential energy difference between the turning points h_1 and h_2 .~~

~~b) Explain briefly in words why it makes sense to fit the function $A \sin[2\pi(x - C)/T]$ to the force peaks when the cart bounces off the spring on the force sensor.~~

~~c) Enter the results of your fit to the second bounce into this table:~~

A (N)	k (N/m)	$A^2/2k$ (mJ)

~~and use them to find the force constant of the spring. Compute the potential energy stored in the compressed spring, $A^2/2k$, and compare it with the average of $U(h_1)$ and $U(h_2)$ from the table above.~~

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Problem 4

a) By solving the equation of motion $m \ddot{x} = -kx$ with the initial conditions $x(0) = -x_0$ and $\dot{x}(0) = v_0$ we get $x(t) = -A \cos(\sqrt{\frac{k}{m}} t + \phi)$ with the parameters A and ϕ such that:

$$A \cos(\phi) = x_0$$

$$A \sqrt{\frac{k}{m}} \sin(\phi) = v_0$$

from which $\tan(\phi) = \frac{v_0}{x_0} \sqrt{\frac{m}{k}}$ and $A = \sqrt{x_0^2 + \frac{m}{k} v_0^2}$

The period is given by: $T = 2\pi \sqrt{\frac{m}{k}}$

b) the mass will return to the equilibrium position after a time t_1 such that $t_1 \sqrt{\frac{k}{m}} = \frac{\pi}{2} - \phi$ that is $t_1 = \sqrt{\frac{m}{k}} (\frac{\pi}{2} - \phi) = T (\frac{1}{4} - \frac{\phi}{2\pi})$

c) The mass will be completely extended after a time t_2 such that $t_2 \sqrt{\frac{k}{m}} = \pi - \phi$ that is $t_2 = \sqrt{\frac{m}{k}} (\pi - \phi) = T (\frac{1}{2} - \frac{\phi}{2\pi})$

