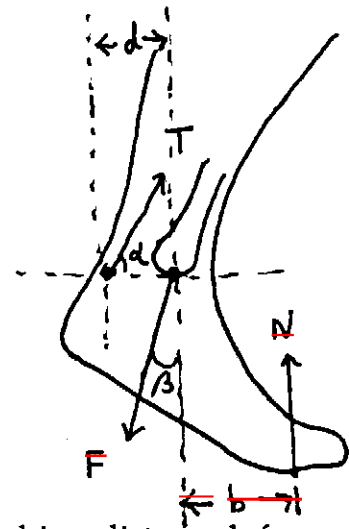


Problem 3: Static Equilibrium: The ankle

Consider one foot of a man who is crouching on the ground with his weight evenly distributed on his two feet. The mass of the man is m . In this position on tiptoes, the Achilles tendon is under considerable tension T and makes an angle α with the horizontal. The tibia acts on the ankle with a force \vec{F} at a point that is at the same height above the ground as the point where the Achilles tendon acts on the ankle. These points are separated by a distance d . This force \vec{F} makes an angle β with the vertical. Assume the contact point of the foot on the ground is a distance b from a vertical line passing through the point of contact between the tibia and the ankle. Assume the center of mass of the foot lies directly below the contact point between the tibia and the ankle. The foot has a mass m_f .



- Find the magnitude of the tension, T , in the Achilles tendon.
- Find an expression that the angle β makes with the vertical.
- Find an expression for the magnitude of the force, \vec{F} , of the tibia on the foot.

Problem 4: Experiments

Part A: Experiment Centripetal Force

Consider an ideal massless rubber-band which has an unstretched length l_0 . A roll of 50 pennies of total mass m_p is attached to the rubber band. The new stretched length of the rubber band with the pennies hanging down from the end of the rubber-band is l .

- Assume that the rubber band satisfies Hooke's Law when it is stretched. What is the spring constant of the rubber-band?

A small nut with mass m_n is then attached to the rubber band and both are rotated by a motor at an unknown constant frequency f . The rubber band is stretched to a length d .

- Find an expression for the frequency f in terms of r , m_p , l , l_0 , g , and m .
- How long does it take for the nut to complete one rotation?
- What is the angular frequency of the nut?

$$\sum F_x = 0$$

$$T \cos \alpha - F \sin \beta = 0$$

$$\sum F_y = 0$$

$$T \sin \alpha - F \cos \beta + N - m_j g = 0$$

By rearranging terms and dividing the two equations to eliminate F , we can solve for the unknown angle:

$$\frac{\sin \beta}{\cos \beta} = \frac{T \cos \alpha}{T \sin \alpha + N - m_j g}$$

$$\tan \beta = \frac{\frac{mgb}{2d \sin \alpha} \cos \alpha}{\frac{mgb}{2d \sin \alpha} \sin \alpha + \frac{mg}{2} - m_j g}$$

$$\beta = \arctan \left(\frac{mb \cot \alpha}{mb + md - 2m_j d} \right)$$

e.) We can plug this back into one of our force equations to find the magnitude of the force:

$$T \cos \alpha - F \sin \beta = 0$$

$$F = \frac{T \cos \alpha}{\sin \beta}$$

Actually plugging in the torque and angle don't do much for the algebra, so we can just leave our answer in this form, as all the quantities are known from previous parts of the problem.

Problem 4A:

a.) Considering the force balance on the pennies, we have their gravitational weight balanced by the spring force of the rubber band. Taking upwards to be positive:

$$-m_p g + k(l - l_0) = 0$$

$$k = \frac{m_p g}{l - l_0}$$

b.) For the purposes of this problem, we can neglect the mass of the rubber band and the effects of gravity. In this case, we have only the spring force of the rubber band providing the centripetal force for the circling nut. Taking radially outwards to be positive:

$$F = m_n a$$

$$-k(d - l_0) = m_n (-4\pi^2 r f^2)$$

Now we can plug in our previous expression for k and solve for the frequency f :

$$f^2 = \left(\frac{m_p g}{l - l_0} \right) \left(\frac{d - l_0}{m_n 4\pi^2 r} \right)$$

$$f = \sqrt{\left(\frac{m_p g}{4\pi^2 m_n r} \right) \left(\frac{d - l_0}{l - l_0} \right)}$$

c.) The frequency is the number of cycles per second, so the period (the number of seconds per cycle) is just the inverse of it:

$$T = \frac{1}{f}$$

d.) The angular frequency is the number of radians per second. Since there are 2π radians per complete cycle, the angular frequency is just:

$$\omega = 2\pi f$$

For simplicity, we can leave our answers in this form instead of substituting in an explicit expression for f , which is already solved for.

~~Problem 4B:~~

~~Starting from our assumed power-law force equation, we can take the log of both sides to isolate the power-law exponent:~~

$$F_m = a(S_c)^b$$

$$\log F_m = \log a + b(\log S_c)$$

~~We see that under the log transform, the data should follow a straight line with slope b and intercept $\log a$. Examining the data, the straight line fit passes approximately through the first and the last data point, so we can extract the slope from those two:~~

$$b = \frac{\log y_2 - \log y_1}{\log x_2 - \log x_1}$$

$$b = \frac{\log 0.055 - \log 0.34}{\log 25 - \log 10} = -2.0$$

~~Note that the particular units of the measurements do not matter, since a difference of logs is simply the log of the ratio of the quantities.~~