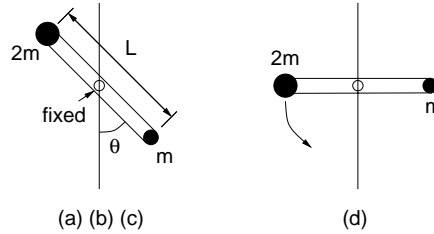


Problem 2: Dynamics (15 pts)

Two balls of mass m and $2m$ are connected by a rod of length L . The mass of the rod is small and can be treated as zero. The size of the balls can also be neglected. We also assume the center of the rod is fixed, but the rod can rotate about its center in the vertical plane without friction.



- (a) Find the center of mass of the two balls. That is find the distance between the center of mass and the ball of mass $2m$.
- (b) Find the moment of inertia of the two balls about the rotation axis.
- (c) Find the gravity induced angular acceleration of the rod when the angle between the rod and the vertical line is θ as shown.
- (d) If the rod starts its swing from the horizontal position, what is the angular velocity of the rod when it reaches the vertical position?

Solution:

(a) Let d be the distance between the center of mass and the ball of mass $2m$. We have $(2m)d = m(L - d)$. We find $d = L/3$.

(b) Moment of inertia about the axis (the center of the rod): $I = 2m(L/2)^2 + m(L/2)^2 = \frac{3}{4}mL^2$.

(c) Torque about the axis (the center of the rod): $T = 2mg\frac{L}{2} \sin \theta - mg\frac{L}{2} \sin \theta$. Angular acceleration = $T/I = \frac{2g}{3L} \sin \theta$.

(d) Change in the potential energy $\Delta U = 2mg(L/2) - mg(L/2)$. From the energy conservation $\Delta U = K = I\omega^2/2$, we find the angular velocity $\omega = \sqrt{\frac{2\Delta U}{I}} = \sqrt{\frac{4g}{3L}}$.