

~~Nov 24~~

~~Hour One: Archimedes Principle~~

~~Reading: YF 14.1-14.3~~

~~Hour Two: Archimedes Principle PRS Contest~~

~~Nov 26 No Class~~

~~Problem Set 12: Due Fri Dec 3 at 4:00 pm.~~

~~**Problem 1 : Bohr hydrogen atom**~~

~~The Bohr hydrogen atom models the atom as an electron orbiting the proton under the influence of an electric force producing uniform circular motion with radius a_0 . The mass of the electron is $m_e = 9.1 \times 10^{-31} \text{ kg}$; the electric charge is $e = 1.6 \times 10^{-19} \text{ C}$; the Planck constant is $h = 6.63 \times 10^{-34} \text{ J} \cdot \text{s}$, and the magnitude of the electric force is given by Coulomb's Law $|\vec{F}| = ke^2/r^2$ where $k = 9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$. The angular momentum is quantized according to $L = nh/2\pi$ where $n = 1, 2, \dots$~~

- ~~a) Write down the equation that arises from the application of Newton's 2nd Law to the electron.~~
- ~~b) What is the angular momentum of the electron about the center of the atom?~~
- ~~c) Using your results from parts a) and b) derive an equation for the radius a_0 of the atom as a function of n, e, h, m_e , and k .~~
- ~~d) What is the energy E_n for the atom? Express your answer in terms of n, e, h, m_e , and k .~~
- ~~e) A hydrogen atom emits a photon which arise from an energy transition from the $n = 3$ to $n = 1$ energy level. Calculate the frequency of the light emitted~~

$$~~f = \Delta E/h = -(E_4 - E_3)/h.~~$$

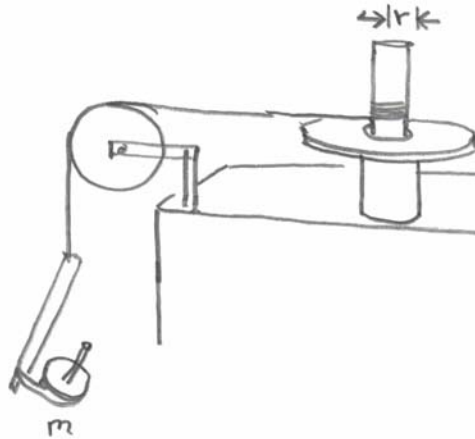
- ~~f) What is the wavelength of the emitted light from part e)?~~

Problem 2: Experiment 11 Angular Momentum: Analysis

Part One: Rotor Moment of Inertia

Enter the results from your experiment report into the table below.

α_{down} [rad · s ⁻²]	α_{up} [rad · s ⁻²]	a [m · s ⁻²]	T [N]	τ_{up} [N · m]



Note: The rotational dynamics for the two stages are given by

$$RT - \tau_f = I\alpha_{up}$$

$$-\tau_f = I\alpha_{down}$$

Note that $\alpha_{down} < 0$.

The first two equations above imply that

$$RT + I\alpha_{down} = I\alpha_{up}.$$

The force equation for the first stage is given by

$$mg - T = m\alpha_{up}.$$

The linear acceleration and angular acceleration are constrained by

$$\alpha_{up} = R\alpha_{up}.$$

Combining these last two equations and solving for the tension yields

$$T = mg - m\alpha R_{up}$$

Substituting the tension into the combined torque equation yields

$$(mg - m\alpha_{up}R)R + I\alpha_{down} = I\alpha_{up}$$

We can solve this for the moment of inertia

$$I = \frac{mgR - m\alpha_{up}R^2}{\alpha_{up} - \alpha_{down}} = \frac{mgR - m\alpha_{up}R^2}{\alpha_{up} + |\alpha_{down}|}$$

(Here $m = 0.055$ kg is the mass of the weight, $r = 0.0127$ m, α_{up} and α_{down} are obtained from your measurements).

What is your numerical value for I_R ?

Part Two: Inelastic Collision:

Write your measurement results into the table below.

ω_1 [rad · s ⁻¹]	ω_2 [rad · s ⁻¹]	δt [s]

What is your numerical value for $I_W = \frac{1}{2}m_w(r_0^2 + r_i^2)$?

1. Use the moments of inertia I_R and I_W along with ω_i and ω_f to calculate the angular momentum before and after the collision and compare them.
2. Use the values you found for the friction torque τ_f and δt to estimate the angular impulse of τ_f during the collision. Compare it to the angular momentum difference that you just calculated.
3. Calculate the rotational kinetic energies $K_1 = \frac{1}{2}I_R\omega_1^2$, before, and $K_2 = \frac{1}{2}(I_R + I_W)\omega_2^2$, after the collision.

Part Three: Slow Inelastic Collision:

Write your measurement results into the table below.

ω_1 [rad · s ⁻¹]	ω_2 [rad · s ⁻¹]	δt [s]	α_c [rad · s ⁻²]

1. Use the moments of inertia I_R and I_W along with ω_1 and ω_2 to calculate the angular momentum before and after the collision and compare them.
2. Use the value you found for the friction torque τ_f and δt to estimate the angular impulse of τ_f during the collision. Compare it to the difference in angular momentum before and after the collision.
3. Use the value you found for the angular acceleration during the collision, α_c , to estimate the total torque τ_c on the rotor during the collision.
4. The torque τ_c is made of two parts: the friction torque τ_f from the bearings and the torque τ_{RW} due exerted by the washer you dropped on the rotor. By the 3rd law, the rotor exerts an equal and opposite torque on the washer. Since you know τ_f , subtract it from τ_c to find an estimate for τ_{RW} .

Part Four: Nonconservative Work in the Slow Collision

You can find τ_{RW} a different way, because it produces the angular acceleration of the dropped washer, whose average value is $\alpha_W = \omega_2/\delta t$. Use this relation to estimate τ_{RW} and compare it to the previous estimate you made:

The torque τ_{RW} comes from the sliding friction between the washer on the rotor and the washer you dropped. Thus there must be some non-conservative work. You may calculate it if you know the angular “distance” the washer slides over the rotor before it reaches the same angular velocity as the rotor, and you do have enough information to find that. The angle the rotor rotates through during the collision is (analogous to linear motion with constant acceleration)

$$\Delta\theta_R = \omega_1\delta t - \frac{1}{2}|\alpha_c|\delta t^2$$

while the dropped washer rotates through an angle

$$\Delta\theta_W = \frac{1}{2}|\alpha_W|\delta t^2.$$

Thus the non-conservative work done by the sliding friction between the two washers will be

$$W_{NC,W} = \tau_{RW}(\Delta\theta_R - \Delta\theta_W).$$

You can also calculate the non-conservative work done by the bearing friction during the collision

$$W_{NC,B} = \tau_f\Delta\theta_R.$$

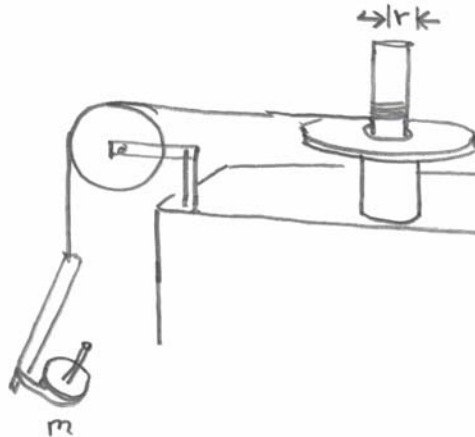
Calculate these two amounts of non-conservative work and compare their sum to the change in rotational kinetic energy $\frac{1}{2}I_R\omega_1^2 - \frac{1}{2}(I_R + I_W)\omega_2^2$ during the collision.

Problem 2: Experiment 11 Angular Momentum: Analysis

Part One: Rotor Moment of Inertia

Enter the results from your experiment report into the table below.

α_{down} [rad·s ⁻²]	α_{up} [rad·s ⁻²]	a [m·s ⁻²]	T [N]	τ_{up} [N·m]	τ_f [N·m]	I_R [kg·m ²]
-34.2	82.5	1.0505	.482	6.12×10^{-3}	-1.79×10^{-3}	5.24×10^{-5}



Note: The rotational dynamics for the two stages are given by

$$RT - \tau_f = I_R \alpha_{up}$$

$$-\tau_f = I_R \alpha_{down}$$

Note that $\alpha_{down} < 0$.

The first two equations above imply that

$$RT + I_R \alpha_{down} = I_R \alpha_{up} .$$

The force equation for the first stage is given by

$$mg - T = ma_{up} .$$

The linear acceleration and angular acceleration are constrained by

$$a_{up} = R\alpha_{up}.$$

Combining these last two equations and solving for the tension yields

$$T = mg - m\alpha_{up}R$$

Substituting the tension into the combined torque equation yields

$$(mg - m\alpha_{up}R)R + I_R\alpha_{down} = I_R\alpha_{up}$$

We can solve this for the moment of inertia

$$I_R = \frac{mgR - m\alpha_{up}R^2}{\alpha_{up} - \alpha_{down}} = \frac{mgR - m\alpha_{up}R^2}{\alpha_{up} + |\alpha_{down}|}$$

(Here $m = 0.055$ kg is the mass of the weight, $r = 0.0127$ m, α_{up} and α_{down} are obtained from your measurements).

Answer: What is your numerical value for I_R ?

$$I_R = \frac{mR(g - \alpha_{up}R)}{\alpha_{up} - \alpha_{down}} = \frac{(0.055 \text{ kg})(0.0127 \text{ m})((9.805 \text{ m} \cdot \text{s}^{-2}) - (82.5 \text{ rad} \cdot \text{s}^{-2})(0.0127 \text{ m}))}{((82.5 \text{ rad} \cdot \text{s}^{-2}) - (-34.2 \text{ rad} \cdot \text{s}^{-2}))}$$

$$I_R = 5.24 \times 10^{-5} \text{ kg} \cdot \text{m}^2$$

The frictional torque is then

$$\tau_f = I_R\alpha_{down} = (5.24 \times 10^{-5} \text{ kg} \cdot \text{m}^2)(-34.2 \text{ rad} \cdot \text{s}^{-2}) = -1.79 \times 10^{-3} \text{ J}$$

Part Two: Inelastic Collision:

Write your measurement results into the table below. Note you may not have included the last two columns since the report do not specify to include these. I included my values.

ω_i [rad · s ⁻¹]	ω_f [rad · s ⁻¹]	δt [s]	α_i [rad · s ⁻²]	α_f [rad · s ⁻²]
281	147	0.003	41.3	18.9

What is your numerical value for $I_W = \frac{1}{2}m_w(r_0^2 + r_i^2)$?

Answer: A 1" US Standard Washer has inner radius $r_1 = 1.35 \times 10^{-2}$ m and an outer radius $r_2 = 3.20 \times 10^{-2}$ m. The moment of inertia of a washer is given by

$$I_{cm} = \frac{1}{2}m_w(r_0^2 + r_i^2).$$

What was the mass of the washer you dropped? The mass is printed on the washer. The mass of my washer is $m_w = 84.4$ g.

What was the moment of inertia of the washer you dropped?

$$I_W = \frac{1}{2}m_w(r_0^2 + r_i^2) = 5.09 \times 10^{-5} \text{ kg} \cdot \text{m}^2$$

1. Use the moments of inertia I_R and I_W along with ω_i and ω_f to calculate the angular momentum before and after the collision and compare them.

Answer:

$$L_{cm,before} = 1.48 \times 10^{-2} \text{ kg} \cdot \text{m}^2 \cdot \text{s}^{-1}, \quad L_{cm,after} = 1.52 \times 10^{-2} \text{ kg} \cdot \text{m}^2 \cdot \text{s}^{-1}.$$

This is reasonably good agreement and shows that angular momentum is essentially conserved.

2. Use the values you found for the friction torque τ_f and δt to estimate the angular impulse of τ_f during the collision. Compare it to the angular momentum difference that you just calculated.

Answer: $J = \tau_f \Delta t = (-1.79 \times 10^{-3} \text{ J})(0.003 \text{ s}) = -5.3 \times 10^{-6} \text{ kg} \cdot \text{m}^2 \cdot \text{s}^{-1}$. **This does not agree with my difference in angular momentum**

$$L_{cm,after} - L_{cm,before} = 4 \times 10^{-4} \text{ kg} \cdot \text{m}^2 \cdot \text{s}^{-1}.$$

3. Calculate the rotational kinetic energies $K_1 = \frac{1}{2}I_R\omega_1^2$, before, and $K_2 = \frac{1}{2}(I_R + I_W)\omega_2^2$, after the collision.

Answer:

$$K_1 = \frac{1}{2} I_R \omega_1^2 = \frac{1}{2} (5.24 \times 10^{-5} \text{ kg} \cdot \text{m}^2) (281 \text{ rad} \cdot \text{s}^{-1})^2 = 2.07 \text{ J}$$

$$K_2 = \frac{1}{2} (I_R + I_W) \omega_2^2 = \frac{1}{2} ((5.24 \times 10^{-5} \text{ kg} \cdot \text{m}^2) + (5.09 \times 10^{-5} \text{ kg} \cdot \text{m}^2)) (147 \text{ rad} \cdot \text{s}^{-1})^2 = 1.12 \text{ J}.$$

$$\Delta K = K_2 - K_1 = \frac{1}{2} (I_R + I_W) \omega_2^2 - \frac{1}{2} I_R \omega_1^2 = 1.12 \text{ J} - 2.07 \text{ J} = 0.95 \text{ J}$$

Mechanical energy is not constant since the collision is inelastic.

Part Three: Slow Inelastic Collision:

Write your measurement results into the table below. Note I included my values for α_i and α_f but the lab report did not ask you for these values.

ω_1 [rad · s ⁻¹]	ω_2 [rad · s ⁻¹]	δt [s]	α_i [rad · s ⁻²]	α_c [rad · s ⁻²]	α_f [rad · s ⁻²]
271	127	0.79	-40.5	-185	-18.4

- Use the moments of inertia I_R and I_W along with ω_1 and ω_2 to calculate the angular momentum before and after the collision and compare them.

Answer:

$$L_{cm,before} = I_R \omega_1 = 1.42 \times 10^{-2} \text{ kg} \cdot \text{m}^2 \cdot \text{s}^{-1}, \quad L_{cm,after} = (I_R + I_W) \omega_2 = 1.31 \times 10^{-2} \text{ kg} \cdot \text{m}^2 \cdot \text{s}^{-1}.$$

These are close to each other in value.

- Use the value you found for the friction torque τ_f and δt to estimate the angular impulse of τ_f during the collision. Compare it to the difference in angular momentum before and after the collision.

Answer:

$$J = \tau_f \Delta t = (-1.79 \times 10^{-3} \text{ J}) (0.79 \text{ s}) = -1.41 \times 10^{-3} \text{ kg} \cdot \text{m}^2 \cdot \text{s}^{-1},$$

$$L_{cm,after} - L_{cm,before} = -1.1 \times 10^{-3} \text{ kg} \cdot \text{m}^2 \cdot \text{s}^{-1}.$$

This gives even better agreement than in the fast collision.

3. Use the value you found for the angular acceleration during the collision, α_c , to estimate the total torque τ_{total} on the rotor during the collision.

Answer: The total torque on the rotor during the collision is

$$\tau_{total} = I_R \alpha_c = (5.24 \times 10^{-5} \text{ kg} \cdot \text{m}^2)(-185 \text{ rad} \cdot \text{s}^{-2}) = -9.70 \times 10^{-3} \text{ J}$$

The angular impulse is

$$J = \tau_{total} \Delta t = I_R \alpha_c \Delta t$$

$$J = I_R \alpha_c \Delta t = (5.24 \times 10^{-5} \text{ kg} \cdot \text{m}^2)(-185 \text{ rad} \cdot \text{s}^{-2})(0.79 \text{ s}) = -7.66 \times 10^{-3} \text{ kg} \cdot \text{m}^2 \cdot \text{s}^{-1}$$

The change in angular momentum is

$$\Delta L = -6.5 \times 10^{-3} \text{ kg} \cdot \text{m}^2 \cdot \text{s}^{-1}$$

Reasonable agreement!

4. The torque τ_c is made of two parts: the friction torque τ_f from the bearings and the torque τ_{RW} due exerted by the washer you dropped on the rotor. By the 3rd law, the rotor exerts an equal and opposite torque on the washer. Since you know τ_f , subtract it from τ_c to find an estimate for τ_{RW} .

Answer: The total torque is equal to the frictional torque and the torque τ_{RW} exerted by the washer

$$\tau_{total} = \tau_f + \tau_{RW}$$

So the torque τ_{RW} exerted by the washer is

$$\tau_{RW} = \tau_{total} - \tau_f = (-9.70 \times 10^{-3} \text{ J}) - (-1.79 \times 10^{-3} \text{ J}) = -7.91 \times 10^{-3} \text{ J}.$$

Note: this torque causes the dropped washer to increase its angular speed. It produces an angular impulse

$$J_W = -\tau_{RW} \Delta t = -(-7.91 \times 10^{-3} \text{ J})(0.79 \text{ s}) = 6.25 \times 10^{-3} \text{ kg} \cdot \text{m}^2 \cdot \text{s}^{-1}.$$

The change in angular momentum of the dropped washer is

$$\Delta L_W = I_W \omega_2 = (5.09 \times 10^{-5} \text{ kg} \cdot \text{m}^2)(127 \text{ rad} \cdot \text{s}^{-1}) = 6.46 \times 10^{-3} \text{ kg} \cdot \text{m}^2 \cdot \text{s}^{-1}$$

in excellent agreement.

Part Four: Nonconservative Work in the Slow Collision

You can find τ_{RW} a different way, because it produces the angular acceleration of the dropped washer, whose average value is $\alpha_W = \omega_2 / \delta t$. Use this relation to estimate τ_{RW} and compare it to the previous estimate you made:

The torque τ_{RW} comes from the sliding friction between the washer on the rotor and the washer you dropped. Thus there must be some non-conservative work. You may calculate it if you know the angular “distance” the washer slides over the rotor before it reaches the same angular velocity as the rotor, and you do have enough information to find that. The angle the rotor rotates through during the collision is (analogous to linear motion with constant acceleration)

$$\Delta \theta_R = \omega_1 \delta t - \frac{1}{2} |\alpha_c| \delta t^2$$

while the dropped washer rotates through an angle

$$\Delta \theta_W = \frac{1}{2} |\alpha_W| \delta t^2.$$

Thus the non-conservative work done by the sliding friction between the two washers will be

$$W_{NC,W} = \tau_{RW} (\Delta \theta_R - \Delta \theta_W).$$

You can also calculate the non-conservative work done by the bearing friction during the collision

$$W_{NC,B} = \tau_f \Delta \theta_R.$$

Calculate these two amounts of non-conservative work and compare their sum to the change in rotational kinetic energy $\frac{1}{2} I_R \omega_1^2 - \frac{1}{2} (I_R + I_W) \omega_2^2$ during the collision.

Answer: The torque from the lower washer on the dropped washer causes the dropped washer to increase its angular speed.

If we average the angular acceleration

$$\alpha_W = \omega_2 / \Delta t = (127 \text{ rad} \cdot \text{s}^{-1}) / (0.79 \text{ s}) = 1.61 \times 10^2 \text{ rad} \cdot \text{s}^{-2}.$$

So the torque is then

$$\tau_w = I_w \alpha_w = (5.09 \times 10^{-5} \text{ kg} \cdot \text{m}^2)(1.61 \times 10^2 \text{ rad} \cdot \text{s}^{-2}) = -8.20 \times 10^{-3} \text{ J}$$

The angular displacement for the washer is

$$\Delta\theta_w = \frac{1}{2} \alpha_w \Delta t^2 = \frac{1}{2} (1.61 \times 10^2 \text{ rad} \cdot \text{s}^{-2})(0.79 \text{ s})^2 = 5.02 \times 10^1 \text{ rad} .$$

The angular displacement for the rotor is

$$\Delta\theta_R = \omega_1 \Delta t - \frac{1}{2} \alpha_c \Delta t^2 = (271 \text{ rad} \cdot \text{s}^{-1})(0.79 \text{ s}) - \frac{1}{2} (185 \text{ rad} \cdot \text{s}^{-2})(0.79 \text{ s})^2 = 1.56 \times 10^2 \text{ rad} .$$

So the relative angular displacement between the washer and the rotor is

$$(\Delta\theta_R - \Delta\theta_w) = (1.56 \times 10^2 \text{ rad}) - (5.02 \times 10^1 \text{ rad}) = 1.06 \times 10^2 \text{ rad}$$

Thus the torque between the washers produces a negative non-conservative work

$$W_{NC,W} = \tau_{RW} (\Delta\theta_R - \Delta\theta_w) = (-7.91 \times 10^{-3} \text{ J})(1.06 \times 10^2 \text{ rad}) = -8.46 \times 10^{-1} \text{ J} .$$

The bearing introduces a frictional torque on the rotor which also contributes a negative non-conservative work slowing down the rotor

$$W_{NC,R} = \tau_f (\Delta\theta_R) = (-1.79 \times 10^{-3} \text{ J})(1.56 \times 10^2 \text{ rad}) = -2.79 \times 10^{-1} \text{ J}$$

So the total non-conservative work is

$$W_{NC,W} + W_{NC,R} = \tau_{RW} (\Delta\theta_R - \Delta\theta_w) + \tau_f (\Delta\theta_R) = (\tau_{RW} + \tau_f) (\Delta\theta_R) - \tau_{RW} \Delta\theta_w = \tau^{total} (\Delta\theta_R) - \tau_{RW} \Delta\theta_w .$$

As we can see, the non-conservative work can be divided into the negative work done on the rotor and the positive work done on the washer since $\tau^{total} < 0$ and $\tau_{RW} < 0$.

This is equal to

$$W_{NC,W} + W_{NC,R} = (-8.46 \times 10^{-1} \text{ J}) + (-2.79 \times 10^{-1} \text{ J}) = -1.12 \text{ J} .$$

The rotational kinetic energy is given by

$$K_2 = \frac{1}{2}(I_R + I_W)\omega_2^2 = \frac{1}{2}((5.24 \times 10^{-5} \text{ kg} \cdot \text{m}^2) + (5.09 \times 10^{-5} \text{ kg} \cdot \text{m}^2))(127 \text{ rad} \cdot \text{s}^{-1})^2 = 8.3 \times 10^{-1} \text{ J}$$

$$K_1 = \frac{1}{2}I_R\omega_1^2 = \frac{1}{2}(5.24 \times 10^{-5} \text{ kg} \cdot \text{m}^2)(271 \text{ rad} \cdot \text{s}^{-1})^2 = 1.92 \text{ J}$$

So the change in rotational kinetic energy is given by

$$\Delta K = K_2 - K_1 = \frac{1}{2}(I_R + I_W)\omega_2^2 - \frac{1}{2}I_R\omega_1^2 = 0.83 \text{ J} - 1.92 \text{ J} = -1.10 \text{ J}$$

The agreement is excellent with the error due to rounding.