

**Massachusetts Institute of Technology
Department of Physics**

8.01X

Fall 2002

Practice Final Exam

Problem 1: (Experiment AM)

In experiment AM, a circular steel washer, of mass m_1 and moment of inertia $I_w = 4.0 \times 10^{-5} \text{ kg}\cdot\text{m}^2$, is mounted on the shaft of a small motor. Assume that the moment of inertia of the motor is negligible. The washer is set into motion. When it reaches 200 rad/sec, at $t = 0$ the power to the motor is shut off, and the washer slows down until it reaches an angular velocity of 140 rad/sec at $t = 6$ sec. At that moment a second identical circular steel washer of mass $m_2 = m_1$ is dropped on the first. The collision takes place over a time 0.2 sec.

- a) What is the initial angular momentum of the washer at $t=0$?
- b) What is the initial kinetic energy of the washer at $t=0$?
- c) What is the magnitude of the frictional torque acting from $t = 0$ to 6 sec?
- d) What is the angular velocity of both washers at $t= 6.2$ sec?
- e) What is the average torque which the dropped washer exerts on the spinning washer during the time between 6.0 sec and 6.2 sec?

Problem 2: (Bernoulli's Principle)

A water bottle of area $A = 3.8 \times 10^{-3} \text{ m}^2$ containing water at atmospheric pressure is sealed shut with an initial volume of air $V_0 = 1.5 \times 10^{-4} \text{ m}^3$ and brought to the top of Mount Washington where the pressure is 0.88 atm. A hole is punched in the side of the bottle a distance $h_0 = 10.0$ cm below the water level. Assume the flow is steady out of the hole, and ignore any effects due to the hole. Note that $1 \text{ atm} = 1.013 \times 10^5 \text{ N/m}^2$.

- a) What is the magnitude of the velocity of the efflux through the hole immediately after the hole is punched?
- b) Assume the air is an ideal gas, what is the air pressure in the bottle as a function of the height h of the water level above the hole?
- c) What is the magnitude of the velocity of the efflux through the hole as a function of the height h of the water level above the hole?

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Practice Final Exam SOLUTIONS

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- What is the initial angular momentum of the washer at $t=0$?
- What is the initial kinetic energy of the washer at $t=0$?
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- What is the angular velocity of both washers at $t = 6.2$ sec?
- What is the average torque which the dropped washer exerts on the spinning washer during the time between 6.0 sec and 6.2 sec?

$$a) \quad L_0 = I_{cm} \omega_0 = (4.0 \times 10^{-5} \text{ kg}\cdot\text{m}^2) \left(\frac{200 \text{ rad}}{\text{sec}} \right) = 8.0 \times 10^{-3} \frac{\text{kg}\cdot\text{m}^2}{\text{sec}}$$

$$b) \quad K_0 = \frac{1}{2} I_{cm} \omega_0^2 = \frac{1}{2} (4.0 \times 10^{-5} \text{ kg}\cdot\text{m}^2) \left(\frac{200 \text{ rad}}{\text{sec}} \right)^2 = 8.0 \times 10^{-1} \text{ J}$$

$$c) \quad \tau = I_{cm} \left| \frac{\Delta \omega}{\Delta t} \right| = (4.0 \times 10^{-5} \text{ kg}\cdot\text{m}^2) \left(\frac{140 \text{ rad} - 200 \text{ rad}}{6 \text{ sec}} \right) = 4 \times 10^{-4} \text{ J}$$

$$d) \quad -\tau \Delta t = \Delta L = 2 I_{cm} \omega_f - I_{cm} \omega_b$$

$$\omega_f = \frac{-\tau \Delta t + I_{cm} \omega_b}{2 I_{cm}} = \frac{(-4 \times 10^{-4} \text{ J})(0.2 \text{ sec}) + (4.0 \times 10^{-5} \text{ kg}\cdot\text{m}^2) \left(\frac{140 \text{ rad}}{\text{sec}} \right)}{(2)(4.0 \times 10^{-5} \text{ kg}\cdot\text{m}^2)}$$

$$\omega_f = 69 \frac{\text{rad}}{\text{sec}}$$

$$e) \quad -\tau_{total} \Delta t = I_{cm} (\omega_f - \omega_b)$$

$$-\tau_{total} = \frac{I_{cm} (\omega_f - \omega_b)}{\Delta t} = \frac{(4 \times 10^{-5} \text{ kg}\cdot\text{m}^2) \left(\frac{69 \text{ rad}}{\text{sec}} - \frac{140 \text{ rad}}{\text{sec}} \right)}{0.2 \text{ sec}} = -1.42 \times 10^{-2} \text{ J}$$

$$\tau_{total} = \tau_{frictional} + \tau_{dropper washer}$$

$$\tau_{dropper washer} = \tau_{total} - \tau_{fr} = -1.42 \times 10^{-2} \text{ J} - -4 \times 10^{-4} \text{ J} = -1.38 \times 10^{-2} \text{ J}$$