

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Physics 8.01 TEAL

Fall Term 2004

Exam 2: Equation Summary

One Dimensional Kinematics:

$$\vec{v} = d\vec{r} / dt, \quad \vec{a} = d\vec{v} / dt$$

$$v_x(t) - v_{x,0} = \int_{t'=0}^{t'=t} a_x(t') dt' \quad x(t) - x_0 = \int_{t'=0}^{t'=t} v_x(t') dt'$$

Constant Acceleration:

$$x(t) = x_0 + v_{x,0}(t-t_0) + \frac{1}{2} a_x(t-t_0)^2 \quad v_x(t) = v_{x,0} + a_x(t-t_0)$$
$$y(t) = y_0 + v_{y,0}(t-t_0) + \frac{1}{2} a_y(t-t_0)^2 \quad v_y(t) = v_{y,0} + a_y(t-t_0)$$

where $x_0, v_{x,0}, y_0, v_{y,0}$ are the initial position and velocities components at $t = t_0$

Newton's Second Law: Force, Mass, Acceleration

$$\vec{F} \equiv m\vec{a} \quad \vec{F}^{total} = \vec{F}_1 + \vec{F}_2 \quad F_x^{total} = ma_x \quad F_y^{total} = ma_y \quad F_z^{total} = ma_z$$

Newton's Third Law:

$$\vec{F}_{1,2} = -\vec{F}_{2,1}$$

Force Laws:

Universal Law of Gravity: $\vec{F}_{1,2} = -G \frac{m_1 m_2}{r_{1,2}^2} \hat{r}_{1,2}$, attractive

Gravity near surface of earth: $\vec{F}_{grav} = m_{grav} \vec{g}$, towards earth

Contact force: $\vec{F}_{contact} = \vec{N} + \vec{f}$, depends on applied forces

Static Friction: $0 \leq f_s \leq f_{s,max} = \mu_s N$ direction depends on applied forces

Kinetic Friction: $f_k = \mu_k N$ opposes motion

Hooke's Law: $F = k|\Delta x|$, restoring

Kinematics Circular Motion: arc length: $s = R\theta$; angular velocity: $\omega = d\theta/dt$
 tangential velocity: $v = R\omega$; angular acceleration: $\alpha = d\omega/dt = d^2\theta/dt^2$; tangential
 acceleration $a_\theta = R\alpha$.

Period: $T = \frac{2\pi R}{v} = \frac{2\pi}{\omega}$; **frequency:** $f = \frac{1}{T} = \frac{\omega}{2\pi}$,

Radial Acceleration: $|a_r| = R\omega^2$; $|a_r| = \frac{v^2}{R}$; $|a_r| = 4\pi^2 R f^2$; $|a_r| = \frac{4\pi^2 R}{T^2}$

Center of Mass: $\vec{R}_{cm} = \sum_{i=1}^{i=N} m_i \vec{r}_i / m^{total} \rightarrow \int_{body} dm \vec{r} / m^{total}$;

Velocity of Center of Mass: $\vec{V}_{cm} = \sum_{i=1}^{i=N} m_i \vec{v}_i / m^{total} \rightarrow \int_{body} dm \vec{v} / m^{total}$

Torque: $\vec{\tau}_S = \vec{r}_{S,P} \times \vec{F}_P$ $|\vec{\tau}_S| = |\vec{r}_{S,P}| |\vec{F}_P| \sin \theta = r_\perp F = r F_\perp$

Static Equilibrium: $\vec{F}_{total} = \vec{F}_1 + \vec{F}_2 + \dots = \vec{0}$; $\vec{\tau}_S^{total} = \vec{\tau}_{S,1} + \vec{\tau}_{S,2} + \dots = \vec{0}$.

Kinetic Energy: $K = \frac{1}{2}mv^2$; $\Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2$

Work: $W = \int_{r_0}^{r_f} \vec{F} \cdot d\vec{r}$; **Work- Kinetic Energy:** $W^{total} = \Delta K$

Power: $P = \vec{F} \cdot \vec{v} = dK/dt$

Potential Energy: $\Delta U = -W_{conservative} = -\int_A^B \vec{F}_c \cdot d\vec{r}$

Potential Energy Functions with Zero Points:

Constant Gravity: $U(y) = mgy$ with $U(y_0 = 0) = 0$.

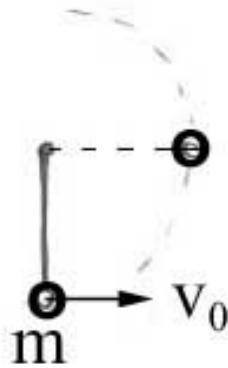
Inverse Square Gravity: $U_{gravity}(r) = -\frac{Gm_1 m_2}{r}$ with $U_{gravity}(r_0 = \infty) = 0$.

Hooke's Law: $U_{spring}(x) = \frac{1}{2}kx^2$ with $U_{spring}(x = 0) = 0$.

Work- Mechanical Energy: $W_{nc} = \Delta K + \Delta U^{total} = \Delta E_{mech} = (K_f + U_f^{total}) - (K_0 + U_0^{total})$,

Problem 5: (20 points)

A ball of negligible radius and mass m hangs from a string of length l . It is hit in such a way that it then travels in a vertical circle (i.e., the tension in the string is always greater than zero). The initial speed of the ball after being struck is v_0 . You may ignore air resistance. Let g denote the gravitational constant.

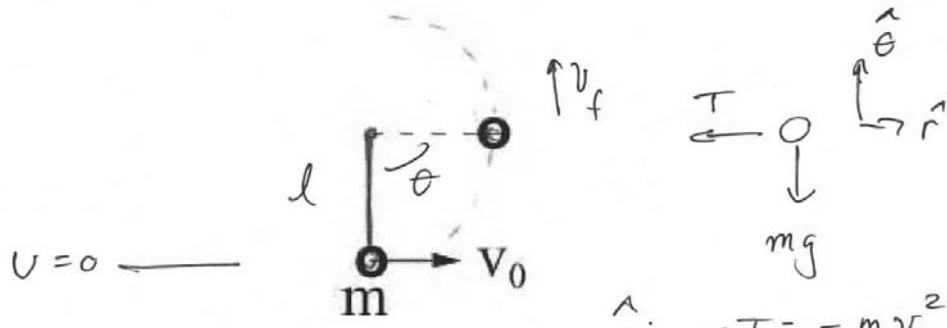


- a) Find the speed of the ball when the string is horizontal.
- b) Find the tension in the string when the string is horizontal.

Answer:

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a) Find the speed of the ball when the string is horizontal.

b) Find the tension in the string when the string is horizontal.

$$\hat{r}: -T = -\frac{mv_f^2}{l}$$

$$\Rightarrow T = \frac{mv_f^2}{l}$$

Answer: Since $\vec{T} \perp d\vec{r}$, $W_T = \vec{T} \cdot d\vec{r} = 0$.
 So tension does no work, hence mechanical energy is constant.

initial state: bottom of circle, choose $U = 0$ at bottom.
 $E_0 = \frac{1}{2}mv_0^2$

final state: half-way around, $\theta = \pi$

$$E_f = \frac{1}{2}mv_f^2 + mgl$$

$$E_0 = E_f \Rightarrow \frac{1}{2}mv_0^2 = \frac{1}{2}mv_f^2 + mgl$$

$$\Rightarrow \frac{mv_f^2}{l} = \frac{mv_0^2}{l} - 2mg$$

$$T = \frac{mv_f^2}{l} = \frac{mv_0^2}{l} - 2mg$$

$$v_f = (v_0^2 - 2gl)^{1/2}$$