

Nov 17

Hour One: Problem Solving Session 18: Galactic Black Hole

Reading: YF 12.5, 12.8

Hour Two: Test Review

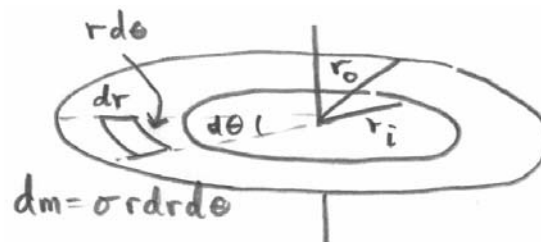
Nov 18 QUIZ THREE: Energy, Momentum, and Rotational Motion 7:30-9:30 pm.

Nov 19 No Class

Problem Set 11: Due Tues Nov 23 at 4:00 pm.

Problem 1: (Moment of Inertia)

A 1" US Standard Washer has inner radius $r_1 = 1.35 \times 10^{-2} m$ and an outer radius $r_2 = 3.10 \times 10^{-2} m$. The washer is approximately $d = 4.0 \times 10^{-3} m$ thick. The density of the washer is $\rho = 7.8 \times 10^3 kg/m^3$. Calculate the moment of inertia of the washer about an axis passing through the center of mass and show that it is equal to $I_{cm} = \frac{1}{2} m_w (r_0^2 + r_i^2)$.



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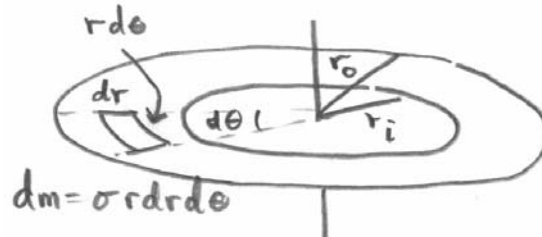
Physics 8.01T

Fall Term 2004

Problem Set 10: Torque, Rotational Dynamics, Physical Pendulum, Angular Momentum; Solutions

Problem 1: (Moment of Inertia)

A 1" US Standard Washer has inner radius $r_1 = 1.35 \times 10^{-2} m$ and an outer radius $r_2 = 3.10 \times 10^{-2} m$. The washer is approximately $d = 4.0 \times 10^{-3} m$ thick. The density of the washer is $\rho = 7.8 \times 10^3 kg/m^3$. Calculate the moment of inertia of the washer about an axis passing through the center of mass and show that it is equal to $I_{cm} = \frac{1}{2} m_w (r_0^2 + r_i^2)$.



We need to calculate the integral

$$I_{cm} = \int r^2 dm.$$

There are several ways of approaching this problem. To calculate the integral, we need to choose the mass element, i.e. we need to split the washer into elementary parts. Probably the simplest choice is to take the mass element to be a thin ring of width dr . The mass of this ring is

$$dm = \rho d dS = \sigma 2\pi r dr,$$

where d is the thickness, $\sigma = \rho d$ is the mass per unit surface area and we used the fact that the ring has small width to approximate its surface area as $dS = 2\pi r dr$. Then the integral that we need to calculate is

$$I_{cm} = \int r^2 dm = 2\pi\sigma \int_{r_1}^{r_2} r^2 r dr = \frac{\pi\sigma}{2} (r_2^4 - r_1^4) \approx 4.4 \times 10^{-5} kg \times m^2.$$

To put the answer into the form given in the problem, we need to find the mass of the washer. The mass of the whole is the sum of the masses of the parts, so it is

$$m_w = \int dm = 2\pi\sigma \int_{r_1}^{r_2} r dr = \pi\sigma (r_2^2 - r_1^2) \approx 76 g.$$

Since $(r_2^4 - r_1^4) = (r_2^2 - r_1^2)(r_1^2 + r_2^2)$, the final answer is

$$I_{cm} = \frac{1}{2} m_w (r_1^2 + r_2^2).$$

Another way of solving the problem is to choose the mass element to be a small "rectangle" in polar coordinates of radial width dr and angular size $d\theta$. Then the surface area of this element is $dS = r d\theta dr$ and the moment of inertia is

$$I_{cm} = \int r^2 dm = \sigma \int_0^{2\pi} d\theta \int_{r_1}^{r_2} dr r^2 r$$

The integral over θ is easy to calculate since the integrand does not depend on θ and after that we are left with exactly the same expression as in the previous case.

Yet another way to calculate the moment of inertia is to use the following trick. Consider the washer to be made of two solid disk, one of positive mass and radius r_2 and the other one of negative mass and radius r_1 . If you recall that for a solid disk of radius R the moment of inertia is $I = \pi\sigma R^4/2$, you again recover the previous answer.