

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Physics 8.01 TEAL

Fall Term 2004

Exam 2: Equation Summary

One Dimensional Kinematics:

$$\vec{v} = d\vec{r} / dt, \quad \vec{a} = d\vec{v} / dt$$

$$v_x(t) - v_{x,0} = \int_{t'=0}^{t'=t} a_x(t') dt' \quad x(t) - x_0 = \int_{t'=0}^{t'=t} v_x(t') dt'$$

Constant Acceleration:

$$x(t) = x_0 + v_{x,0}(t-t_0) + \frac{1}{2} a_x(t-t_0)^2 \quad v_x(t) = v_{x,0} + a_x(t-t_0)$$
$$y(t) = y_0 + v_{y,0}(t-t_0) + \frac{1}{2} a_y(t-t_0)^2 \quad v_y(t) = v_{y,0} + a_y(t-t_0)$$

where $x_0, v_{x,0}, y_0, v_{y,0}$ are the initial position and velocities components at $t = t_0$

Newton's Second Law: Force, Mass, Acceleration

$$\vec{F} \equiv m\vec{a} \quad \vec{F}^{total} = \vec{F}_1 + \vec{F}_2 \quad F_x^{total} = ma_x \quad F_y^{total} = ma_y \quad F_z^{total} = ma_z$$

Newton's Third Law:

$$\vec{F}_{1,2} = -\vec{F}_{2,1}$$

Force Laws:

Universal Law of Gravity: $\vec{F}_{1,2} = -G \frac{m_1 m_2}{r_{1,2}^2} \hat{r}_{1,2}$, attractive

Gravity near surface of earth: $\vec{F}_{grav} = m_{grav} \vec{g}$, towards earth

Contact force: $\vec{F}_{contact} = \vec{N} + \vec{f}$, depends on applied forces

Static Friction: $0 \leq f_s \leq f_{s,max} = \mu_s N$ direction depends on applied forces

Kinetic Friction: $f_k = \mu_k N$ opposes motion

Hooke's Law: $F = k|\Delta x|$, restoring

Kinematics Circular Motion:

arc length: $s = R\theta$; **angular velocity:** $\omega = d\theta/dt$ **tangential velocity:** $v = R\omega$; **angular acceleration:** $\alpha = d\omega/dt = d^2\theta/dt^2$; **tangential acceleration** $a_\theta = R\alpha$.

Period: $T = \frac{2\pi R}{v} = \frac{2\pi R}{R\omega} = \frac{2\pi}{\omega}$; **frequency:** $f = \frac{1}{T} = \frac{\omega}{2\pi}$,

Radial Acceleration: $|a_r| = R\omega^2$; $|a_r| = \frac{v^2}{R}$; $|a_r| = 4\pi^2 R f^2$; $|a_r| = \frac{4\pi^2 R}{T^2}$

Center of Mass: $\vec{R}_{cm} = \frac{\sum_{i=1}^{i=N} m_i \vec{r}_i}{\sum_{i=1}^{i=N} m_i} \rightarrow \frac{\int_{body} dm \vec{r}}{\int_{body} dm}$;

Velocity of Center of Mass: $\vec{V}_{cm} = \frac{\sum_{i=1}^{i=N} m_i \vec{v}_i}{\sum_{i=1}^{i=N} m_i} \rightarrow \frac{\int_{body} dm \vec{v}}{\int_{body} dm}$

Torque: $\vec{\tau}_S = \vec{r}_{S,P} \times \vec{F}_P$ $|\vec{\tau}_S| = |\vec{r}_{S,P}| |\vec{F}_P| \sin \theta = r_\perp F = r F_\perp$

Static Equilibrium: $\vec{F}_{total} = \vec{F}_1 + \vec{F}_2 + \dots = \vec{0}$; $\vec{\tau}_S^{total} = \vec{\tau}_{S,1} + \vec{\tau}_{S,2} + \dots = \vec{0}$.

Kinetic Energy: $K = \frac{1}{2}mv^2$; $\Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2$

Work: $W = \int_{r_0}^{r_f} \vec{F} \cdot d\vec{r}$; **Work-Change in Kinetic Energy:** $W^{total} = \Delta K$

Power: $P = \vec{F} \cdot \vec{v} = dK/dt$

Potential Energy: $\Delta U = -W_{conservative} = -\int_A^B \vec{F}_c \cdot d\vec{r}$

Potential Energy Functions with Zero Points:

Constant Gravity: $U(y) = mgy$ with $U(y_0 = 0) = 0$.

Inverse Square Gravity: $U_{gravity}(r) = -\frac{Gm_1 m_2}{r}$ with $U_{gravity}(r_0 = \infty) \equiv 0$.

Hooke's Law: $U_{spring}(x) = \frac{1}{2}kx^2$ with $U_{spring}(x = 0) \equiv 0$.

Work- Change in Mechanical Energy: $\vec{F}^{total} = \vec{F}_c^{total} + \vec{F}_{nc}^{total}$;

$W^{total} = W_c^{total} + W_{nc}^{total} = \Delta K$; $W_{nc} = \Delta K + \Delta U^{total} = \Delta E_{mech}$, $E_{mech} = K + U^{total}$

$W_{nc} = \Delta E_{mech} = E_f - E_0 = (K_f + U_f^{total}) - (K_0 + U_0^{total})$

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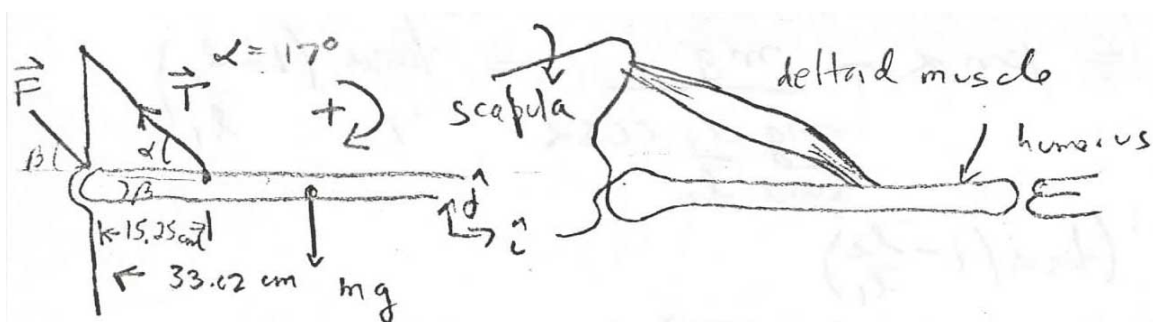
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Practice Exam 2

~~Note: The Exam 2 will have a few concept questions on the test. Review all the concept tests from class to prepare for this.~~

~~Problem 1: static equilibrium~~

~~You are able to hold out your arm in an outstretched horizontal position thanks to the action of the deltoid muscle. Assume the humerus bone has a mass of m , the center of mass of the humerus is a distance d from the scapula, the deltoid muscle attaches to the humerus a distance s from the scapula and the angle the deltoid muscle makes with the horizontal is α . A schematic representation of this action looks as follows:~~



- ~~a) What is the tension T in the deltoid muscle?~~
- ~~b) What are the vertical and horizontal components of the force exerted by the scapula (shoulder blade) on the humerus?~~

Problem 2: Water Bucket

A water bucket attached to a rope and spun in a vertical circle of radius r . Suppose the bucket has mass m and that the bucket can be approximated as a point mass at the end of the rope. The rope breaks when the bucket is moving vertically upwards. The bucket rises to a height h above the release point.

- a) What was the velocity of the bucket when it was released?
- b) What was the tension in the cord when it broke?

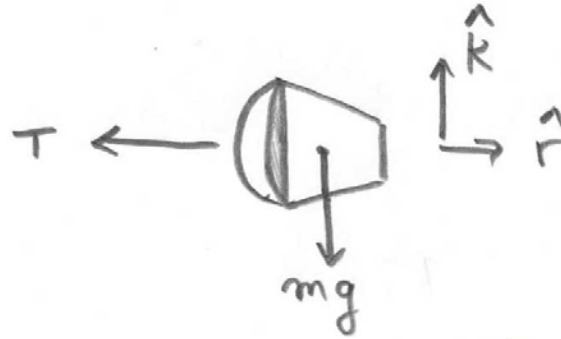
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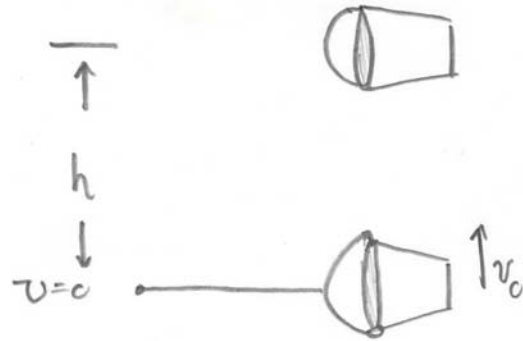
- What was the velocity of the bucket when it was released?
- What was the tension in the cord when it broke?

Solution:

The force diagram on the bucket is shown in the figure.



Choose the initial state, at time t_0 , when the string has just broken. Choose the final state, at time t_f , when the bucket has reached its maximum height. Choose zero for potential energy at the point where the string breaks.



Initial Energy: The initial kinetic energy is $K_0 = \frac{1}{2}mv_0^2$. The initial potential energy is $U_0 = 0$. So the initial mechanical energy is

$$E_0 = K_0 + U_0 = \frac{1}{2}mv_0^2.$$

Final Energy: The final kinetic energy is $K_f = 0$. The final potential energy is $U_f = mgh$. So the final mechanical energy is

$$E_f = K_f + U_f = mgh.$$

Non-conservative work: There is no non-conservative work.

Change in Mechanical Energy:

The change in mechanical energy

$$0 = W_{nc} = \Delta E_{mech},$$

is then

$$0 = mgh - \frac{1}{2}mv_0^2$$

This equation can be solved for the escape velocity, v_0 ,

$$v_0 = \sqrt{2gh}.$$

Just before the string broke the radial force equation is

Therefore Newton's Second Law in the radial direction, \hat{r} , is

$$-T = -mv_0^2/r.$$

So the tension in the string is

$$T = 2mgh/r.$$