

MASSACHUSETTS INSTITUTE OF TECHNOLOGY  
Department of Physics

Physics 8.01 TEAL

Fall Term 2004

**Exam 3: Equation Summary**

**Momentum:**

$$\vec{p} = m\vec{v}, \quad \vec{F}_{ave} \Delta t = \Delta \vec{p}, \quad \vec{F}_{ext}^{total} = \frac{d\vec{p}^{total}}{dt}$$

$$\text{Impulse: } \vec{I} \equiv \int_{t=0}^{t=t_f} \vec{F}(t) dt = \Delta \vec{p}$$

$$\text{Torque: } \vec{\tau}_S = \vec{r}_{S,P} \times \vec{F}_P \quad |\vec{\tau}_S| = |\vec{r}_{S,P}| |\vec{F}_P| \sin \theta = r_{\perp} F = r F_{\perp}$$

**Static Equilibrium:**

$$\vec{F}_{total} = \vec{F}_1 + \vec{F}_2 + \dots = \vec{0}; \quad \vec{\tau}_S^{total} = \vec{\tau}_{S,1} + \vec{\tau}_{S,2} + \dots = \vec{0}.$$

$$\text{Rotational dynamics: } \vec{\tau}_S^{total} = \frac{d\vec{L}_S}{dt}$$

$$\text{Angular Velocity: } \vec{\omega} = (d\theta/dt) \hat{k}$$

$$\text{Angular Acceleration: } \vec{\alpha} = (d^2\theta/dt^2) \hat{k}$$

$$\text{Fixed Axis Rotation: } \vec{\tau}_S = I_S \vec{\alpha}$$

$$\tau_S^{total} = I_S \alpha = I_S \frac{d\omega}{dt}$$

$$\text{Moment of Inertia: } I_S = \int_{body} dm (r_{\perp})^2$$

$$\text{Angular Momentum: } \vec{L}_S = \vec{r}_{S,m} \times m\vec{v},$$

$$|\vec{L}_S| = |\vec{r}_{S,m}| |m\vec{v}| \sin \theta = r_{\perp} p = r p_{\perp}$$

**Angular Impulse:**

$$\vec{J}_S = \int_{t_0}^{t_f} \vec{\tau}_S dt = \Delta \vec{L}_S = \vec{L}_{S,f} - \vec{L}_{S,0}$$

**Rotation and Translation:**

$$\vec{L}_S^{total} = \vec{L}_S^{orbital} + \vec{L}_{cm}^{spin},$$

$$\vec{L}_S^{orbital} = \vec{r}_{S,cm} \times \vec{p}^{total},$$

$$\vec{L}_{cm}^{spin} = I_{cm} \vec{\omega}_{spin}$$

$$\vec{\tau}_S^{orbit} = \frac{d\vec{L}_S^{orbit}}{dt}, \quad \vec{\tau}_{cm}^{spin} = \frac{d\vec{L}_{cm}^{spin}}{dt}$$

**Rotational Energy:**  $K_{cm} = \frac{1}{2} I_{cm} \omega_{cm}^2$

**Rotational Power:**  $P_{rot} \equiv \frac{dW_{rot}}{dt} = \vec{\tau}_S \cdot \vec{\omega} = \tau_S \omega = \tau_S \frac{d\theta}{dt}$

**One Dimensional Kinematics:**  $\vec{v} = d\vec{r} / dt$  ,  $\vec{a} = d\vec{v} / dt$

$$v_x(t) - v_{x,0} = \int_{t'=0}^{t'=t} a_x(t') dt' \quad x(t) - x_0 = \int_{t'=0}^{t'=t} v_x(t') dt'$$

**Constant Acceleration:**

$$x(t) = x_0 + v_{x,0}(t-t_0) + \frac{1}{2} a_x(t-t_0)^2 \quad v_x(t) = v_{x,0} + a_x(t-t_0)$$

$$y(t) = y_0 + v_{y,0}(t-t_0) + \frac{1}{2} a_y(t-t_0)^2 \quad v_y(t) = v_{y,0} + a_y(t-t_0)$$

where  $x_0, v_{x,0}, y_0, v_{y,0}$  are the initial position and velocities components at  $t = t_0$

**Newton's Second Law: Force, Mass, Acceleration**

$$\vec{F} \equiv m\vec{a} \quad \vec{F}^{total} = \vec{F}_1 + \vec{F}_2 \quad F_x^{total} = ma_x \quad F_y^{total} = ma_y \quad F_z^{total} = ma_z$$

**Newton's Third Law:**  $\vec{F}_{1,2} = -\vec{F}_{2,1}$

**Force Laws:**

Universal Law of Gravity:  $\vec{F}_{1,2} = -G \frac{m_1 m_2}{r_{1,2}^2} \hat{r}_{1,2}$  , attractive

Gravity near surface of earth:  $\vec{F}_{grav} = m_{grav} \vec{g}$  , towards earth

Contact force:  $\vec{F}_{contact} = \vec{N} + \vec{f}$  , depends on applied forces

Static Friction:  $0 \leq f_s \leq f_{s,max} = \mu_s N$  direction depends on applied forces

Kinetic Friction:  $f_k = \mu_k N$  opposes motion

Hooke's Law:  $F = k |\Delta x|$  , restoring

**Kinematics Circular Motion:** arc length:  $s = R\theta$  ; angular velocity:  $\omega = d\theta/dt$   
 tangential velocity:  $v = R\omega$  ; angular acceleration:  $\alpha = d\omega/dt = d^2\theta/dt^2$  ; tangential acceleration  $a_\theta = R\alpha$  .

**Period:**  $T = \frac{2\pi R}{v} = \frac{2\pi}{\omega}$ ; **frequency:**  $f = \frac{1}{T} = \frac{\omega}{2\pi}$ ,

**Radial Acceleration:**  $|a_r| = R\omega^2$ ;  $|a_r| = \frac{v^2}{R}$ ;  $|a_r| = 4\pi^2 R f^2$ ;  $|a_r| = \frac{4\pi^2 R}{T^2}$

**Center of Mass:**  $\vec{R}_{cm} = \sum_{i=1}^{i=N} m_i \vec{r}_i / m^{total} \rightarrow \int_{body} dm \vec{r} / m^{total}$ ;

**Velocity of Center of Mass:**  $\vec{V}_{cm} = \sum_{i=1}^{i=N} m_i \vec{v}_i / m^{total} \rightarrow \int_{body} dm \vec{v} / m^{total}$

**Torque:**  $\vec{\tau}_S = \vec{r}_{S,P} \times \vec{F}_P$   $|\vec{\tau}_S| = |\vec{r}_{S,P}| |\vec{F}_P| \sin \theta = r_{\perp} F = r F_{\perp}$

**Static Equilibrium:**  $\vec{F}_{total} = \vec{F}_1 + \vec{F}_2 + \dots = \vec{0}$ ;  $\vec{\tau}_S^{total} = \vec{\tau}_{S,1} + \vec{\tau}_{S,2} + \dots = \vec{0}$ .

**Kinetic Energy:**  $K = \frac{1}{2}mv^2$ ;  $\Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2$

**Work:**  $W = \int_{r_0}^{r_f} \vec{F} \cdot d\vec{r}$ ; **Work- Kinetic Energy:**  $W^{total} = \Delta K$

**Power:**  $P = \vec{F} \cdot \vec{v} = dK/dt$

**Potential Energy:**  $\Delta U = -W_{conservative} = -\int_A^B \vec{F}_c \cdot d\vec{r}$

**Potential Energy Functions with Zero Points:**

Constant Gravity:  $U(y) = mgy$  with  $U(y_0 = 0) = 0$ .

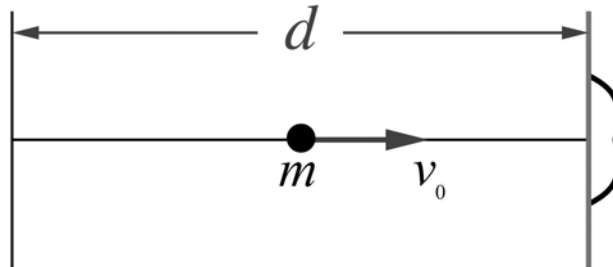
Inverse Square Gravity:  $U_{gravity}(r) = -\frac{Gm_1 m_2}{r}$  with  $U_{gravity}(r_0 = \infty) = 0$ .

Hooke's Law:  $U_{spring}(x) = \frac{1}{2}kx^2$  with  $U_{spring}(x = 0) = 0$ .

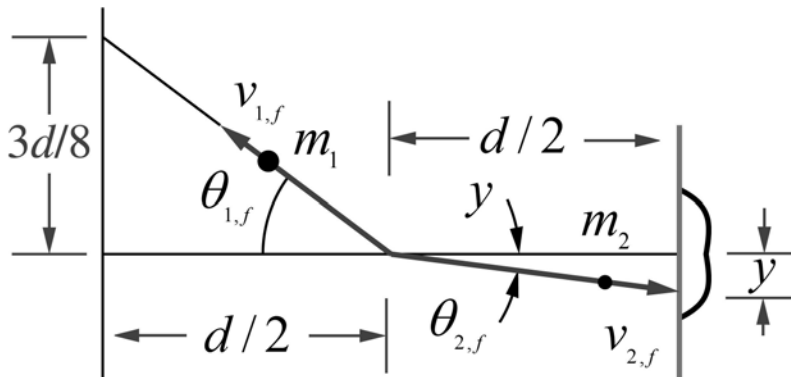
**Work- Mechanical Energy:**  $W_{nc} = \Delta K + \Delta U^{total} = \Delta E_{mech} = (K_f + U_f^{total}) - (K_0 + U_0^{total})$

## Problem 2: Exploding Puck

A hockey player shoots a “trick” hockey puck along the ice towards the center of the goal from a position  $d$  directly in front of the goal. The initial speed of the puck is  $v_0$  and the puck has a mass  $m$ .



Half way to the goal the puck explodes into two fragments. One piece of mass  $m_1 = (3/5)m$  comes back towards the player and passes  $3d/8$  to the side of the spot it was initially shot from with a speed  $v_{1,f} = (5/12)v_0$ . The other piece of the puck with mass  $m_2 = (2/5)m$  continues on towards the goal with a speed  $v_{2,f}$ .

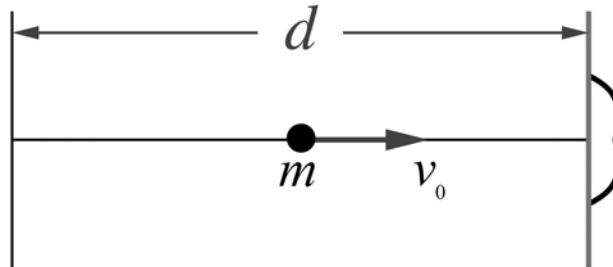


Assume that there is no friction as the puck slides along the ice and that the mass of explosive in the puck is negligible.

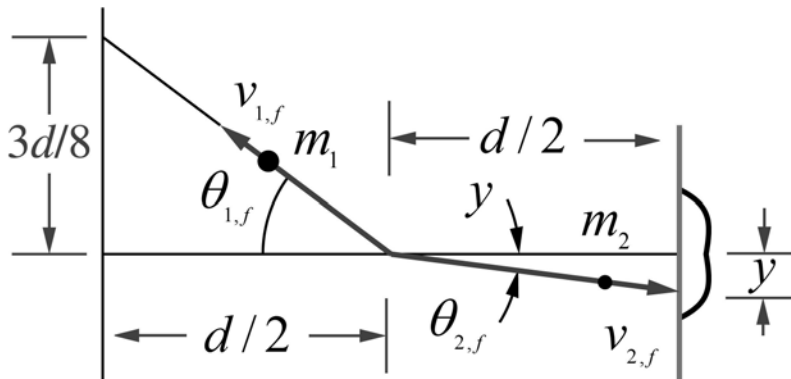
- Write down the equations for conservation of momentum of the puck and fragments in terms of the quantities shown in the figure above.
- By what distance,  $y$ , does the piece that continues towards the goal miss the center of the goal? Express your answer in terms of  $d$ .

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$$\hat{i}: m v_0 = -m_1 v_{1,f} \cos \theta_{1,f} + m_2 v_{2,f} \cos \theta_{2,f}$$

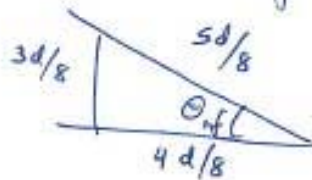
$$\hat{j}: 0 = m_1 v_{1,f} \sin \theta_{1,f} - m_2 v_{2,f} \sin \theta_{2,f}$$

$$m_1 = \frac{3}{5} m, \quad m_2 = \frac{2}{5} m, \quad v_{1,f} = \frac{5}{12} v_0$$

$$\Rightarrow \hat{i}: v_0 = -\left(\frac{3}{5}\right)\left(\frac{5}{12} v_0\right) \cos \theta_{1,f} + \frac{2}{5} v_{2,f} \cos \theta_{2,f}$$

$$\hat{j}: 0 = \frac{3}{5} \frac{5}{12} v_0 \sin \theta_{1,f} - \frac{2}{5} v_{2,f} \sin \theta_{2,f}$$

$$\cos \theta_{1,f} = \frac{4}{5}, \quad \sin \theta_{1,f} = \frac{3}{5}$$



$$\hat{i}: v_0 = \left(-\frac{1}{4} v_0\right) \left(\frac{4}{5}\right) + \frac{2}{5} v_{2,f} \cos \theta_{2,f}$$

$$\hat{j}: 0 = \left(\frac{1}{4}\right) \left(\frac{3}{5}\right) v_0 - \frac{2}{5} v_{2,f} \sin \theta_{2,f}$$

solve for  $\tan \theta_{2,f}$

$$\tan \theta_{2,f} = \frac{\frac{2}{5} v_{2,f} \sin \theta_{2,f}}{\frac{2}{5} v_{2,f} \cos \theta_{2,f}} = \frac{\frac{3}{20} v_0}{\frac{6}{5} v_0} = \frac{1}{8} = \frac{y}{d/2}$$

$$\Rightarrow y = d/16$$