

Massachusetts Institute of Technology
Department of Physics

8.01X

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Practice Exam 3

Problem 1: A skydiver of $m = 8.0 \times 10^1$ kg reaches a terminal velocity of 5.0×10^1 m/s. Suppose the diver falls 1.6 km. Assume there is a drag force acting on the skydiver.

- How long does it take the skydiver to fall 1.6 km?
- How much work does the gravitational force do on the falling diver?
- How much work does the drag force do on the falling diver?
- What is the power corresponding to the work done by the drag force? What is the total energy generated by the drag force? Where does this energy go?

Problem 2: A ball of mass 0.1 kg hangs from a string of length 0.5 m. It is hit horizontally so that it starts from its lowest position with a speed of 7.0 m/s. It then continues to travel freely in a vertical circle with negligible loss of energy.

- Find the speed of the ball at the top of the circle.
- Find the tension in the string when the ball is at the top of the circle.

Problem 3: A person, abandoned on a small asteroid of mass $m_1 = 2.6 \times 10^{15}$ kg and radius $r = 5.0 \times 10^3$ m, has a spring that has a spring constant $k = 400$ N/m. The person has a small pen of mass $m_2 = 10$ g which can carry a message.

- How far must the person compress the spring in order for the pen to escape from the gravitational field of the asteroid? Ignore all rotational motion of the pen.
- A spacecraft is orbiting the asteroid once every two hours. What is the radius of the spacecraft's orbit?
- What is the speed of the spacecraft?
- If the pen were launched straight up, what is the speed of the pen at the orbit radius of the spacecraft?

Problem 4 : Suppose you want to jump up by first crouching down and then pushing off the ground. You initially lower your center of mass by 0.2 m. The average force exerted by the floor on you is three times your weight. What is your upward speed as you pass through your standing position just as you are leaving the floor?

a)

Med. Energy is conserved since T is always perpendicular to the displacement

$$\Delta K + \Delta P.E. = W_{n.c.}$$

$$\frac{1}{2} m(v_f^2 - v_0^2) + mg(2l) = 0$$

solving for $v_f^2 = v_0^2 - 2g(2l)$

$$v_f = \left((7.0 \text{ m/s})^2 - (2)(9.8 \frac{\text{m}}{\text{s}^2})(2)(.5 \text{ m}) \right)^{1/2} = 5.4 \frac{\text{m}}{\text{s}}$$

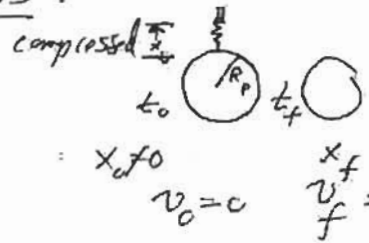
$$b) T = \frac{mv_f^2}{l} - mg = \frac{m(v_0^2 - 2g(2l))}{l} - mg$$

$$T = \frac{mv_0^2}{l} - 5mg = \frac{(.1 \text{ kg})(7.0 \frac{\text{m}}{\text{s}})^2}{(.5 \text{ m})} - (5)(.1 \text{ kg})(9.8 \frac{\text{m}}{\text{s}^2})$$

$$T = 4.9 \text{ N}$$

Problem 3:

the pen should have zero velocity at ∞



Energy is conserved

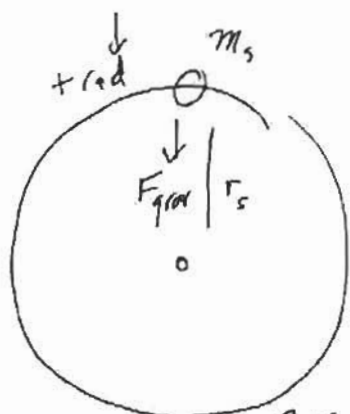
a)

$$\Delta K + \Delta P.E. = W_{n.c.}$$

$$0 + \frac{1}{2} k(x_f^2 - x_0^2) - Gm_1 m_2 \left(\frac{1}{\infty} - \frac{1}{R_p} \right) = 0$$

$$-\frac{1}{2} kx_0^2 + \frac{Gm_1 m_2}{R_p} = 0 \Rightarrow x_0^2 = \frac{2Gm_1 m_2}{R_p k}$$

$$x_0 = \left(\frac{(2)(6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2})(.01 \text{ kg})(2.6 \times 10^{15} \text{ kg})}{(5.0 \times 10^3 \text{ m})(400 \text{ N/m})} \right)^{1/2} = .042 \text{ m} = 4.2 \text{ cm}$$



$$\frac{F_{\text{rad}}}{\frac{G m_s m_p}{r_s^2}} = \frac{m_s a_{\text{rod}}}{m_s r_s \left(\frac{2\pi}{T}\right)^2}$$

solve for

$$r_s^3 = \frac{G m_p T^2}{4\pi^2}$$

$$r_s = \left(\frac{(6.67 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2})(2.6 \times 10^{15} \text{kg}) \left((2)(3.6 \times 10^3 \text{sec}) \right)^2}{4\pi^2} \right)^{1/3}$$

$$r_s = 6.1 \times 10^3 \text{ m}$$

$$v_s = \frac{2\pi r_s}{T} = \frac{(2\pi)(6.1 \times 10^3 \text{ m})}{(7.2 \times 10^3 \text{ sec})} = 5.3 \text{ m/s}$$

d) At the radius r_s , the conservation of mechanical energy can be used to calculate the velocity of the pen



$$t_0, \quad v_0 = 0 \\ x_0 = 4.2 \text{ cm} \\ \text{from part a)} \\ r_0 = R_p$$



$$t', \quad v' \neq 0 \\ x_f = 0 \\ r' = r_s \\ \text{from part c)}$$

$$\frac{\Delta K + \Delta P.E.}{\Delta W_{nc}} = 0 \quad (1)$$

recall (from part a) that

$$\frac{1}{2} k x_0^2 = \frac{G m_1 m_2}{R_p}$$

So eq (1) becomes $\frac{1}{2} m_2 v'^2 - \frac{G m_1 m_2}{r_s} = 0$

$$v' = \left(\frac{2 G m_1}{r_s} \right)^{1/2} = \left(\frac{(2)(6.67 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2})(2.6 \times 10^{15} \text{kg})}{6.1 \times 10^3 \text{ m}} \right)^{1/2}$$

$$v' = 7.54 \text{ m/s}$$