

**Problem 3: (Torque and Conservation of Angular Momentum)** A bowling ball of mass  $m$  and radius  $R$  is thrown down an alley with an initial velocity  $v_0$  and no initial angular velocity. The moment of inertia of the ball about the center of mass is  $I = \frac{2}{5} mR^2$ . The coefficient of friction between the ball and the alley is  $\mu$ .

- What is the angular velocity of the ball when the ball just starts to roll without slipping down the alley?
- What is the kinetic energy of the ball when the ball just starts to roll without slipping down the alley?
- What is the change in kinetic energy ?

~~**Problem 4: (Torque and angular acceleration)** A pulley of mass  $m$ , radius  $R$ , and moment of inertia  $I = \frac{1}{2} mR^2$  about the center of mass is hung from a ceiling with a massless string that winds around the rim of the pulley. At time  $t = 0$  the pulley is released from rest.~~

- ~~Draw the free body diagram on the pulley.~~
- ~~Write down the equation describing the motion of the center of mass.~~
- ~~Write down the rotational equation of motion.~~
- ~~Find the direction and magnitude of the translational acceleration of the pulley.~~
- ~~How long does it take for the pulley to fall a distance  $L$ ?~~
- ~~What is the tension on the string?~~

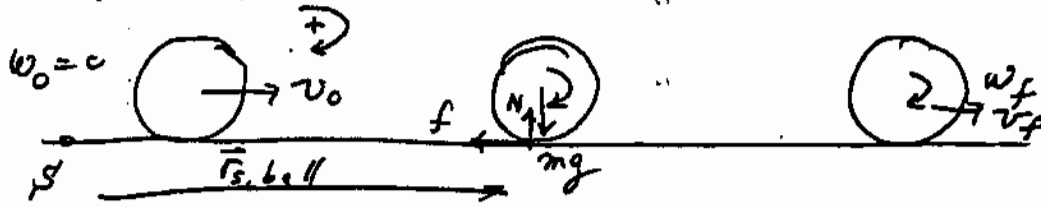
~~**Problem 5: (Conservation of angular momentum)** A long stick of length  $L$  and mass  $m$  lies motionless on a frictionless surface. A putty of the same mass  $m$  moves without friction on the floor with a speed of  $v_0$  toward the stick, hits one of the ends of the stick perpendicularly and is stuck to the end.~~

- ~~How far from the midpoint of the stick is the center of mass of the stick plus the putty at the time of the collision?~~
- ~~What is the momentum of the stick plus putty after the collision?~~
- ~~What is the angular velocity of the stick plus putty after the collision?~~
- ~~How far does the stick's center of mass move during one rotation of the stick?~~

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- What is the angular velocity of the ball when the ball just starts to roll without slipping down the alley?
- What is the kinetic energy of the ball when the ball just starts to roll without slipping down the alley?
- What is the change in kinetic energy?



Choose any point  $S$  along the alley. Since

$$\vec{\tau}_S = \vec{r}_{S, ball} \times (m\vec{g} + \vec{N} + \vec{f}) = \vec{r}_{S, ball} \times \vec{f} \text{ since}$$

$$m\vec{g} = -\vec{N} \text{ so those torques cancel.}$$

Since  $\vec{r}_{S, ball}$  is antiparallel to  $\vec{f}$ ,  
 $\vec{r}_{S, ball} \times \vec{f} = 0$ . Thus  $\vec{L}_S = \text{constant}$

$$\vec{L}_0 = \vec{r}_{S, cm} \times m \vec{v}_0 = R m v_0 \text{ (into page)}$$

$$\vec{L}_f = I_{cm} \omega_f \text{ (into page)} + \vec{r}_{S, cm} \times m \vec{v}_f$$

$$= (I_{cm} \omega_f + R m v_f) \text{ (into page)}$$

Thus the rolling without slipping condition

$$v_f = R \omega_f$$

$$L_f = I_{cm} \omega_f + R m R \omega_f = L_0 = R m v_0$$

$$\frac{2}{5} m R^2 \omega_f + m R^2 \omega_f = R m v_0$$

$$\frac{7}{5} m R^2 \omega_f = R m v_0 \Rightarrow \omega_f = \frac{5}{7} \frac{v_0}{R}$$

$$v_f = \frac{5}{7} v_0$$

b) The kinetic energy of the ball

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$$K = \frac{1}{2} m v_f^2 + \frac{1}{2} I \omega_f^2 = \frac{1}{2} m v_f^2 + \frac{1}{2} \frac{2}{5} m R^2 \left( \frac{5}{7} \frac{v_0}{R} \right)^2$$

$$K = \frac{1}{2} m \left( \frac{5}{7} v_0 \right)^2 + \frac{5 m v_0^2}{49} = \frac{5}{14} m v_0^2$$

$$\Delta K = K_0 - K_f = \frac{1}{2} m v_0^2 - \frac{5}{14} m v_0^2 = \frac{3}{7} m v_0^2$$