

### Spinning Object: $L$ Conserved

$$L_1 = I_1 \omega_1 = m r_1^2 \omega_1$$

$$L_2 = I_2 \omega_2 = m r_2^2 \omega_2$$

$$L_1 = L_2$$

$$\omega_2 = \left(\frac{r_1}{r_2}\right)^2 \omega_1$$



$$E_1 = \frac{1}{2} I_1 \omega_1^2 = \frac{1}{2} m r_1^2 \omega_1^2$$

$$E_2 = \frac{1}{2} I_2 \omega_2^2 = \frac{1}{2} m r_2^2 \omega_2^2$$

$$E_2/E_1 = \frac{\omega_2^2 r_2^2}{\omega_1^2 r_1^2} = \left(\frac{r_1}{r_2}\right)^4 > 1$$

### Example: Stool + Bicycle Wheel

- Stool at rest

Initial Ang Mom:  $L_0$  : up

System = Person + Wheel + Stool

$$\text{Initial: } \vec{L}_{\text{system}} = \vec{L}_0 = I_0 \omega_0 \hat{k}$$

$$\text{Final: } \vec{L}_{\text{system}} = \vec{L}_{\text{Person+Stool}} + \vec{L}_{\text{wheel}}$$

$$L_{\text{sup}} = \vec{L}_{\text{Person+Stool}} - \vec{L}_0$$

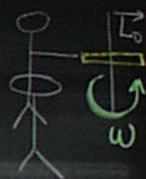
$$L_i = L_f$$

$$\vec{L}_0 = \vec{L}_{\text{Person+Stool}} - \vec{L}_0$$

$$\vec{L}_{\text{Person+Stool}} = 2\vec{L}_0$$

$$I_p \omega_p = 2 I_0 \omega_0$$

$$\omega_p = \frac{2 I_0 \omega_0}{I_p}$$



### Work and Energy in Rotational Motion

Force acts on body.

Rotate object through angle  $\theta$

$ds = r d\theta$  in a time  $(dt)$

$$dW = \vec{F} \cdot d\vec{s} = (F \sin \phi) r d\theta$$

$$F \sin \phi = F_t \text{ (tangential comp. of } \vec{F})$$

$$F \cos \phi = F_r \text{ (} \perp ds, \text{ radial no work)}$$

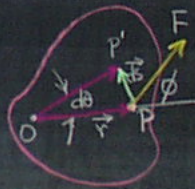
$$(F \sin \phi) r = \tau_z \text{ (Torque about axis)}$$

$$dW = \tau_z d\theta$$

$$W = \int_{\theta_1}^{\theta_2} \tau_z d\theta$$

$$\text{If } \tau_z = \text{const.}; W = \tau_z (\theta_2 - \theta_1)$$

$$\text{Power: } P = \frac{dW}{dt} = \tau_z \frac{d\theta}{dt} = \tau_z \omega$$



### Work-Energy Theorem: Rotations

Work by torque  $\tau \rightarrow$  Changes KE

$$\tau \rightarrow \alpha \quad \omega_1 \rightarrow \omega_2$$

$$\tau = I \alpha = I \frac{d\omega}{dt} = I \frac{d\omega}{d\theta} \frac{d\theta}{dt} = I \frac{d\omega}{d\theta} \omega$$

$$\tau d\theta = dW = I \omega d\omega$$

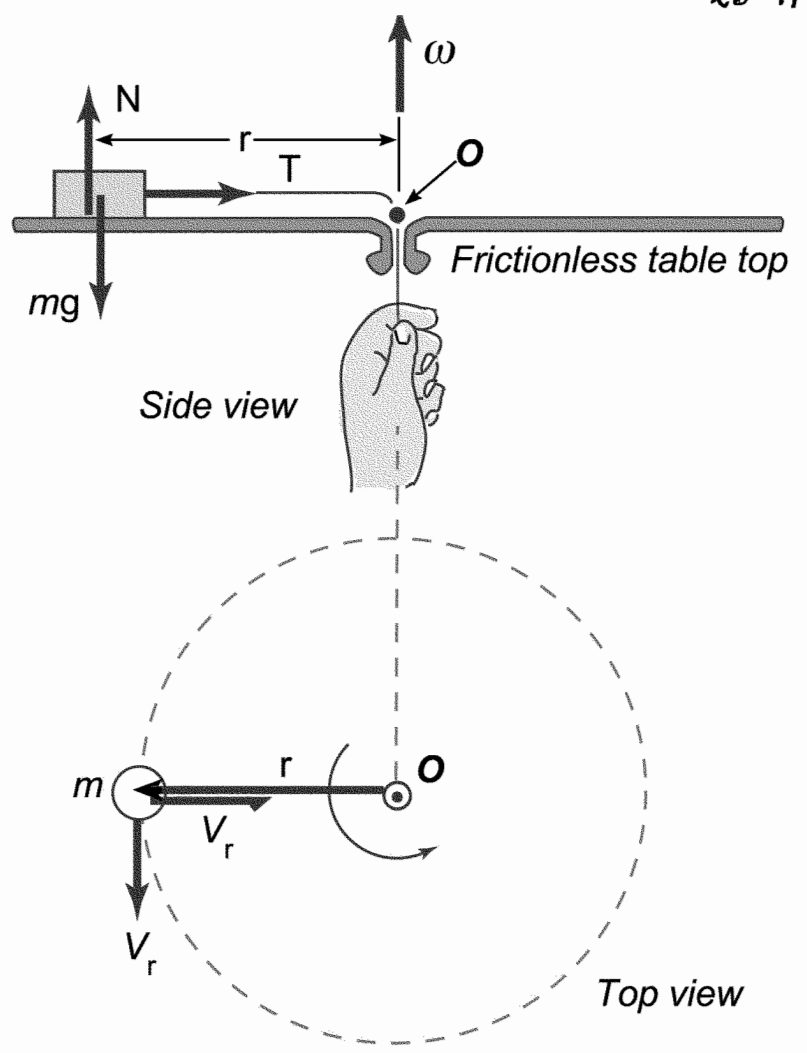
$$W = \int_{\omega_1}^{\omega_2} \tau d\theta = \int_{\omega_1}^{\omega_2} I \omega d\omega$$

Example

Particle on a string at radius  $r_1$  is moving in a circle with angular velocity  $\omega_1$ .

String shortened to new radius  $r_2$ .  
 What is new angular velocity  $\omega_2$ ?

Ans: Shortening string requires a radial force only on particle — no torques.

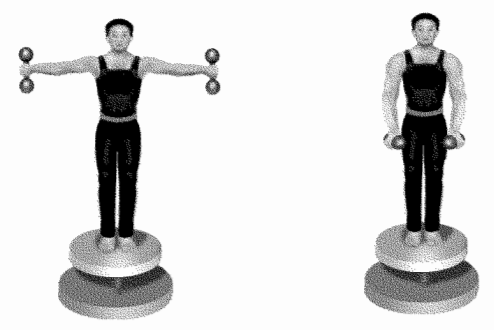


$\therefore h_i = h_f$  [Ang. Mom. Conserved]

$I_i \omega_i = I_f \omega_f$

$m r_1^2 \omega_1 = m r_2^2 \omega_2$

$\omega_2 = \left(\frac{r_1}{r_2}\right)^2 \omega_1$



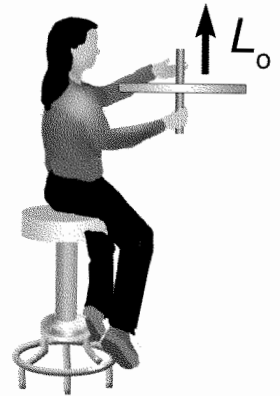
Conservation of angular momentum about a fixed axis.

Ex: skaters, gymnasts, ballet dancers change ang. velocity by altering moment-of-inertia by extending arms etc.

## Example: Spinning Bicycle Wheel

25-12

student holds a spinning bicycle wheel. Initially student and stool at rest. wheel spins in horizontal plane with initial ang. mom.  $\vec{L}_0$  up.



Wheel is inverted through center by  $180^\circ$ .  
What happens?

System = student + wheel + stool.

Total Ang Mom =  $\vec{L}_0$  [wheel only spinning]

As wheel is inverted, student supplies a torque — it is internal to the system.

No external torques acting on the system about vertical axis. Free spinning axel.  
 $\therefore$  Total angular momentum is conserved.

$$\text{Initial: } \vec{L}_{\text{system}} = \vec{L}_0$$

$$\text{Finally: } \vec{L}_{\text{system}} = \vec{L}_{\text{student+stool}} + \vec{L}_{\text{wheel}}$$

$$\therefore \vec{L}_0 = \vec{L}_{\text{student+stool}} + \vec{L}_{\text{wheel}}$$

$$\vec{L}_0 = \vec{L}_{\text{st+stool}} - \vec{L}_0$$

$$\therefore \vec{L}_{\text{st+stool}} = 2\vec{L}_0$$

If  $I_p$  = moment of inertia (student + stool)

$$\therefore I_p \omega_p = 2 I_0 \omega_0$$

If wheel is held at an angle  $\theta$  measured from the vertical axis, the student acquires an ang. mom given by

$$L_0 (1 - \cos \theta)$$

In this case the ang. mom. precesses about axis of rotation of platform. A horizontal torque supplied by the student then accounts for this precession.